QCD vs. Monte Carlo Event Generators

http://www.terascale.de/mc

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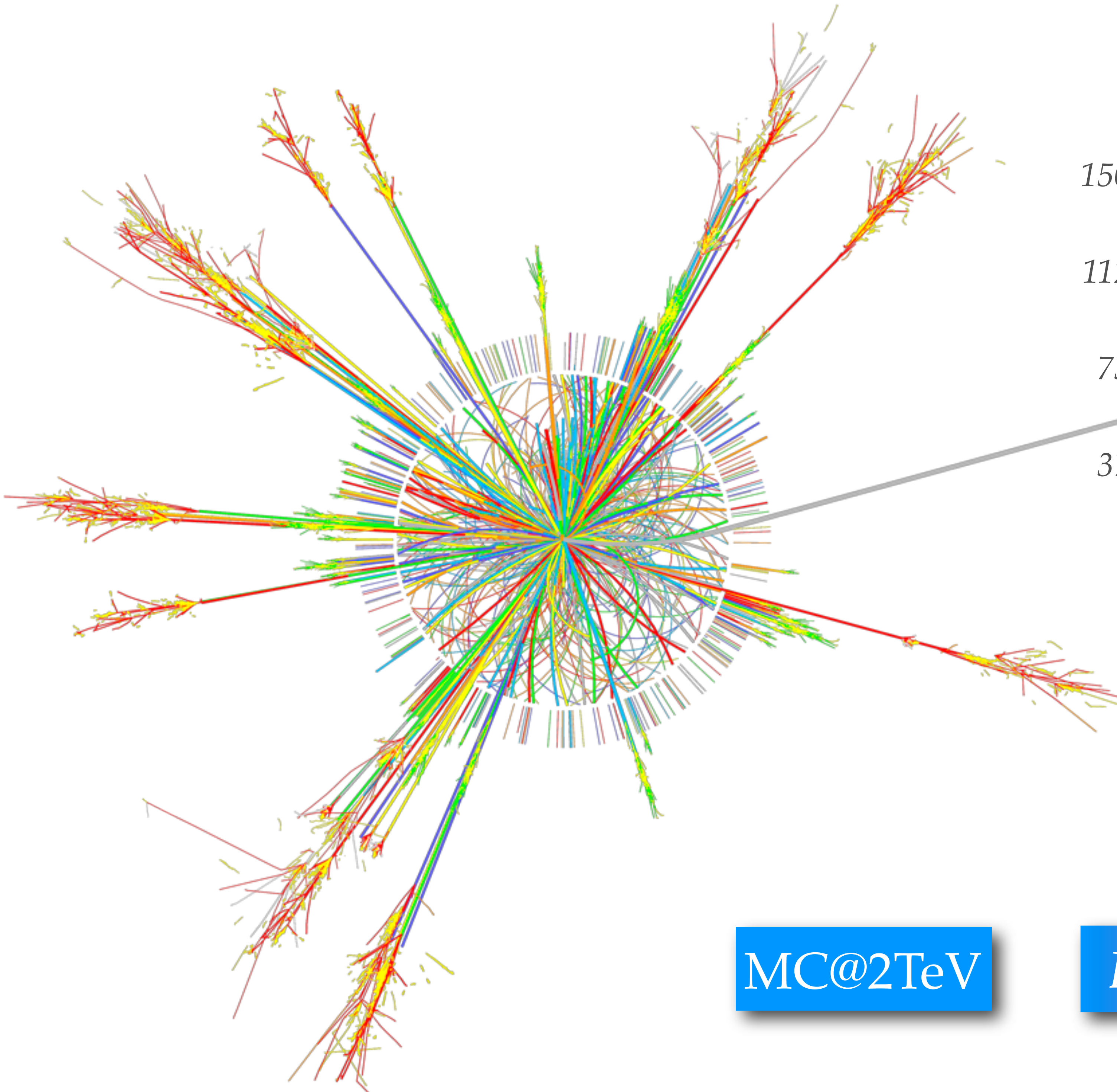
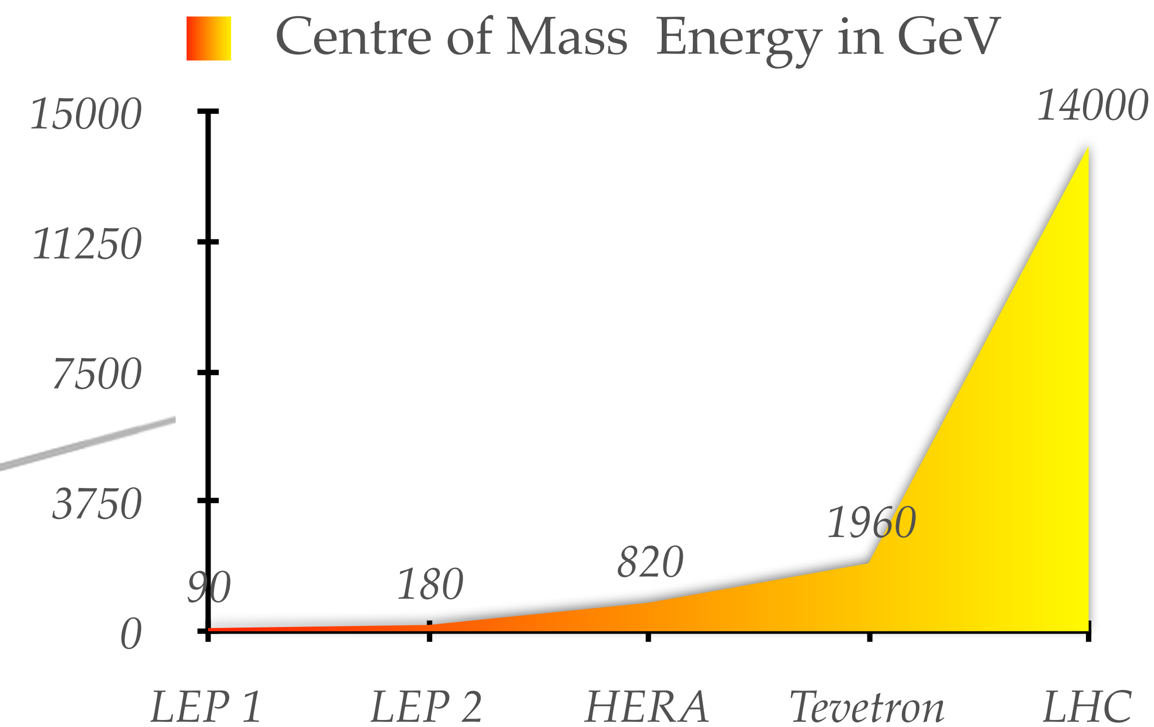
Introduction

The LHC is almost running and we will have to deal with the data soon.



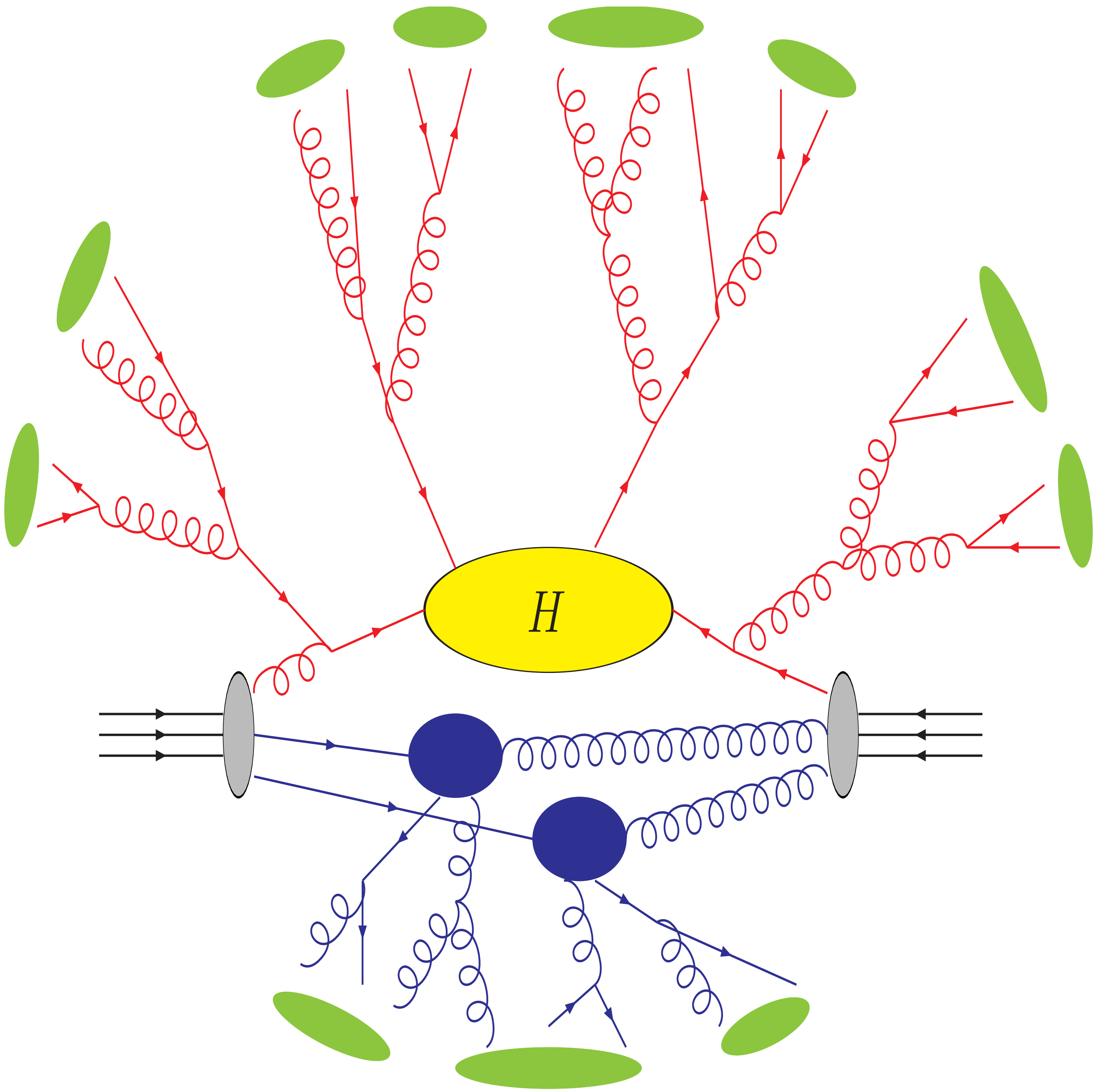


MC@2TeV → *Extrapolation* → MC@14TeV

Picture: ATLAS simulation

Introduction

The structure of the Monte Carlo event generators



1. Incoming hadron *(gray bubbles)*
   - Parton distribution function

2. Hard part of the process *(yellow bubble)*
   - Matrix element calculation, cross sections at LO, NLO, NNLO level

3. Radiations *(red graphs)*
   - Parton shower calculation
   - Matching to the hard part

4. Underlying event *(blue graphs)*
   - Models based on multiple interaction

5. Hardonization *(green bubbles)*
   - Universal models

Introduction

Master equation for LHC discovery:

New Physics = Data (experimental) - Background (theory)

EXPERIMENT
- Collecting raw data
- Detector corrections
- Converting to hadron level
- Converting to parton level

MC event generators

THEORY
- Calculate at least at NLO level *(if it is available)*
- Resum the large logarithms and match it to NLO *(if it is available)*

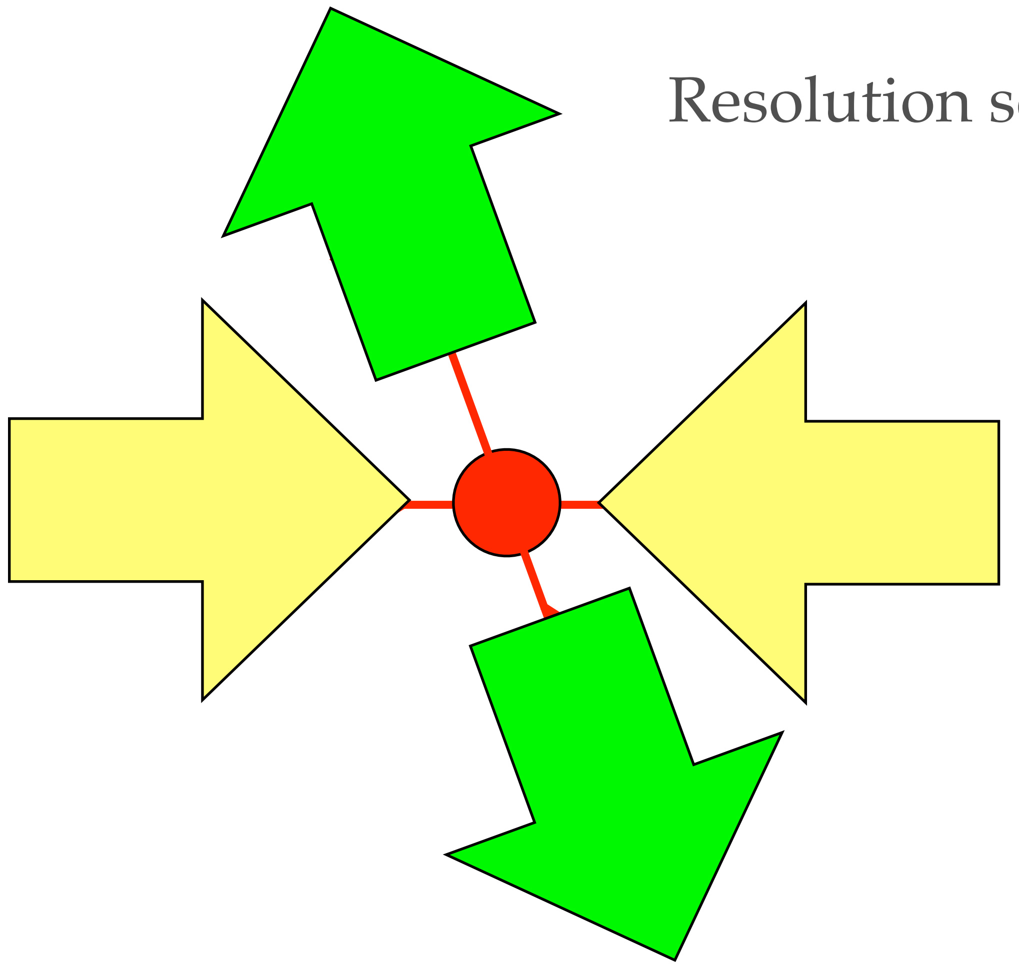
Master equation of the Monte Carlo program:

Data (no new physics) = [Hard part $\otimes$ Shower + MPI $\otimes$ Shower] $\otimes$ Hadronization

Well defined | Needs some work | Only model

Iterative Algorithm

The parton shower evolution starts from the simplest hard configuration, that is usually 2→2 like.



Resolution scale: 400 GeV

Decreasing the resolution scale, more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

This intuitive picture is usually called *Wilsonian renormalisation technique* in theoretical physics.

$$\mathcal{U}(t_\mathrm{f},t_2)|\mathcal{M}_2) = \underbrace{\mathcal{N}(t_\mathrm{f},t_2)|\mathcal{M}_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_\mathrm{f}} dt_3\, \mathcal{U}(t_\mathrm{f},t_3)\mathcal{H}(t_3)\mathcal{N}(t_3,t_2)|\mathcal{M}_2)}^{\text{“Something happens”}}$$

Iterative Algorithm

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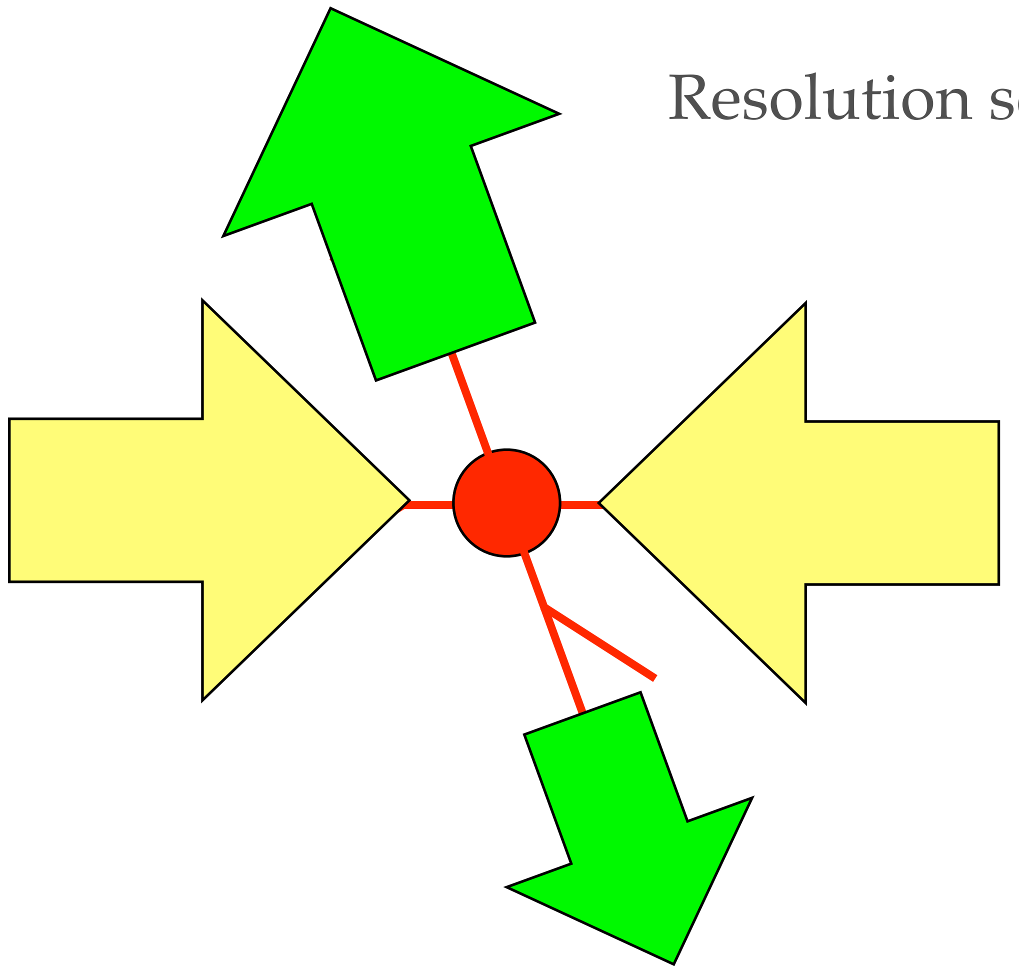

Resolution scale: 350 GeV

Decreasing the resolution scale, more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

This intuitive picture is usually called *Wilsonian renormalisation technique* in theoretical physics.

$$\mathcal{U}(t_{\mathrm{f}}, t_2)|\mathcal{M}_2) = \underbrace{\mathcal{N}(t_{\mathrm{f}}, t_2)|\mathcal{M}_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_{\mathrm{f}}} dt_3\, \mathcal{U}(t_{\mathrm{f}}, t_3)\mathcal{H}(t_3)\mathcal{N}(t_3, t_2)|\mathcal{M}_2)}^{\text{“Something happens”}}$$

Iterative Algorithm

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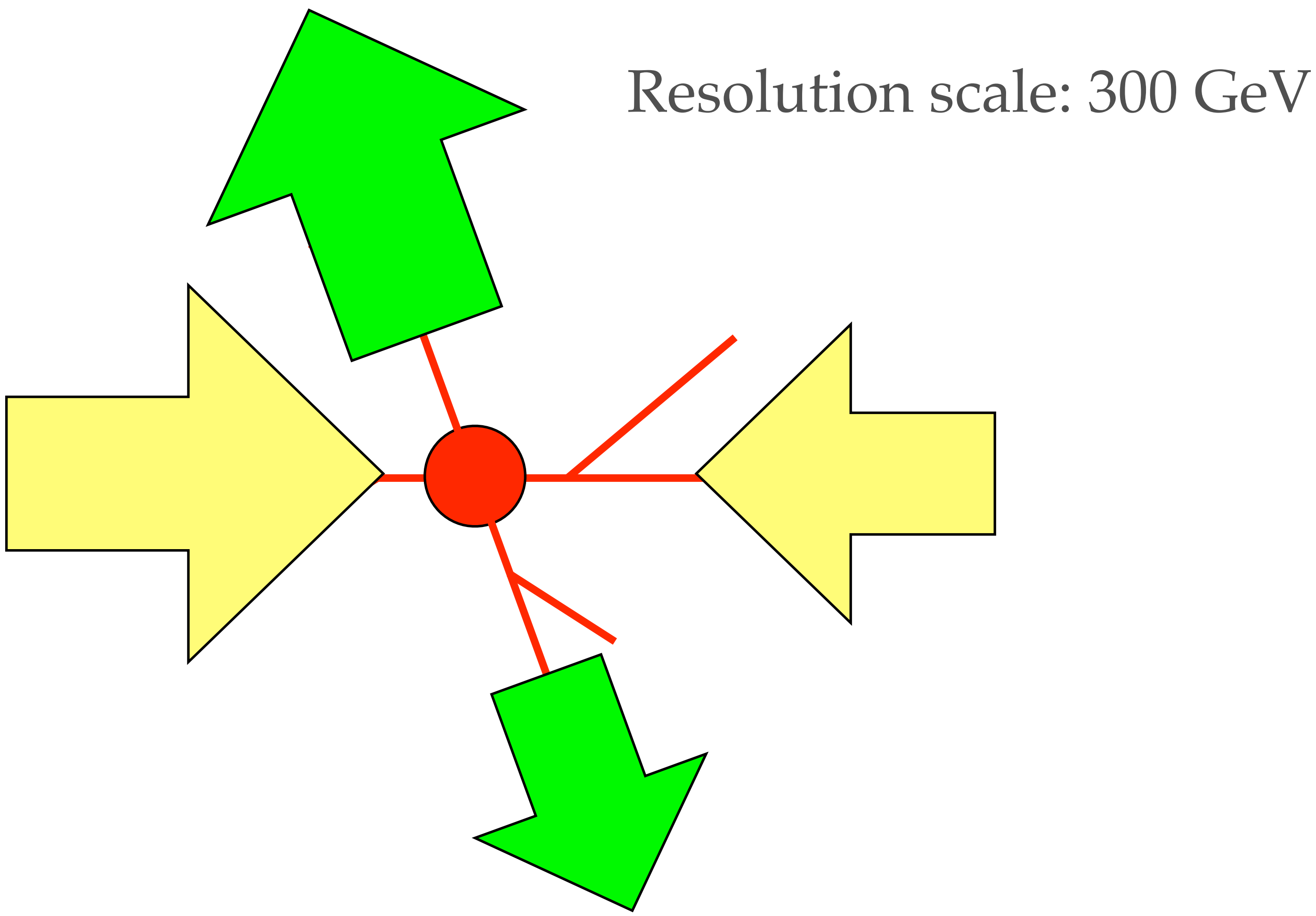

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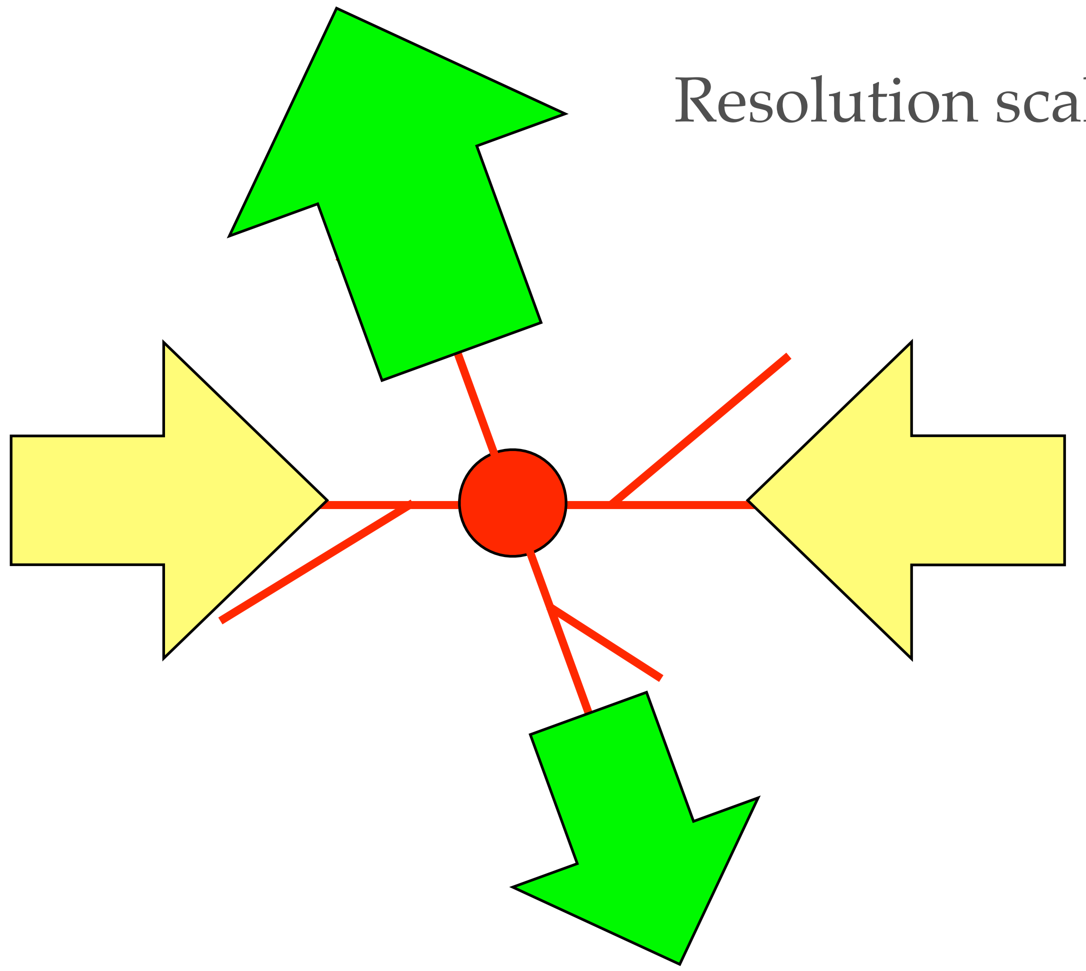
Iterative Algorithm

The parton shower evolution starts from the simplest hard configuration, that is usually $2 \to 2$ like.



Resolution scale: 250 GeV

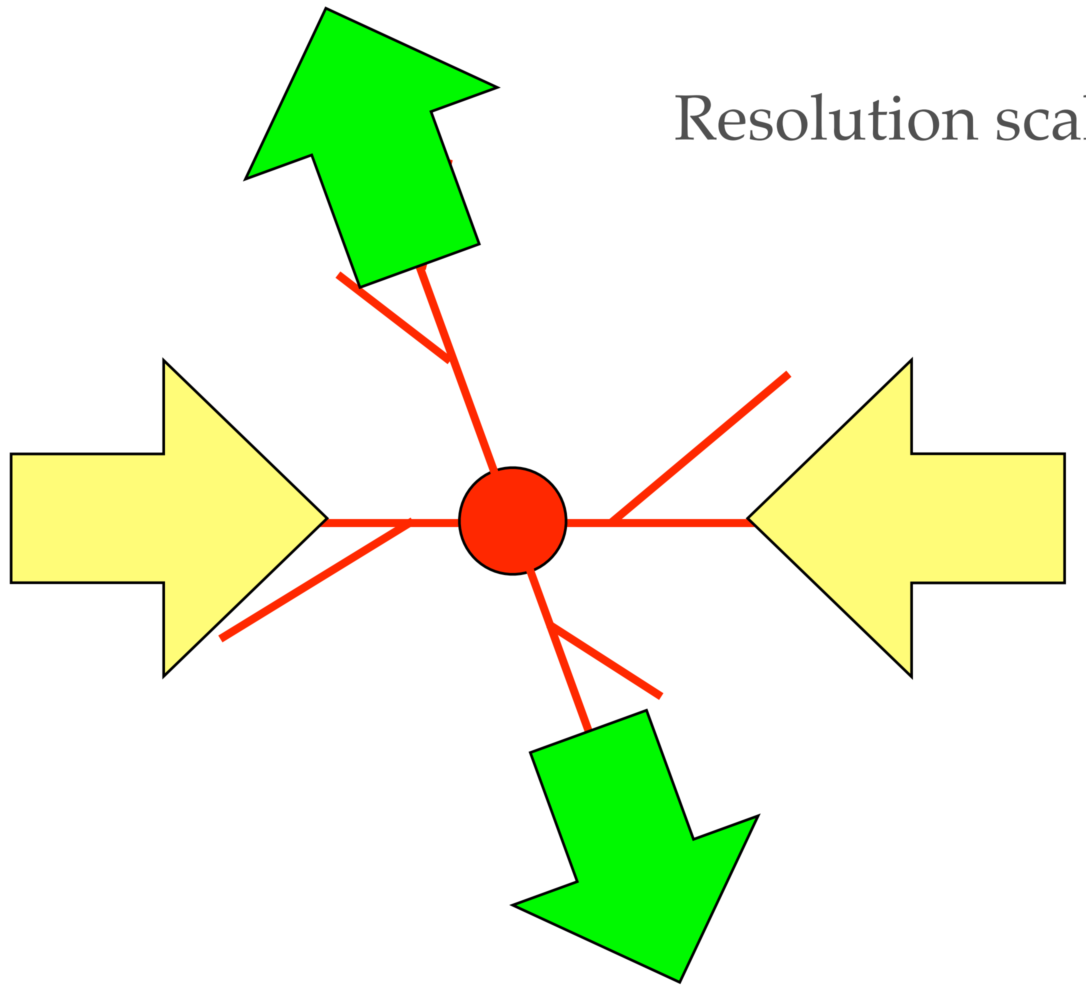
Decreasing the resolution scale, more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

This intuitive picture is usually called *Wilsonian renormalisation technique* in theoretical physics.

$$\mathcal{U}(t_{\mathrm{f}}, t_2)|\mathcal{M}_2) = \underbrace{\mathcal{N}(t_{\mathrm{f}}, t_2)|\mathcal{M}_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_{\mathrm{f}}} dt_3\, \mathcal{U}(t_{\mathrm{f}}, t_3)\mathcal{H}(t_3)\mathcal{N}(t_3, t_2)|\mathcal{M}_2)}^{\text{“Something happens”}}$$

Iterative Algorithm

The parton shower evolution starts from the simplest hard configuration, that is usually 2→2 like.



Resolution scale: 200 GeV

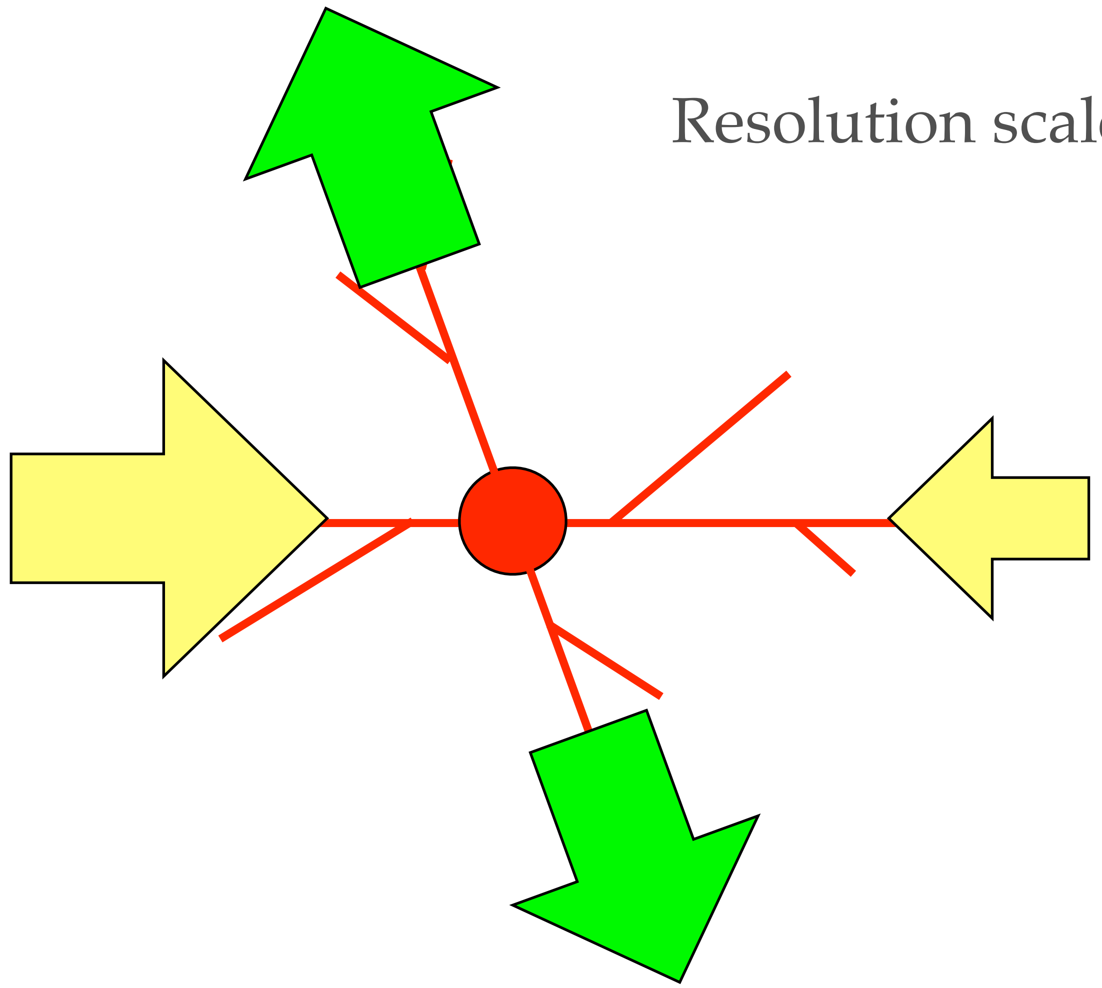
Decreasing the resolution scale, more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

This intuitive picture is usually called *Wilsonian renormalisation technique* in theoretical physics.

$$\mathcal{U}(t_\mathrm{f},t_2)\big|\mathcal{M}_2\big) = \underbrace{\mathcal{N}(t_\mathrm{f},t_2)\big|\mathcal{M}_2\big)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_\mathrm{f}} dt_3\,\mathcal{U}(t_\mathrm{f},t_3)\mathcal{H}(t_3)\mathcal{N}(t_3,t_2)\big|\mathcal{M}_2\big)}^{\text{“Something happens”}}$$

Iterative Algorithm

The parton shower evolution starts from the simplest hard configuration, that is usually 2→2 like.



Resolution scale: 150 GeV

Decreasing the resolution scale, more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

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$$\mathcal{U}(t_{\mathrm{f}}, t_2)|\mathcal{M}_2) = \underbrace{\mathcal{N}(t_{\mathrm{f}}, t_2)|\mathcal{M}_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_{\mathrm{f}}} dt_3\, \mathcal{U}(t_{\mathrm{f}}, t_3)\mathcal{H}(t_3)\mathcal{N}(t_3, t_2)|\mathcal{M}_2)}^{\text{“Something happens”}}$$

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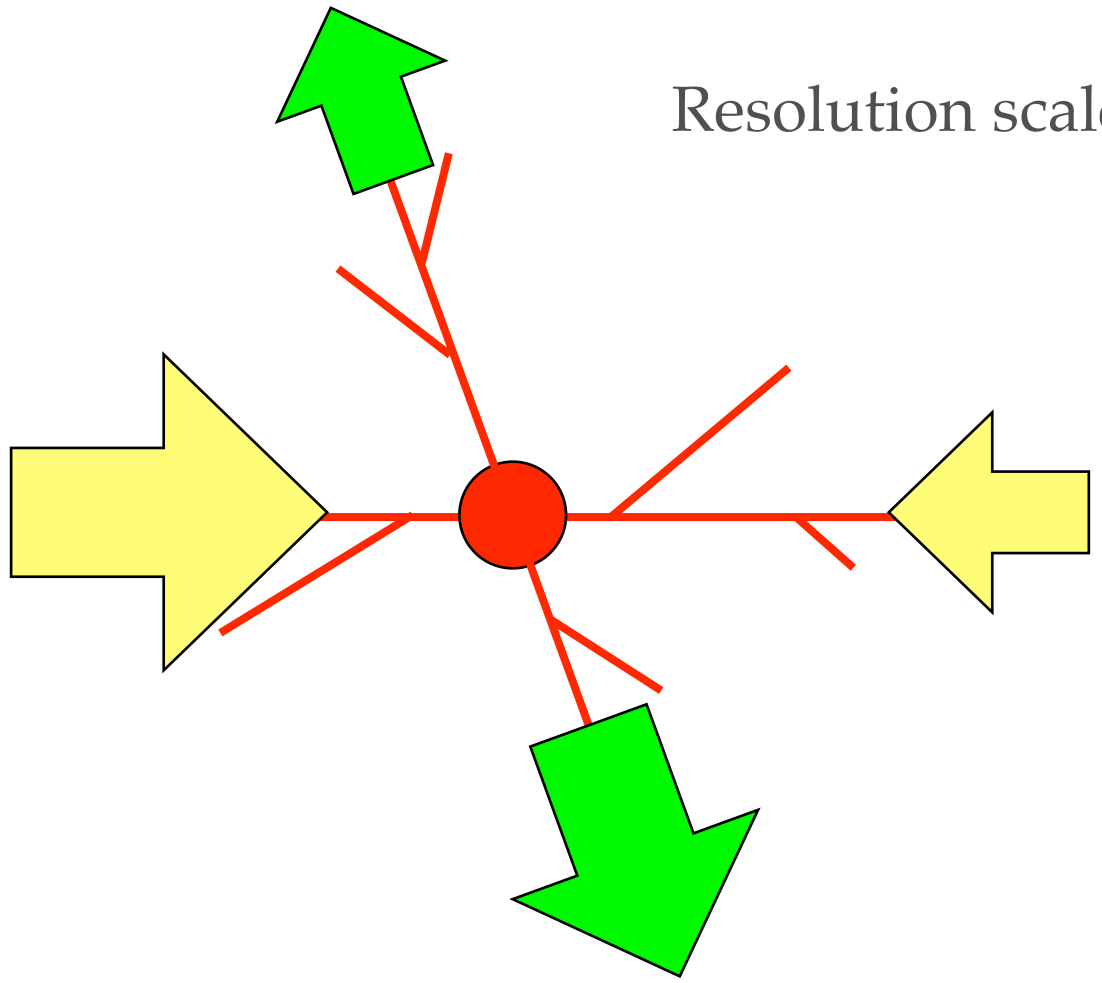

Resolution scale: 100 GeV

Decreasing the resolution scale, more and more partons are visible and less absorbed by the incoming hadrons and the final state jets.

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$$\mathcal{U}(t_{\mathrm{f}}, t_2)|\mathcal{M}_2) = \underbrace{\mathcal{N}(t_{\mathrm{f}}, t_2)|\mathcal{M}_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_{\mathrm{f}}} dt_3\, \mathcal{U}(t_{\mathrm{f}}, t_3)\mathcal{H}(t_3)\mathcal{N}(t_3, t_2)|\mathcal{M}_2)}^{\text{“Something happens”}}$$

Iterative Algorithm

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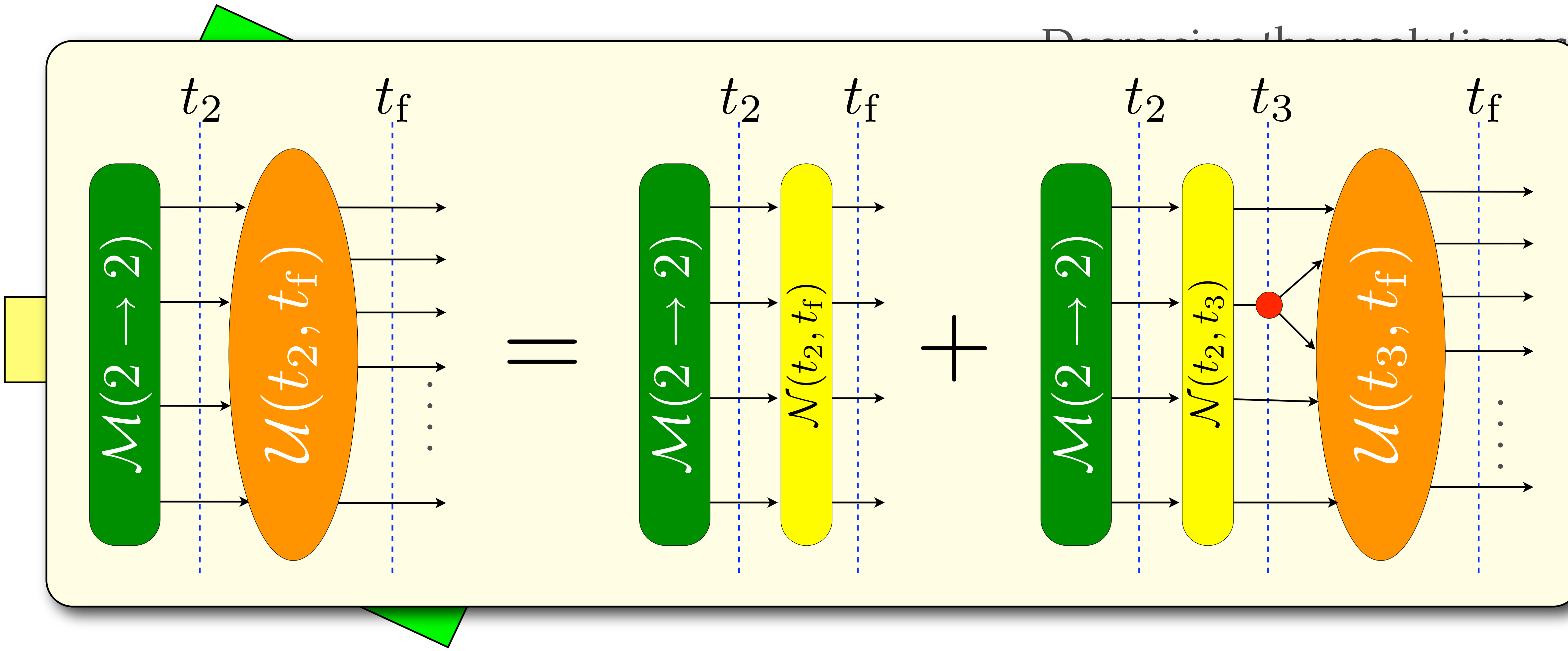

$$\mathcal{U}(t_\mathrm{f}, t_2)|\mathcal{M}_2) = \underbrace{\mathcal{N}(t_\mathrm{f}, t_2)|\mathcal{M}_2)}_{\text{“Nothing happens”}} + \overbrace{\int_{t_2}^{t_\mathrm{f}} dt_3\, \mathcal{U}(t_\mathrm{f}, t_3)\mathcal{H}(t_3)\mathcal{N}(t_3, t_2)|\mathcal{M}_2)}^{\text{“Something happens”}}$$

Statistical space

In QCD a m-patron system is described by the density operator

$$\begin{aligned}\rho(\{p,f\}_m) &= \big|\mathcal{M}(\{p,f\}_m)\big\rangle\big\langle\mathcal{M}(\{p,f\}_m)\big| \\ &= \sum_{s,c,s',c'} \big|\{s',c'\}_m\big\rangle\big(\{p,f,s',c',s,c\}_m\big|\rho\big)\big\langle\{s,c\}_m\big|\end{aligned}$$

In the statistical space it is represented by a vector

$$\big|\rho\big) = \sum_m \frac{1}{m!}\int\big[d\{p,f,s',c',s,c\}_m\big]\,\big|\{p,f,s',c',s,c\}_m\big)\,\big(\{p,f,s',c',s,c\}_m\big|\rho\big)$$

Measurement operators can be also represented by vectors in the statistical space

$$\big|F\big) = \sum_m \frac{1}{m!}\int\big[d\{p,f,s',c',s,c\}_m\big]\,\big|\{p,f,s',c',s,c\}_m\big)\,F(\{p,f\}_m)$$

E.g.: Total cross section

$$\big|1\big) \Leftrightarrow F(\{p,f\}_m) = 1$$

Transverse momentum in Drell-Yan:

$$\big|\boldsymbol{p}_\perp\big) \Leftrightarrow F(\{p,f\}_m) = \delta(\boldsymbol{p}_\perp - \boldsymbol{p}_{\perp,Z})$$

QCD vs. MC

SHOWER CROSS SECTION

$$\sigma^{S}[F] = \big(F\big|\rho\big)$$

- It is an all order but approximated calculation
- Based on soft and collinear factorization of the amplitudes
- Usually more approximation considered (e.g: large Nc,...)
- Implemented in general purpose MC programs (HERWIG, PHYTIA,...)
- *Sums up large logarithms*

QCD CROSS SECTION

$$\sigma^{QCD}[F]$$

- It is an all order but approximated calculation
- Based on soft and collinear factorization of the amplitudes
- Precise in color
- Case-by-case rather elaborate calculation
- *Sums up large logarithms, correctly*

Let us compare them!

QCD vs. MC

SHOWER CROSS SECTION

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QCD CROSS SECTION

$$\sigma^{QCD}[F]$$

- It is an all order but approximated
- and collinear
  of the amplitudes
- or
- rather elaborate
  calculation
- *Sums up large logarithms, correctly*

Herwig has been tested for

- e+e-: thrust, C-parameter, Durham jet rates, jet mass distribution, ...
- DIS, DY: large x

Let us compare them!

QCD vs. Parton Shower

Recent paper by Marchesini and Dokshitzer indicates that the color dipole based showers are not consistent with the parton evolution picture. They studied the quark energy distribution.

This has been checked both analytically and numerically and the shower is consistent with the DGALP equation.

Z.N, D.E. Soper: JHEP 0905:088,2009;

P. Skands, S. Weinzierl: arXiv:0903:2150

From shower equation

$$\frac{d}{dt}\big(x,q\big|\mathcal{U}(t,t')\big|M_2\big) = \big(x,q\big|[\mathcal{H}_I(t)-\mathcal{V}(t)]\,\mathcal{U}(t,t')\big|M_2\big)$$

No approximation and assumptions.
Only algebraic manipulations.

to DGLAP

$$\frac{d}{dt}D_q(t,t',x) = \int_x^1 \frac{dz}{z} P_{qq}(z) D_q(t,t',x/z) + \mathcal{O}(e^{-t})$$

Drell-Yan pT distribution

Building a shower based on the Catani-Seymour splitting functions and mappings can lead to the loss of accuracy.

$$\big(\boldsymbol{p}_\perp\big|\mathcal{U}(t,0)\big|M_2\big) = \big(\boldsymbol{p}_\perp\big|\mathcal{N}(t,0)\big|M_2\big) + \int_0^t d\tau\,\big(\boldsymbol{p}_\perp\big|\mathcal{H}(\tau)\mathcal{N}(\tau,0)\big|M_2\big)$$

This is effectively an approximated NLO calculation with summation of the virtual emissions. No resummation of the large logarithms correctly. We got wrong equation because of the choice of the momentum mapping.

The correct equation is

$$\big(\boldsymbol{p}_\perp\big|\mathcal{U}(t,0)\big|M_2\big) = \big(\boldsymbol{p}_\perp\big|\mathcal{N}(t,0)\big|M_2\big) + \int_0^t d\tau\,\big(\boldsymbol{p}_\perp\big|\mathcal{U}(t,\tau)\mathcal{H}(\tau)\mathcal{N}(\tau,0)\big|M_2\big)$$

We have to study analytically and test against known QCD results.

QCD: Drell-Yan pT distribution

The NLL expression of the pT distribution was obtained using the renormalization group technique and the result is

$$\frac{d\sigma}{d\boldsymbol{p}_\perp dY} \approx \int \frac{d^2\boldsymbol{b}}{(2\pi)^2}\, e^{i\boldsymbol{b}\cdot\boldsymbol{p}_\perp}$$

$$\times \sum_{a,b} \int_{x_a}^{1} \frac{d\eta_a}{\eta_a} \int_{x_b}^{1} \frac{d\eta_b}{\eta_b}\; f_{a/A}(\eta_a, C^2/\boldsymbol{b}^2)\; f_{b/B}(\eta_b, C^2/\boldsymbol{b}^2)$$

$$\times \exp\left(-\int_{C^2/\boldsymbol{b}^2}^{M^2} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2}\left[A(\alpha_s(\boldsymbol{k}_\perp^2)) \log\left(\frac{M^2}{\boldsymbol{k}_\perp^2}\right) + B(\alpha_s(\boldsymbol{k}_\perp^2))\right]\right)$$

$$\times \sum_{a',b'} H^{(0)}_{a'b'}\; C_{a'a}\left(\frac{x_a}{\eta_a}, \alpha_s\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) C_{b'b}\left(\frac{x_b}{\eta_b}, \alpha_s\left(\frac{C^2}{\boldsymbol{b}^2}\right)\right) \quad .$$

$$x_A = \sqrt{\frac{M^2}{s}}\, e^{Y}$$

$$x_B = \sqrt{\frac{M^2}{s}}\, e^{-Y}$$

$$C = 2e^{-\gamma_E}$$

where

$$A(\alpha_s) = 2\, C_F\, \frac{\alpha_s}{2\pi} + 2\, C_F \left\{ C_A \left[\frac{67}{18} - \frac{\pi^2}{6}\right] - \frac{5\, n_f}{9} \right\} \left(\frac{\alpha_s}{2\pi}\right)^2 + \cdots \quad ,$$

$$B(\alpha_s) = -4\, \frac{\alpha_s}{2\pi} + \left[-\frac{197}{3} + \frac{34 n_f}{9} + \frac{20\pi^2}{3} - \frac{8 n_f \pi^2}{27} + \frac{8\zeta(3)}{3}\right] \left(\frac{\alpha_s}{2\pi}\right)^2 + \cdots \quad ,$$

$$C_{a'a}(z, \alpha_s) = \delta_{a'a}\delta(1-z) + \frac{\alpha_s}{2\pi}\left[\delta_{a'a}\left\{\frac{4}{3}(1-z) + \frac{2}{3}\,\delta(1-z)\,(\pi^2 - 8)\right\} + \delta_{ag}\, z(1-z)\right]$$

MC: Drell-Yan process

The result and the derivation strongly depends on the shower algorithm, so it is useful to stick at one. My choice an shower algorithm with quantum interference.

Z.N, D.E. Soper: JHEP 0709:114,2007; JHEP 0803:030,2008; JHEP 0807:025,2008

Now, the shower equation is

$$\frac{d}{dt}\big(\hat{\boldsymbol{p}}, Y\big|\rho(t)\big) = \big(\hat{\boldsymbol{p}}, Y\big|\mathcal{H}_{\mathrm{I}}(t) - \mathcal{V}(t)\big|\rho(t)\big)$$

- Fully exclusive and systematical formulation of the parton shower
- Quantum interferences are considered properly
  - Color evolution
  - Spin correlations
- Full control over the kinematics
  - Mapping based on exact phase space factorization
  - Ordering in virtuality (this is the most natural ordering variable)

MC: Drell-Yan process

Now, the shower equation is

$$\frac{d}{dt}\big(\hat{\boldsymbol{p}}, Y\big|\rho(t)\big) = \big(\hat{\boldsymbol{p}}, Y\big|\mathcal{H}_{\mathrm{I}}(t) - \mathcal{V}(t)\big|\rho(t)\big)$$

After some harmless approximations, algebraic manipulations and about 2 months of hard work the result is

$$\frac{d\sigma}{d\boldsymbol{p}_\perp\, dY} = \int \frac{d\boldsymbol{b}}{(2\pi)^2}\, e^{\mathrm{i}\boldsymbol{p}_\perp\cdot\boldsymbol{b}} \exp\left\{-C_{\mathrm{F}}\int_{C^2/\boldsymbol{b}^2}^{M^2}\frac{d\boldsymbol{k}^2}{\boldsymbol{k}^2}\,\frac{\alpha_{\mathrm{s}}(\lambda\boldsymbol{k}^2)}{\pi}\left[\log\frac{M^2}{\boldsymbol{k}^2}-\frac{3}{2}\right]\right\}$$
$$\times\sum_{a,b} H^{(0)}_{a,b}\, f_{a/A}\left(x_{\mathrm{A}},\frac{C^2}{\boldsymbol{b}^2}\right)\, f_{b/B}\left(x_{\mathrm{A}},\frac{C^2}{\boldsymbol{b}^2}\right)$$

With the support of the *standard DGLAP equation* for the PDFs:

$$\mu_F^2\frac{d}{d\mu_F^2} f_{a/\mathrm{A}}\big(x,\mu_F^2\big) = \sum_{\hat{a}}\int_0^1\frac{dz}{z}\,\frac{\alpha_{\mathrm{s}}(\mu_{\mathrm{F}}^2)}{2\pi}\,P_{\hat{a},a}(z)\, f_{\hat{a}/\mathrm{A}}\big(x/z,\mu_F^2\big)$$

MC: Drell-Yan process

The result is strongly depends on the choice of the *argument of the α_s in the shower*:

$$\frac{\alpha_{\mathrm{s}}(\lambda \boldsymbol{k}_\perp^2)}{2\pi} = \frac{\alpha_{\mathrm{s}}(\boldsymbol{k}_\perp^2)}{2\pi} - 2\beta_1 \log(\lambda) \left(\frac{\alpha_{\mathrm{s}}(\boldsymbol{k}_\perp^2)}{2\pi} \right)^2 + \mathcal{O}(\alpha_{\mathrm{s}}^3)$$

Using scaled transverse momentum for the argument of strong coupling with

$$\lambda = \exp\left(- \frac{C_{\mathrm{A}} \left[67 - 3\pi^2 \right] - 15 n_{\mathrm{f}}}{3\,(33 - 2\,n_{\mathrm{f}})} \right)$$

we can reproduce the QCD cross section at NLL level

$$A^{MC}(\alpha_{\mathrm{s}}) = 2\,C_{\mathrm{F}}\,\frac{\alpha_{\mathrm{s}}}{2\pi} + 2\,C_{\mathrm{F}} \left\{ C_{\mathrm{A}} \left[\frac{67}{18} - \frac{\pi^2}{6} \right] - \frac{5\,n_{\mathrm{f}}}{9} \right\} \left(\frac{\alpha_{\mathrm{s}}}{2\pi} \right)^2 + \cdots \quad ,$$

$$B^{MC}(\alpha_{\mathrm{s}}) = -4\,\frac{\alpha_{\mathrm{s}}}{2\pi} + \cdots \quad ,$$

$$C_{a'a}^{MC}(z, \alpha_{\mathrm{s}}) = \delta_{a'a}\delta(1 - z) + \cdots$$

Modified LO PDF

Do we need modified PDF? No, we don't.

In the derivation we used only that the PDF obeys the following equation (at LO level):

$$\frac{d}{dt} f_{a/A}(\eta_{\mathrm{a}}, M^2 e^{-t}) = -\int_0^1 dz \sum_{f'} \frac{\alpha_{\mathrm{s}}(\lambda \boldsymbol{k}_\perp^2)}{2\pi} \left\{ \frac{1}{z} P_{a,a+f'}(z)\, f_{(a+f')/A}(\eta_a/z, M^2 e^{-t}) - \delta_{f',\mathrm{g}} \left[\frac{2C_a}{1-z} - \gamma_a \right] f_{a/A}(\eta_{\mathrm{a}}, M^2 e^{-t}) \right\} + \mathcal{O}(\alpha_s^2)$$

Every PDF (LO, NLO, NNLO) satisfies this equation at this level of precision.

- *Don't use LO* and LO** PDFs. They are not physical.*
- To improve parton shower performance there are better ways to do.
  - Color evolution, spin correlations,...., quantum interference
  - Higher order corrections
  - ....

Other choices

✗ The shower based on Catani-Seymour factorization fails to reproduce the analytic answer. *The shower result doesn't exponentiate in b-space.* Bad choice of the momentum mapping.

✗ What happens when we don't change anything (splitting function, mapping, correct interference terms, ...), but we use the *transverse momentum as evolution variable* (like in PHYTIA,...).

➡ The result is *correct at LL level* but very like that *it fails at NLL level*. Lack of angular ordering.

✓ What happens when we don't change anything (splitting function, mapping, correct interference terms, ...), but we use the *emission angle as evolution variable* (like in HERWIG,...).

➡ This gives the *right answer at NLL level*

Here we studied algorithms those are more advanced than HERWIG or PHYTHIA, we changed only the evolution parameter to mimic the main features of the standard MC programs, but they differ basically in everything.

Summary

- It is important to test parton shower against resummed QCD calculation.
- This can help us to treat it more systematically.

✓ Our parton shower can sum up the pT logs at NLL level

✗ Unfortunately this algorithm hasn't been implemented,yet.

✗ *Don't use modified LO PDFs (LO* & LO**) and don't produce such creatures!*

- Need more work on testing parton showers systematically against known QCD results.
- Need more work on color evolution, spin correlations, non-global effect, higher order corrections, ..., more theory work required.
- In principle shower *has a chance to* sum up all the *LL and the LO NLL* contributions.

"has a chance to" ≠ "does"

- Shower is only an "exponentiated LO" (*one can call it to eLO*) calculation.