On qualitative aspects of the choice of factorization schemes

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Outline

Introduction

As a simple illustration, consider a non-singlet nucleon structure function $F_{\mathrm{NS}}(x, Q^2)$, whose Mellin moments are given as the product

$$F_{\mathrm{NS}}(n, Q^2) = C_{\mathrm{NS}}\left(n, \frac{Q}{M}, \mathrm{FS}, \mathrm{RS}\right) q_{\mathrm{NS}}(n, M, \mathrm{FS}, \mathrm{RS}) \,.$$

Both the coefficient function $C_{\mathrm{NS}}(n, Q/M, \mathrm{FS}, \mathrm{RS})$ and the non–singlet parton distribution function $q_{\mathrm{NS}}(n, M, \mathrm{FS}, \mathrm{RS})$ depend on

- a factorization scale M,
- a factorization scheme FS,
- a renormalization scheme RS,[1]

but the structure function $F_{\mathrm{NS}}(n, Q^2)$ is independent of them.

[1] The factorization procedure has to be preceded by the renormalization procedure.

The coefficient function $C_{\rm NS}(n, Q/M, {\rm FS}, {\rm RS})$ can be expanded in powers of the QCD coupling parameter $a \equiv \alpha_s/\pi$

$$C_{\rm NS}\left(n, \frac{Q}{M}, {\rm FS}, {\rm RS}\right) = \sum_{k=0}^{\infty} a^k(\mu, {\rm RS})\, C_{\rm NS}^{(k)}\left(n, \frac{Q}{M}, {\rm FS}, \mu, {\rm RS}\right).$$

- $C_{\rm NS}(n, Q/M, {\rm FS}, {\rm RS})$ independent of μ if summed to all orders
- The renormalization scale μ is in principle different from the factorization scale M.

The non-singlet parton distribution function $q_{\rm NS}(n, M, {\rm FS}, {\rm RS})$ satisfies the evolution equation

$$\frac{{\rm d}q_{\rm NS}(n, M, {\rm FS}, {\rm RS})}{{\rm d}\ln M} = a(M, {\rm RS})\, P_{\rm NS}(n, M, {\rm FS}, {\rm RS})\, q_{\rm NS}(n, M, {\rm FS}, {\rm RS}),$$

where

$$P_{\rm NS}(n, M, {\rm FS}, {\rm RS}) = \sum_{k=0}^{\infty} a^k(M, {\rm RS})\, P_{\rm NS}^{(k)}(n, {\rm FS}, {\rm RS}).$$

At NLO

$$C_{\rm NS}\left(n,\frac{Q}{M},{\rm FS}\right) \;=\; C^{(0)}_{\rm NS}(n)+a(\mu)\,C^{(1)}_{\rm NS}\left(n,\frac{Q}{M},{\rm FS}\right),$$

$$P_{\rm NS}(n,M,{\rm FS}) \;=\; P^{(0)}_{\rm NS}(n)+a(M)\,P^{(1)}_{\rm NS}(n,{\rm FS})\,.$$

The NLO couplant $a(\mu)$ obeys

$$\frac{{\rm d}a(\mu)}{{\rm d}\ln\mu}=-ba^2(\mu)\Big(1+ca(\mu)\Big).$$

The LO terms $C^{(0)}_{\rm NS}(n)$ and $P^{(0)}_{\rm NS}(n)$ are universal — independent of any unphysical quantities (renormalization and factorization scales and schemes).

Both the NLO coefficent function $C^{(1)}_{\rm NS}(n, Q/M, {\rm FS})$ and the NLO splitting function $P^{(1)}_{\rm NS}(n, {\rm FS})$ are free, but they are coupled by the equation

$$C^{(1)}_{\rm NS}\left(n, \frac{Q}{M}, {\rm FS}\right) = C^{(0)}_{\rm NS}(n)\left[\kappa(n) + P^{(0)}_{\rm NS}(n)\ln\frac{Q}{M} + \frac{1}{b}P^{(1)}_{\rm NS}(n, {\rm FS})\right],$$

where $\kappa(n)$ is a scale and scheme factorization invariant.

The splitting function $P^{(1)}_{\rm NS}(n, {\rm FS})$ can be used for labeling factorization schemes. The freedom in the choice of a factorization scheme is thus enormous.

The investigation of the dependence of NLO theoretical predictions on a factorization scheme is much more complicated than that in the case of the factorization scale.

Motivations for studying factorization schemes

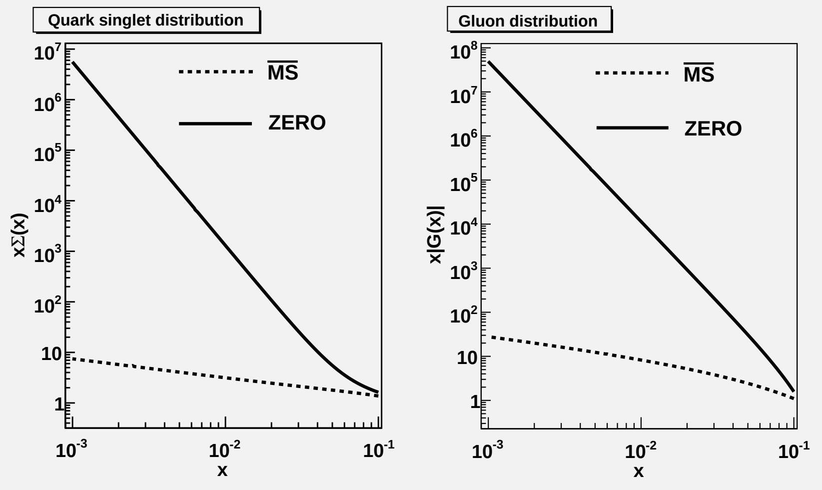
- looking for a factorization scheme in which finite order predictions are stable against small variations of parameters used for specifying factorization schemes
- an improvement in the estimation of theoretical uncertainties
- phenomenologically motivated choices of factorization schemes
  - the DIS factorization scheme,[2] in which the structure function $F_2(x, Q^2)$ has to all orders the same form as in the parton model
  - the ZERO factorization scheme defined by setting the NLO splitting functions $P^{(1)}(x)$ to zero

[2] introduced in G. Altarelli, R.K. Ellis and G. Martinelli, Nucl. Phys. **B143**, 521 (1978)

The ZERO factorization scheme

- suitable for generating NLO initial state parton showers because of their formal equivalence to the LO ones
- optimal for using in NLO Monte Carlo event generators since the current algorithms for parton showering and for attaching initial state parton showers to NLO QCD cross–sections need not be changed
- close to the factorization scheme determined by the Principle of Minimal Sensitivity
- however, the ZERO singlet parton distributions have an unexpected and surprising behaviour for low x

Comparison of the ZERO and $\overline{\text{MS}}$ parton distributions at $M = 50\,\text{GeV}$ when only three light flavours are taken into account ($n_\text{f} = 3$).



NLO partonic cross–sections in the ZERO factorization scheme diverge in a similar way as the parton distributions.

In practice, the ZERO factorization scheme is in general inapplicable:

- The divergent terms cause problems in numerical computations.
- It is likely that the mutual cancellation of the divergent terms in expressions for physical quantities is incomplete at NLO $\longrightarrow$ unreasonable predictions.

But there are no problems with applicability of the ZERO factorization scheme in the non–singlet case.

Specification of factorization schemes

Within the framework of dimensional regularization, the relation between the dressed and bare distribution functions is given by the formula:

$$D_i(x, M, \mathrm{FS}) = \sum_j \int_x^1 \frac{\mathrm{d}y}{y} D_j^{(0)}\left(\frac{x}{y}\right) \left[\delta_{ij}\delta(1-y) + a(M)\left(\frac{1}{\epsilon}A_{ij}^{(11)}(y) + A_{ij}^{(10)}(y)\right)\right.$$
$$\left. + a^2(M)\left(\frac{1}{\epsilon^2}A_{ij}^{(22)}(y) + \frac{1}{\epsilon}A_{ij}^{(21)}(y) + A_{ij}^{(20)}(y)\right) + \cdots\right].$$

- $A^{(k0)}(x)$ fully specifies the factorization scheme and can be chosen arbitrarily.
- The factorization scheme can also be specified by higher orders of the corresponding splitting functions $P^{(k)}(x)$, which we can choose at will.
- A sufficient condition for practical applicability of a factorization scheme: Mellin moments $A^{(k0)}(n)$ are holomorphic for $\mathrm{Re}\, n > 1$.

The relation between $A^{(k0)}(n)$ and $P^{(k)}(n)$:

$$P^{(k)}(n) = \left[A^{(k0)}(n), P^{(0)}(n)\right] - kbA^{(k0)}(n) +$$
$$+ \text{a polynomial expression in } \left\{ \left\{A^{(l1)}(n, \text{MS})\right\}_{l=1}^{k+1}, \left\{A^{(l0)}(n)\right\}_{l=1}^{k-1} \right\}.$$

- $A^{(k0)}(n) \to P^{(k)}(n)$: only addition and multiplication; the condition $A^{(k0)}(n)$ are holomorphic for $\operatorname{Re} n > 1$ is thus sufficient for $P^{(k)}(n)$ to be holomorphic for $\operatorname{Re} n > 1$
- $P^{(k)}(n) \to A^{(k0)}(n)$: addition, multiplication, but also division due to the presence of the commutator; zeros of the denominators then cause that the condition $P^{(k)}(n)$ are holomorphic for $\operatorname{Re} n > 1$ is not sufficient for $A^{(k0)}(n)$ to be holomorphic for $\operatorname{Re} n > 1$, but is only necessary for that

$A^{(10)}(n)$ is holomorphic for $\mathrm{Re}\, n > 1$ if and only if $P^{(1)}(n)$ is holomorphic for $\mathrm{Re}\, n > 1$ and

$$
\begin{aligned}
& P^{(0)}_{Gq}(n)\left(P^{(0)}_{qq}(n) - P^{(0)}_{GG}(n) - b\right)\left(P^{(1)}_{qG}(n) - P^{(1)}_{qG}(n,\overline{\mathrm{MS}})\right) + \\
+ & P^{(0)}_{qG}(n)\left(P^{(0)}_{qq}(n) - P^{(0)}_{GG}(n) + b\right)\left(P^{(1)}_{Gq}(n) - P^{(1)}_{Gq}(n,\overline{\mathrm{MS}})\right) - \\
- & 2P^{(0)}_{qG}(n)P^{(0)}_{Gq}(n)\left(P^{(1)V}_{qq}(n) + P^{(1)V}_{q\bar{q}}(n) + 2n_{\mathrm{f}}P^{(1)S}_{qq}(n) - P^{(1)}_{GG}(n) - \right. \\
- & \left. P^{(1)V}_{qq}(n,\overline{\mathrm{MS}}) - P^{(1)V}_{q\bar{q}}(n,\overline{\mathrm{MS}}) - 2n_{\mathrm{f}}P^{(1)S}_{qq}(n,\overline{\mathrm{MS}}) + P^{(1)}_{GG}(n,\overline{\mathrm{MS}})\right) = 0
\end{aligned}
$$

for $n \in \mathcal{N}$ where

$$
\begin{aligned}
\mathcal{N}_{n_{\mathrm{f}}=3} &= \{1.7329, 4.6306\}, \\
\mathcal{N}_{n_{\mathrm{f}}=4} &= \{1.7995, 3.8458\}, \\
\mathcal{N}_{n_{\mathrm{f}}=5} &= \{1.9001, 3.1798\}.
\end{aligned}
$$

An arbitrary five NLO splitting functions can be chosen without any restriction. The remaining function then has to be selected with a constraint on two of its Mellin moments.

Summary and conclusion

- Not all NLO splitting functions $P^{(1)}(x)$ that appear at first sight as reasonable specify practically applicable factorization schemes.
- The practical applicability of a factorization scheme is assured if the corresponding NLO splitting functions satisfy some nontrivial condition, which can be easily formulated in the space of Mellin moments.
- This condition is unfortunately not satisfied in the ZERO factorization scheme, which would otherwise be optimal for NLO Monte Carlo event generators.
- Searching for a suitable factorization scheme which is close to the ZERO factorization scheme and satisfies the condition of applicability has already been started.