New physics sensitivity of the decay \( B \rightarrow K^*\ell^+\ell^- \)

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in collaboration

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Key issue: separation of new physics and hadronic effects

Factorization formulae based on soft-collinear effective theory (SCET):

for $B \to K^*$ formfactors

$$F_i = H_i \xi^P(E) + \phi_B \otimes T_i \otimes \phi^P_{K^*} + O(\Lambda/m_b)$$

for the decay amplitudes

$$T^{(i)}_a = C^{(i)}_a \xi_a + \phi_B \otimes T^{(i)}_a \otimes \phi_{a,K^*} + O(\Lambda/m_b)$$

- Separation of perturbative hard kernels from process-independent nonperturbative functions like form factors
- Relations between formfactors in large-energy limit
- Limitation: insufficient information on power-suppressed $\Lambda/m_b$ terms (breakdown of factorization: 'endpoint divergences')

Phenomenologically highly relevant issue

general strategy of LHCb to look at ratios of exclusive modes

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LHCb Strategy: Focus on ratios of exclusive modes

Well-known example: Forward-Backward-Charge-Asymmetry in $B \to K^* \ell^+ \ell^-$

- In contrast to the branching ratio the zero of the FBA is almost insensitive to hadronic uncertainties. At LO the zero depends on the short-distance Wilson coefficients only:

\[ q_0^2 = q_0^2(C_7, C_9), \quad q_0^2 = (3.4 + 0.6 - 0.5) \text{GeV}^2 \quad \text{(LO)} \]

- NLO contribution calculated within QCD factorization approach leads to a large 30%-shift: (Beneke, Feldmann, Seidel 2001)

\[ q_0^2 = (4.39 + 0.38 - 0.35) \text{GeV}^2 \quad \text{(NLO)} \]

- However: Issue of unknown power corrections ($\Lambda/m_b$)!
More opportunities in $B \to K^*(K\pi)\ell^+\ell^-$: angular distributions

- Assuming the $K^*$ to be on the mass shell, the decay $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\ell^+\ell^-$ described by the lepton-pair invariant mass, $s$, and the three angles $\theta_l, \theta_{K^*}, \phi$.

After summing over the spins of the final particles:

$$\frac{d^4\Gamma_{\bar{B}d}}{dq^2 \, d\theta_l \, d\theta_K \, d\phi} = \frac{9}{32\pi} I(q^2, \theta_l, \theta_K, \phi) \sin \theta_l \sin \theta_K$$

LHCb statistics ($\geq 2 fb^{-1}$) allows for a full angular fit!

$$I = I_1 + I_2 \cos 2\theta_l + I_3 \sin^2 \theta_l \cos 2\phi + I_4 \sin 2\theta_l \cos \phi + I_5 \sin \theta_l \cos \phi + I_6 \cos \theta_l$$
$$+ I_7 \sin \theta_l \sin \phi + I_8 \sin 2\theta_l \sin \phi + I_9 \sin^2 \theta_l \sin 2\phi.$$

- Angular distribution functions: 
  depend on the 6 complex $K^*$ spin amplitudes
  $$I_i = I_i(\ A_{\perp L/R}, A_{|| L/R}, A_{0 L/R})$$
  \[ \text{(limit } m_{\text{lepton}} = 0) \]
  12 theoretical independent amplitudes $A_j \leftrightarrow 9$ independent coefficient functions in $I$
  Only 9 amplitudes $A_j$ are independent in respect to the angular distribution

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**Theoretical framework**

- Effective Hamiltonian describing the quark transition $b \rightarrow s \ell^+\ell^-$:

\[
\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} \left[ C_i(\mu) \mathcal{O}_i(\mu) + C_i'(\mu) \mathcal{O}_i'(\mu) \right]
\]

We focus on magnetic and semi-leptonic operators and their chiral partners.

- Hadronic matrix element parametrized in terms of $B \rightarrow K^*$ form factors:

- Crucial input: In the $m_B \rightarrow \infty$ and $E_{K^*} \rightarrow \infty$ limit

7 form factors ($A_i(s)/T_i(s)/V(s)$) reduce to 2 universal form factors ($\xi_\perp, \xi_\parallel$) (Charles, Le Yaouanc, Oliver, Pène, Raynal 1999)

Form factor relations broken by $\alpha_s$ and $\Lambda/m_b$ corrections

- Large Energy Effective Theory $\Rightarrow$ QCD factorization/SCET
  (IR structure of QCD)

- Above results are valid in the kinematic region in which

\[
E_{K^*} \simeq \frac{m_B}{2} \left( 1 - \frac{s}{m_B^2} + \frac{m_{K^*}^2}{m_B^2} \right)
\]

is large.

We restrict our analysis to the dilepton mass region $s \in [1\text{GeV}^2, 6\text{GeV}^2]$
$K^*$ spin amplitudes in the heavy quark and large energy limit

\[ A_{\perp,||} = \frac{(H_{+1} \mp H_{-1})}{\sqrt{2}}, \quad A_0 = H_0 \]

\[
A_{\perp L,R} = N \sqrt{2} \lambda^{1/2} \left[ (C_9^{\text{eff}} \mp C_{10}) \frac{V(s)}{m_B + m_{K^*}} + \frac{2m_b}{s} (C_7^{\text{eff}} + C_7^{\text{eff}'}) T_1(s) \right]
\]

\[
A_{|| L,R} = -N \sqrt{2} (m_B^2 - m_{K^*}^2) \left[ (C_9^{\text{eff}} \mp C_{10}) \frac{A_1(s)}{m_B - m_{K^*}} + \frac{2m_b}{s} (C_7^{\text{eff}} - C_7^{\text{eff}'}) T_2(s) \right]
\]

\[
A_{0 L,R} = -\frac{N}{2m_{K^*} \sqrt{s}} \left[ (C_9^{\text{eff}} \mp C_{10}) \left\{ (m_B^2 - m_{K^*}^2 - s) (m_B + m_{K^*}) A_1(s) - \lambda \frac{A_2(s)}{m_B + m_{K^*}} \right\} \right.
\]
\[
+ 2m_b (C_7^{\text{eff}} - C_7^{\text{eff}'}) \left\{ (m_B^2 + 3m_{K^*}^2 - s) T_2(s) - \frac{\lambda}{m_B^2 - m_{K^*}^2} T_3(s) \right\} \right]
\]

\[
A_{\perp L,R} = +\sqrt{2} N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} \mp C_{10}) + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} + C_7^{\text{eff}'}) \right] \xi_{\perp} (E_{K^*})
\]

\[
A_{|| L,R} = -\sqrt{2} N m_B (1 - \hat{s}) \left[ (C_9^{\text{eff}} \mp C_{10}) + \frac{2\hat{m}_b}{\hat{s}} (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{||} (E_{K^*})
\]

\[
A_{0 L,R} = -\frac{N m_B}{2\hat{m}_{K^*} \sqrt{\hat{s}}} (1 - \hat{s})^2 \left[ (C_9^{\text{eff}} \mp C_{10}) + 2\hat{m}_b (C_7^{\text{eff}} - C_7^{\text{eff}'}) \right] \xi_{||} (E_{K^*})
\]
Careful construction of observables

- Good sensitivity to NP contributions, i.e. to $C_7^{eff'}$

- Small theoretical uncertainties
  - Dependence of soft form factors, $\xi_\perp$ and $\xi_\parallel$, to be minimized!
    form factors should cancel out exactly at LO, best for all $s$
  - Unknown $\Lambda/m_b$ power corrections
    $A_{\perp,\parallel,0} = A_{\perp,\parallel,0}^0 \left(1 + c_{\perp,\parallel,0}\right)$ vary $c_i$ in a range of $\pm10\%$ and also of $\pm5\%$
  - Scale dependence of NLO result
  - Input parameters

- Good experimental resolution

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Interesting observables

- **Forward-backward asymmetry**

\[
A_{FB} \equiv \frac{1}{d\Gamma/dq^2} \left( \int_0^1 d(\cos \theta) \frac{d^2\Gamma[\bar{B} \to K^*\ell^+\ell^-]}{dq^2d\cos \theta} - \int_{-1}^0 d(\cos \theta) \frac{d^2\Gamma[\bar{B} \to K^*\ell^+\ell^-]}{dq^2d\cos \theta} \right)
\]

\[
A_{FB} = \frac{3}{2} \frac{\text{Re}(A_{\parallel}A_{\perp}^*) - \text{Re}(A_{\parallel}A_{\perp}^*)}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}
\]

Form factors cancel out at LO only for Zero.

- **Longitudinal polarisation of** $K^*$

\[
F_L(s) = \frac{|A_0|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}
\]

Form factors do not cancel at LO ($\to$ larger hadronic uncertainties)

- **Transversity amplitude** $A_T^2$  
  (Krüger, Matias 2005)

\[
A_T^{(2)} = \frac{|A_{\perp}|^2 - |A_{\parallel}|^2}{|A_{\perp}|^2 + |A_{\parallel}|^2}
\]

Sensitive to right-handed currents (in LO directly $\sim C_7^{eff}f')$

Formfactor cancel out at LO for all $s$

Zero of $A_T^{(2)}$ (for $C_7^{eff}f' \neq 0$) coincides with the Zero of $A_{FB}$ at LO and is also independent from $C_7^{eff}f'$ as in $A_{FB}$. 

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Projection fit possible for $A_T^{(2)}$, $F_L$, $A_{FB}$

\[
\frac{d\Gamma'}{d\phi} = \frac{\Gamma'}{2\pi} \left( 1 + \frac{1}{2}(1 - F_L) A_T^{(2)} \cos 2\phi + A_{\text{Im}} \sin 2\phi \right), \quad \Gamma' = \frac{d\Gamma}{dq^2}
\]

\[
\frac{d\Gamma'}{d\theta_l} = \Gamma' \left( \frac{3}{4} F_L \sin^2 \theta_l + \frac{3}{8}(1 - F_L)(1 + \cos^2 \theta_l) + A_{FB} \cos \theta_l \right) \sin \theta_l,
\]

\[
\frac{d\Gamma'}{d\theta_K} = \frac{3\Gamma'}{4} \sin \theta_K \left( 2F_L \cos^2 \theta_K + (1 - F_L) \sin^2 \theta_K \right),
\]

Observables appear linearly, fits performed on data binned in $q^2$

First experimental measurements with limited accuracy is possible

But: $A_T^{(2)}$ suppressed by $1 - F_L$

Full angular fit is superior, once the data set is large enough ($\gtrsim 2fb^{-1}$)

much better resolution (factor 3 even in $A_T^{(2)}$)

New observables are available

Unbinned analysis, $q^2$ dependence parametrised by polynomial

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New observables

By inspection of the $K^*$ spin amplitudes in terms of Wilson coefficients and SCET form factors one identifies further observables

- sensitive to $C_7^{eff'}$
- theoretical clean
- invariant under 3 $R-L$ symmetries
- with high experimental resolution

\[ A_T^{(3)} = \frac{|A_{0L}A_{0L}^* + A_{0R}^*A_{0R}|}{\sqrt{|A_0|^2|A_\perp|^2}} \]
\[ A_T^{(4)} = \frac{|A_{0L}A_{0L}^* - A_{0R}^*A_{0R}|}{|A_{0L}^*A_{0L} + A_{0R}A_{0R}^*|} \]

New observables allow crosschecks

Different sensibility to $C_7^{eff'}$ via $A_0$ in $A_T^{(3)}$, $A_T^{(4)}$

Next step: design of observables sensitive to other new physics operators

(see also Buras et al. 2008)
Phenomenological analysis

Analysis of SM and models with additional right handed currents \( C^e_{7f'f} \)

Specific model:

MSSM with non-minimal flavour violation in the down squark sector

4 benchmark points

Diagonal: \( \mu = M_1 = M_2 = M_{H^+} = m_{\tilde{u}_R} = 1 \text{ TeV} \) \( \tan \beta = 5 \)

- **Scenario A:** \( m_{\tilde{g}} = 1 \text{ TeV} \) and \( m_{\tilde{d}} \in [200, 1000] \text{ GeV} \)
  \[-0.1 \leq (\delta^d_{LR})_{32} \leq 0.1\]
  a) \( m_{\tilde{g}}/m_{\tilde{d}} = 2.5, \ (\delta^d_{LR})_{32} = 0.016 \)
  b) \( m_{\tilde{g}}/m_{\tilde{d}} = 4, \ (\delta^d_{LR})_{32} = 0.036. \)

- **Scenario B:** \( m_{\tilde{d}} = 1 \text{ TeV} \) and \( m_{\tilde{g}} \in [200, 800] \text{ GeV} \)
  mass insertion as in Scenario A.
  c) \( m_{\tilde{g}}/m_{\tilde{d}} = 0.7, \ (\delta^d_{LR})_{32} = -0.004 \)
  d) \( m_{\tilde{g}}/m_{\tilde{d}} = 0.6, \ (\delta^d_{LR})_{32} = -0.006. \)

Check of compatibility with other constraints (\( B \) physics, \( \rho \) parameter, Higgs mass, particle searches, vacuum stability constraints)
Results

\[ A_T^{(2)} = \frac{|A_\perp|^2 - |A_\parallel|^2}{|A_\perp|^2 + |A_\parallel|^2} \]

Theoretical sensitivity

- light green \( \pm 5\% \Lambda/m_b \)
- dark green \( \pm 10\% \Lambda/m_b \)

Experimental sensitivity \((10 fb^{-1})\)

- light green 1 \(\sigma\)
- dark green 2 \(\sigma\)

Remark:  SuperLHCB/SuperB can offer more precision
Crucial: theoretical status of \(\Lambda/m_b\) corrections has to be improved

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old observables: data available

Babar FPCP 2008
Belle ICHEP 2008

\[ A_{FB} = \frac{3}{2} \frac{\text{Re}(A_{LL}^* A_{\perp R}) - \text{Re}(A_{LR}^* A_{\perp L})}{|A_0|^2 + |A|^2 + |A_{\perp}|^2} \]

Babar FPCP 2008
Belle ICHEP 2008

\[ F_L(s) = \frac{|A_0|^2}{|A_0|^2 + |A|^2 + |A_{\perp}|^2} \]

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LHCb ($10 fb^{-1}$) will clarify the situation

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Comparison between old and new observables

The experimental errors assuming SUSY scenario (b) with large-gluino mass and positive mass insertion, is compared to the theoretical errors assuming the SM.

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CP violating observables

- Angular distributions allow for the measurement of 7 CP asymmetries
  (Krüger, Seghal, Sinha\textsuperscript{2} 2000, 2005)

- NLO ($\alpha_s$) corrections included: scale uncertainties reduced
  (however, some CP asymmetries start at NLO only)
  (Bobeth, Hiller, Piranishvili 2008)

- New CP-violating phases in $C_{10}, C'_{10}, C_9,$ and $C'_9$ are by now NOT very
  much constrained and enhance the CP-violating observables drastically
  (Bobeth, Hiller, Piranishvili 2008; Buras et al. 2008)

- New physics reach of CP-violating observables of the angular distributions depends on the theoretical and experimental uncertainties:
  - soft/QCD formfactors
  - other input parameters
  - scale dependences
  - $\Lambda/m_b$ corrections
  - experimental sensitivity in the full angular fit

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Appropriate normalization eliminates the uncertainty due to form factors

\[ A^8 = \frac{I^8 - \bar{I}^8}{d(\Gamma + \bar{\Gamma})/dq^2} \]

\[ A_{V8}^8 = \frac{I^8 - \bar{I}^8}{I^8 + \bar{I}^8} \]

Orange bands: scale/input uncertainty including formfactors
Red bands: conservative estimate of uncertainty due to formfactors only

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Appropriate normalization eliminates the uncertainty due to form factors II

\[ A^{6s} = \frac{I^{6s} - \bar{I}^{6s}}{d(\Gamma + \bar{\Gamma})/dq^2} \]

\[ A^{6s}_{V2s} = \frac{I^{6s} - \bar{I}^{6s}}{I^{2s} + \bar{I}^{2s}} \]

Orange bands: scale/input uncertainty including formfactors
Red bands: conservative estimate of uncertainty due to formfactors only

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$\Lambda/m_b$ corrections very small due to small weak SM phase

$$A_{V2}^{6s} = \frac{I^{6s} - I^{6s}}{I^{2s} + I^{2s}}$$

$$A_{V8}^{8} = \frac{I^{8} - I^{8}}{I^{8} + I^{8}}$$

Uncertainty due $\Lambda/m_b$ corrections significantly smaller than error due to input parameters

Ansatz with random strong phases $\Phi_{1/2}$ and $C_{1/2}$ with 5% and 10%

$$A = A_1 (1 + C_1 e^{i\phi_1}) + e^{i\theta} A_2 (1 + C_2 e^{i\phi_2})$$

Will significantly larger in scenarios with large new physics phases

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Possible new physics effects versus experimental uncertainties

\[ |C_{9, NP}| = 2, \Phi_9 = \pi/8; |C_{10, NP}| = 1.5, \Phi_{10} = \pi/8; |C'_{10}| = 2, \Phi_{10'} = \pi/8 \]

New physics not outside the experimental 2 \sigma range.

However, all phases (0 → 2\pi) are compatible with the present data.

In contrast to observables like \( A_T^i \), CP observables call for Super-LHCB.

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- NLO corrections included
- $\Lambda/m_b$ corrections estimated for each amplitude as $\pm 10\%$ and $\pm 5\%$
  this uncertainty fully dominant

- Input parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_B$</td>
<td>$5.27950 \pm 0.00033$ GeV</td>
<td>$\lambda$</td>
<td>$0.2262 \pm 0.0014$</td>
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<tr>
<td>$m_K$</td>
<td>$0.896 \pm 0.040$ GeV</td>
<td>$A$</td>
<td>$0.815 \pm 0.013$</td>
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<td>$M_W$</td>
<td>$80.403 \pm 0.029$ GeV</td>
<td>$\tilde{\rho}$</td>
<td>$0.235 \pm 0.031$</td>
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<tr>
<td>$M_Z$</td>
<td>$91.1876 \pm 0.0021$ GeV</td>
<td>$\tilde{\eta}$</td>
<td>$0.349 \pm 0.020$</td>
</tr>
<tr>
<td>$\hat{m}_t(\hat{m}_t)$</td>
<td>$172.5 \pm 2.7$ GeV</td>
<td>$\Lambda_{QCD}^{(n_f=5)}$</td>
<td>$220 \pm 40$ MeV</td>
</tr>
<tr>
<td>$m_{b,PS}(2$ GeV)</td>
<td>$4.6 \pm 0.1$ GeV</td>
<td>$\alpha_s(M_Z)$</td>
<td>$0.1176 \pm 0.0002$</td>
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<tr>
<td>$m_c$</td>
<td>$1.4 \pm 0.2$ GeV</td>
<td>$\alpha_{em}$</td>
<td>$1/137.035999679$</td>
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<tr>
<td>$f_B$</td>
<td>$200 \pm 30$ MeV</td>
<td>$a_1(K^*)_{\perp,\parallel}$</td>
<td>$0.20 \pm 0.05$</td>
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<tr>
<td>$f_{K^*,(1$ GeV)</td>
<td>$185 \pm 10$ MeV</td>
<td>$a_2(K^*)_{\perp}$</td>
<td>$0.06 \pm 0.06$</td>
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<tr>
<td>$f_{K^*,\parallel}$</td>
<td>$218 \pm 4$ MeV</td>
<td>$a_2(K^*)_{\parallel}$</td>
<td>$0.04 \pm 0.04$</td>
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<tr>
<td>$\xi_{K^*,\parallel}(0)$</td>
<td>$0.16 \pm 0.03$</td>
<td>$\lambda_{B,+}(1.5$ GeV$)$</td>
<td>$0.485 \pm 0.115$ GeV</td>
</tr>
<tr>
<td>$\xi_{K^*,\perp}(0)$</td>
<td>$0.26 \pm 0.02$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\xi_{K^*,\perp}(0)$ has been determined from experimental data.
More on kinematics:

z axis: Direction of anti-$K^{*0}$ in rest frame of anti-$B_d$

$\theta_l$: Angle between $\mu^-$ and $z$ axis in $\mu\mu$ rest frame

$\theta_K$: Angle between $K^-$ and $z$ axis in anti-$K^*$ rest frame

$\phi$: Angle between the anti-$K^*$ and $\mu\mu$ decay planes

\[
e_z = \frac{p_{K^-} + p_{\pi^+}}{|p_{K^-} + p_{\pi^+}|}, \quad e_l = \frac{p_{\mu^-} \times p_{\mu^+}}{|p_{\mu^-} \times p_{\mu^+}|}, \quad e_K = \frac{p_{K^-} \times p_{\pi^+}}{|p_{K^-} \times p_{\pi^+}|}
\]

\[
\cos \theta_l = \frac{q_{\mu^-} \cdot e_z}{|q_{\mu^-}|}, \quad \cos \theta_K = \frac{r_{K^-} \cdot e_z}{|r_{K^-}|}, \quad \sin \phi = (e_l \times e_K) \cdot e_z, \quad \cos \phi = e_K \cdot e_l
\]
New physics phases not very much constrained  

(Bobeth, Hiller, Piranishvili 2008)
Angular distributions functions depend on the 6 complex $K^*$ spin amplitudes

$$I_i = I_i(A_{\perp L/R}, A_{\parallel L/R}, A_{0 L/R})$$

(limit $m_{\text{lepton}} = 0$)

Helicity amplitudes:

$$A_{\perp, \parallel} = (H_{+1} \mp H_{-1})/\sqrt{2}, \quad A_0 = H_0,$$

$$I_1 = \frac{3}{4} (|A_{\perp L}|^2 + |A_{\parallel L}|^2 + (L \rightarrow R)) \sin^2 \theta_K + \left(|A_{0 L}|^2 + |A_{0 R}|^2\right) \cos^2 \theta_K$$

$$\equiv a \sin^2 \theta_K + b \cos^2 \theta_K,$$

$$I_2 = \frac{1}{4} (|A_{\perp L}|^2 + |A_{\parallel L}|^2) \sin^2 \theta_K - |A_{0 L}|^2 \cos^2 \theta_K + (L \rightarrow R)$$

$$\equiv c \sin^2 \theta_K + d \cos^2 \theta_K,$$

$$I_3 = \frac{1}{2} \left[ (|A_{\perp L}|^2 - |A_{\parallel L}|^2) \sin^2 \theta_K + (L \rightarrow R) \right] \equiv e \sin^2 \theta_K,$$

$$I_4 = \frac{1}{\sqrt{2}} \left[ \text{Re}(A_{0 L} A_{\parallel L}^*) \sin 2\theta_K + (L \rightarrow R) \right] \equiv f \sin 2\theta_K,$$

$$I_5 = \sqrt{2} \left[ \text{Re}(A_{0 L} A_{\perp L}^*) \sin 2\theta_K - (L \rightarrow R) \right] \equiv g \sin 2\theta_K,$$

$$I_6 = 2 \left[ \text{Re}(A_{\parallel L} A_{\perp L}^*) \sin^2 \theta_K - (L \rightarrow R) \right] \equiv h \sin^2 \theta_K,$$

$$I_7 = \sqrt{2} \left[ \text{Im}(A_{0 L} A_{\perp L}^*) \sin 2\theta_K - (L \rightarrow R) \right] \equiv j \sin 2\theta_K,$$

$$I_8 = \frac{1}{\sqrt{2}} \left[ \text{Im}(A_{0 L} A_{\parallel L}^*) \sin 2\theta_K + (L \rightarrow R) \right] \equiv k \sin 2\theta_K,$$

$$I_9 = \left[ \text{Im}(A_{\parallel L} A_{\perp L}^*) \sin^2 \theta_K + (L \rightarrow R) \right] \equiv m \sin^2 \theta_K.$$

11 coefficients to be fixed in the full angular fit, but $a = 3c$ and $b = -d$
12 theoretical independent amplitudes $A_j \Leftrightarrow 9$ independent coefficient functions in $I$

**Symmetries of the angular distribution functions** $I_i = I_i( A_{\perp L/R}, A_{\parallel L/R}, A_{0L/R} )$

(angular distribution spin averaged)

- **Global phase transformation of the $L$ amplitudes**

  $$A'_{\perp L} = e^{i\phi_L} A_{\perp L}, \quad A'_{\parallel L} = e^{i\phi_L} A_{\parallel L}, \quad A'_{0L} = e^{i\phi_L} A_{0L}$$

- **Global phase transformations of the $R$ amplitudes**

  $$A'_{\perp R} = e^{i\phi_R} A_{\perp R}, \quad A'_{\parallel R} = e^{i\phi_R} A_{\parallel R}, \quad A'_{0R} = e^{i\phi_R} A_{0R}$$

- **Continuous $L$-$R$ rotation**

  $$
  \begin{align*}
  A'_{\perp L} &= \cos \theta A_{\perp L} + \sin \theta A^*_{\perp R} \\
  A'_{\perp R} &= -\sin \theta A^*_{\perp L} + \cos \theta A_{\perp R} \\
  A'_{0L} &= \cos \theta A_{0L} - \sin \theta A^*_{0R} \\
  A'_{0R} &= \sin \theta A^*_{0L} + \cos \theta A_{0R} \\
  A'_{\parallel L} &= \cos \theta A_{\parallel L} - \sin \theta A^*_{\parallel R} \\
  A'_{\parallel R} &= \sin \theta A^*_{\parallel L} + \cos \theta A_{\parallel R}.
  \end{align*}
  $$

Only 9 amplitudes $A_j$ are independent in respect to the angular distribution

Observables as $F(I_i)$ are also invariant under the 3 symmetries!
Transversity amplitude $A_T^{(1)}$

Defining the helicity distributions $\Gamma_{\pm}$ as

$$\Gamma_{\pm} = |H_{\pm 1}^L|^2 + |H_{\pm 1}^R|^2$$

one can define (Melikhov, Nikitin, Simula 1998)

$$A_T^{(1)} = \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+} \quad A_T^{(1)} = \frac{-2\text{Re}(A_\parallel A_{\perp}^*)}{|A_\perp|^2 + |A_\parallel|^2}$$

Very sensitive to right-handed currents (Lunghi, Matias 2006)

Very insensitive to $\Lambda/m_b$ corrections

Formfactor cancel out at LO for all $s$

Big surprise:

$A_T^{(1)}$ is not invariant under the symmetries of the angular distribution

- $A_T^{(1)}$ cannot be extracted from the full angular distribution

- LHCb: practically not possible to measure the helicity of the final states on a event-by-event basis (neither as statistical distribution)

- Not a principal problem, but $A_T^{(1)}$ not an observable at LHCb or at Super B (measure three-momentum and charge)
• Region $s \leq 1 GeV^2$

  – corresponds to information which is tested out by the $b \rightarrow s\gamma$ mode

  – lower resonances complicate the theoretical description

  – longitudinal amplitude generates a logarithmic divergence in the limit $s \rightarrow 0$ indicating problems in the theoretical description

    transversal amplitude however is fine, so observables based on it free from this theoretical problem

  – electron modes preferable (lower cut)
SLHCb versus SFF  Important role of $\Lambda/m_b$ corrections

Measurement of inclusive modes restricted to $e^+e^-$ machines. (S)LHC experiments: Focus on theoretically clean exclusive modes necessary.

Well-known example: Zero of forward-backward-charge asymmetry in $b \rightarrow s\ell^+\ell^-$

Exclusive Zero:

Theoretical error: $9\% + O(\Lambda/m_b)$ uncertainty

Experimental error at SLHC: $2.1\%$ Libby

Inclusive Zero:

Theoretical error: $O(5\%)$

Experimental error at SFF: $4 - 6\%$ Browder, Ciuchini, Gershon, Hazumi, Hurth, Okada, Stocchi

Egede, Hurth, Matias, Ramon, Reece
arXiv:0807.2589

Huber, Hurth, Lunghi, arXiv:0712.3009

Browder, Ciuchini, Gershon, Hazumi, Hurth, Okada, Stocchi
arXiv:0710.3799
Present bounds on $C_7$ and $C_{7}'$:
Test of allowed region around $C_7' = 0$ in the $C_7$ and $C_7'$ plane

$A_T^{(2)}$
new observables

old observables

$A_T^{(3)}$

$A_T^{(4)}$

$A_{FB}$

$F_L$
Present role of time-dependent CP asymmetry $B \to K^{*}\gamma$

Theoretical status of CP asymmetry

- General folklore: within the SM are small, $O(m_s/m_b)$

$$
\mathcal{O}_{7L} \equiv \frac{e}{16\pi^2} m_b \bar{s}\sigma_{\mu\nu} P_R b F^{\mu\nu} \quad \mathcal{O}_{7R} \equiv \frac{e}{16\pi^2} m_s/d \bar{s}\sigma_{\mu\nu} P_L b F^{\mu\nu}.
$$

Mainly: $\bar{B} \to X_s\gamma_L$ and $B \to X_s\gamma_R \Rightarrow$ almost no interference in the SM

- But: within the inclusive case the assumption of a two-body decay is made, the argument does not apply to $b \to s\gamma$ gluon

Corrections of order $O(\alpha_s)$, mainly due operator $O_2 \Rightarrow \Gamma^{\text{brems}}_{22}/\Gamma_0 \sim 0.025$

$\Rightarrow$ 11% right-handed contamination


- QCD sum rule estimate of the time-dependent CP asymmetry in $B^0 \to K^{*0}\gamma$

including long-distance contributions due to soft-gluon emission from quark loops versus dimensional estimate of the nonlocal SCET operator series:


$$
S = -0.022 \pm 0.015^{+0}_{-0.01}, \quad S^{\text{sgluon}} = -0.005 \pm 0.01 \leftrightarrow |S^{\text{sgluon}}| \approx 0.06
$$

Note: Expansion parameter is $\Lambda_{QCD}/Q$ where $Q$ is the kinetic energy of the hadronic part. There is no contribution at leading order. Therefore, the effect is expected to be larger for larger invariant hadronic mass, thus, the $K^*$ mode has to have the smallest effect, below the ‘average’ 10%

Experiment: $S = 0.19 \pm 0.23$ (HFAG)
Future role of time-dependent CP asymmetry $B \rightarrow K^{*\gamma}$

$$S_{K^{*\gamma}} = -\frac{2|r|}{1 + |r|^2} \sin \left( 2\beta - \arg(C_7^{(0)}C_7') \right), \quad r = C_7'/C_7^{(0)}$$

SuperB: $\Delta S = \pm 0.04$ \hspace{1cm} arXiv:hep-ex/0406071

LHCb: $B_s \rightarrow \Phi\gamma$

$$S_{\Phi\gamma} = 0 \pm 0.002 \hspace{1cm} \sin(\phi_s)! \hspace{1cm} \text{Muheim,Xie,Zwicky,arXiv:0802.0876}$$

$$A_{\Phi\gamma}^{\Delta\Gamma} = 0.047 \pm 0.025 + 0.015 \hspace{1cm} \cos(\phi_s)!$$

LHCb (2$fb^{-1}$): $\Delta A = 0.22$ \hspace{1cm} Golutvin et al., LHCb-PHYS-2007-147