Three- and four-jet production at low-x at HERA

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- Introduction
  - Theoretical models
  - Definition of variables
- Cross section measurements
  - 3-jet sample
  - 3-jet subsample
  - 4-jet subsample
- Summary and conclusions
Multi-jet measurements at HERA

Three- and Four-jet Production at Low x at HERA.

$\Delta\eta \sim 4$

HERA jet measurement covers ~4 pseudorapidity units $\rightarrow$ we expect typically no more than 3-4 hard emissions

At HERA 3, 4 jets are at the limit of hard radiation phase space at small $x$
Physics motivation of this measurement

- **Check in detail** theory predictions of parton level calculations provided by NLOJET++ (fixed order NLO for 3-jet inclusive sample)

- NLOJET++ is theory based on DGLAP approximation, which assumes ordering of parton transverse momenta along cascade. 
  QUESTION: is DGLAP good enough in all available for 3-jet inclusive phase space at HERA?

- Check in detail predictions of LO+parton shower generators with $k_T$-ordered (RAPGAP) and unordered (Color Dipol Model) cascade

- In contrast to inclusive jets and di-jets three-jet final states require at least one gluon radiation in addition to $\gamma^* g \rightarrow q\bar{q} \rightarrow 3$-jet sample ideally suited to study gluon emissions and underlying parton dynamics
NLO parton level MC

\( \text{NLOJET++} \) (Z.Nagi, Z.Trocsanyi)

\[
\mu_r = \mu_f = \frac{1}{m} \sum p^*_T_i
\]

Scale

Uncertainty

\( \mu_f \) and \( \mu_r \) varied by common factor of 2 or 0.5

- NNLO calculation for dijets
- NLO calculation for trijets
- LO calculation for four-jets
- Trijet calculation contains \( \alpha_s \ln(1/x) \) term
QCD models based on DGLAP and Color Dipole Model (CDM)

- RAPGAP: implements DGLAP evolution with $k_\perp$ ordering
- RAPGAP RESOLVED: also evolution from "hadronic photon" side, in a sense breaks ordering, but within DGLAP scheme

- ARIADNE: implements Color Dipoles Model:
  - Quasi-classical color dipoles radiate independently
  - No $k_\perp$ ordering
Event and jet selection

Event selection

- $5 \text{ GeV}^2 < Q^2 < 80 \text{ GeV}^2$
- $10^{-4} < x_{Bj} < 10^{-2}$, $0.1 < y < 0.7$

Jet selection

Jets formed from the tracks and clusters (incl. $k_T$ algorithm in $\gamma p$ CMS, dist. par.=1)

- $\geq 3$ jets with $p_T^{*} > 4 \text{ GeV}$
  (good correlation between jets@detector level and jets@hadron/parton level)
- $p_{T1}^{*} + p_{T2}^{*} > 9 \text{ GeV}$
  (to compare the data to the NLO($O(\alpha_s^3)$) calculat.)
- $-1 < \eta_{jet} < 2.5$ in lab. frame
- $\geq 1$ central jet with $-1 < \eta_{jet} < 1.3$
  (quantities with * measured in $\gamma p$ CMS)

Data sample

Integrated lumi 44.2 pb$^{-1}$
384000 events $\geq 3$ jets
6000 events $\geq 4$ jets
Observables describing 3-jet system

Scaled energies

\[ X_i' = \frac{E_i'}{E_1' + E_2' + E_2'} \]

Two angles \( \theta' \) and \( \psi' \) orientation of 3-jet system with respect to colliding boson-parton system

Jet transverse momenta \( p_{T1}^* > p_{T2}^* > p_{T3}^* \) in \( \gamma^* p \) system

Jet pseudorapidities \( \eta_1 \ \eta_2 \ \eta_3 \) in laboratory system
3-jet cross section and jet multiplicity distribution

- 3-jet cross section well described by NLO($\alpha_s^3$)
- NLO($\alpha_s^3$) underestimates 4-jet rate by factor 2.6
- CDM (unordered radiation) provides excellent description of jet multiplicity distribution up to $N_{\text{jet}} = 6$
- RAPGAP (ordered parton shower) fails to describe jet multiplicity distribution, underestimates 4-jet rate by factor 2.9

Hadronisation corrections uncertainty (model dependence)

Scale + hadronisation uncertainty in quadrature
Here and in all the other plots NLO($\alpha_s^3$) is significant improvement w.r.t. NLO($\alpha_s^2$).

At very small $x < 2 \times 10^{-4}$ NLO($\alpha_s^3$) undershoots the data (upper edge of theoretical error band). Not observed or less accentuated in previous analyses with restricted phase space ($M_{3\text{jet}} > 25$ GeV or higher $E_T$ – jet cut).

RAPGAP fails to describe both shape and normalization (plot normalized by 1.55).

CDM provides excellent description in shape and fair in normalization (here 1.05).
3-jet inclusive sample: pseudorapidities in laboratory

- Data described within large theoretical uncertainties, but tendency to underestimate cross section at large positive pseudorapidities (forward jets)
- Improvement of $O(\alpha_s^3)$ w.r.t. $O(\alpha^2)$ increases with pseudorapidity
3-jet system variables $X_1', X_2', \psi', \theta'$ in NLO($\alpha_s^{2,3}$)

- Good description apart from slight deficit in normalisation, seen in previous plots at large pseudorapidities.
3-jet system variables $X'_2, \cos \theta'$ and $p_{T1}^*$ in MC

- **CDM** provides in general good description of 3-jet system except except transverse momentum of the leading jet
- **RAPGAP** in most cases fails to describe shapes of the distributions
3-jet sample subsamples

Central jets:
$-1 < \eta_{jet} < 1$

Forward jets:
$\eta_{fj1} > 1.73$
$x_{fj1} > 0.035$
$\eta_{fj2} > 1$

All jets:
$E_{t, jet}^* > 4 \text{ GeV}$

2 forward + 1 central

1 forward + 2 central

• The fraction of jets due to gluon radiation is expected (MC) to be larger for forward jets than for central jets

• $f+2c$ sample will have many events with a single radiated gluon (3-jet)

• $2f+1c$ sample has a larger fraction with 2 radiated gluons (4-jet $\rightarrow$ LO)
Bjorken-x distribution of forward jet subsamples

- NLO($\alpha_s^3$) provides rather good description of 1f+2c sample
- For 2f+1c subsample dramatic improvement from $O(\alpha_s^2)$ to $O(\alpha_s^3)$. The large remaining deficiency for $x < 2\cdot10^{-4}$ is significant
- 2f+1c sample in large part is process with 2 radiated gluons → $O(\alpha_s^3)$ is effectively LO calculation
Four-jet sample: comparison with MC

- CDM (LO+ $k_T$-unordered parton shower) provides almost perfect description of the $N_{\text{jet}}>3$ data sample, except $p_T^* > 15$ GeV
- RAPGAP (LO+ $k_T$-ordered parton shower) fails to describe the data (normalization factor 2.9)
Summary and conclusions

• Remarkable success of $\text{NLO}(\alpha_s^3)$ calculation by NLOJET++

• Huge improvement w.r.t. to $O(\alpha_s^2)$ theory especially for large positive rapidities and small $x$

• There are regions of phase space where fixed order NLO DGLAP calculation $O(\alpha_s^3)$ cannot describe the HERA data

• LO+ $p_T$- unordered parton shower (Color Dipol Model) describes the data surprisingly well (except $p_T^*>15$ GeV)

• LO+ $p_T$-ordered parton shower (RAPGAP) fails to describe the data