SM Predictions for the Muon $(g-2)/2$

Simon Eidelman

Budker Institute of Nuclear Physics,
Novosibirsk, Russia

Outline

1. QED and Electroweak terms
2. Progress with e^+e^- data
3. LO hadronic term
4. Prospects
5. Conclusions

Muon Anomalous Magnetic Moment

$$\vec{\mu} = g\frac{e}{2m}\vec{s}, \qquad a = (g-2)/2.$$

In Dirac theory for pointlike particles $g = 2$,
higher-order effects or new physics $\Rightarrow g \neq 2$

Any significant difference of $a_\mu^{\rm exp}$ from $a_\mu^{\rm th}$ indicates
New Physics beyond the Standard Model.

a_μ is much more sensitive to new physics effects than a_e:
the gain is usually $\sim (m_\mu/m_{\rm e})^2 \approx 4.3 \cdot 10^4$.

$$a_\mu^{\rm th} = a_\mu^{\rm SM} + a_\mu^{\rm NP}, \qquad a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}.$$

Experimental Status of a_l

$a_e = 1159652180.73(28) \times 10^{-12}$ 0.24×10^{-9}

D. Hanneke et al., PRL 100, 120801 (2008)
QED test or α determination

$a_\mu = 116592080(63) \times 10^{-11}$ 0.54×10^{-6}

G.W. Bennett et al. (E821), PRD 73, 072003 (2006)
Sensitive test of the Standard Model

$a_\tau = -0.018(17)$ or $-0.052 < a_\tau < 0.013$ 95%CL

J. Abdallah et al. (DELPHI), EPJ C 35, 159 (2004)
Theory: $117721(5) \times 10^{-8}$, SE, M. Passera, MPL A 22, 159 (2007)

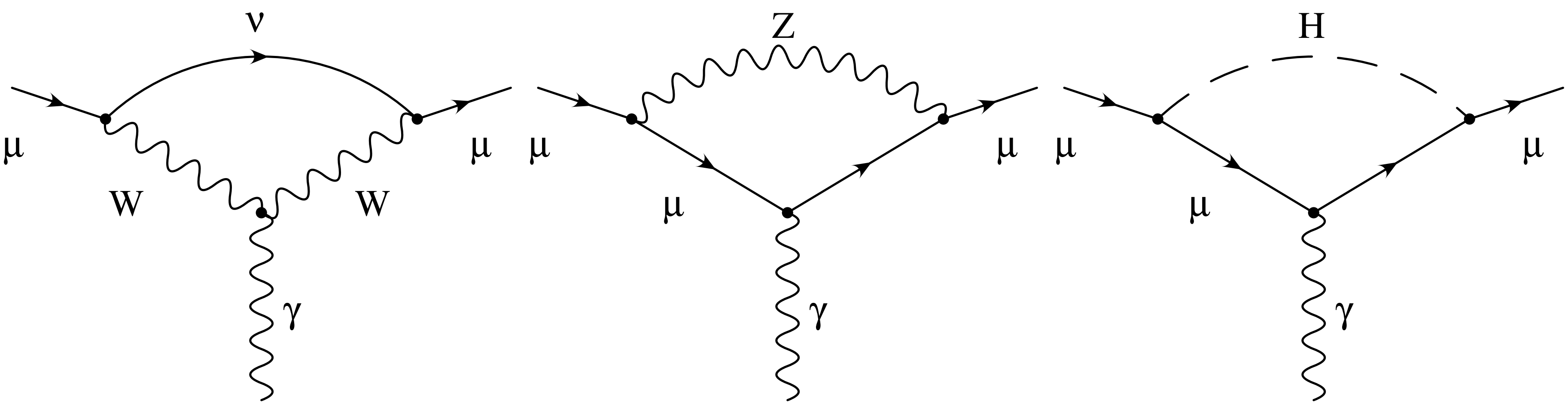
QED Contribution $a_\mu^{\rm QED}$

$$
\begin{array}{rcrl}
a_\mu^{\rm QED} \cdot 10^{10} = \Sigma C_i (\frac{\alpha}{\pi})^i & = & 11614097.3 \text{ (1-loop)} & 1 \text{ diagram} \\
 & + & 41321.8 \text{ (2-loop)} & 9 \\
 & + & 3014.2 \text{ (3-loop)} & > 100 \\
 & + & 38.1 \text{ (4-loop)} & > 1000 \\
 & + & 0.4 \text{ (5-loop)} & > 20000
\end{array}
$$

α^3 terms known analytically (S. Laporta, E. Remiddi, 1993),
α^4 terms – numerically (T. Kinoshita et al., 2003-2008),
L log α^5 (TK et al., 2005,2007; A.L. Kataev, 2006, K. Chetyrkin et al., 2008):
$a_\mu^{\rm QED} = (116584719.4 \pm 1.4) \cdot 10^{-11}$.

From the latest value of a_e (D. Hanneke et al., 2008; M. Passera, 2008):
$\alpha^{-1} = 137.035999084(51)$, $a_\mu^{\rm QED} = (116584718.09 \pm 0.14 \pm 0.04) \cdot 10^{-11}$.
The errors are due to: a/ $\mathcal{O}(\alpha^5)$, b/ α

Electroweak contribution a_μ^{EW}



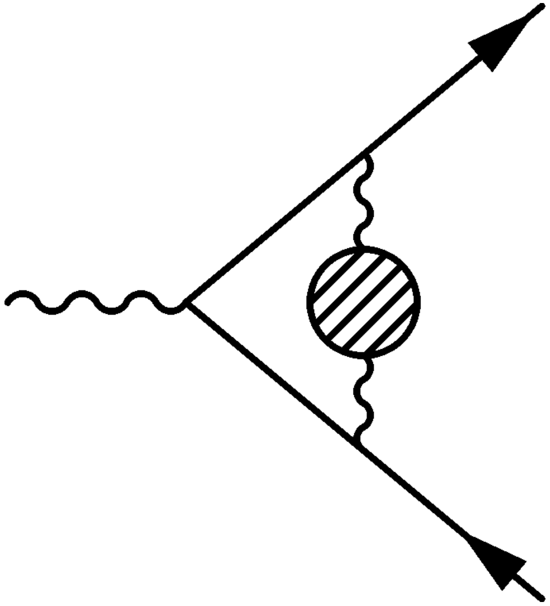
One-loop electroweak contributions

Authors	Year	a_μ^{EW}, 10^{-10}
$\ldots, \ldots, \ldots$	1972	19.5
A. Czarnecki et al.	1996	15.2 ± 0.4
A. Czarnecki et al.	2002	$15.4 \pm 0.1 \pm 0.2$

The errors are due to: a/ hadr. loops, b/ M_H, M_t, 3-loop effects.

Hadronic contribution $a_\mu^{\rm had}$

$$a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,HO} + a_\mu^{\rm had,LBL}$$



$$a_\mu^{\rm had,LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds \, \frac{R(s)\,\hat{K}(s)}{s^2},$$

C. Bouchiat, L. Michel, Bouchiat, 1961;
M. Gourdin, E. de Rafael, 1969

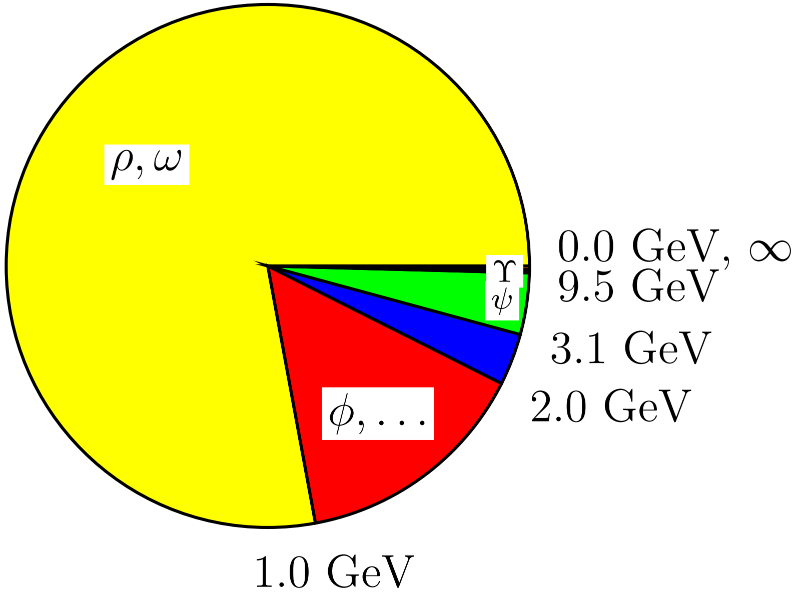
$$R(s) = \frac{\sigma(e^+e^- \to {\rm hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)},$$

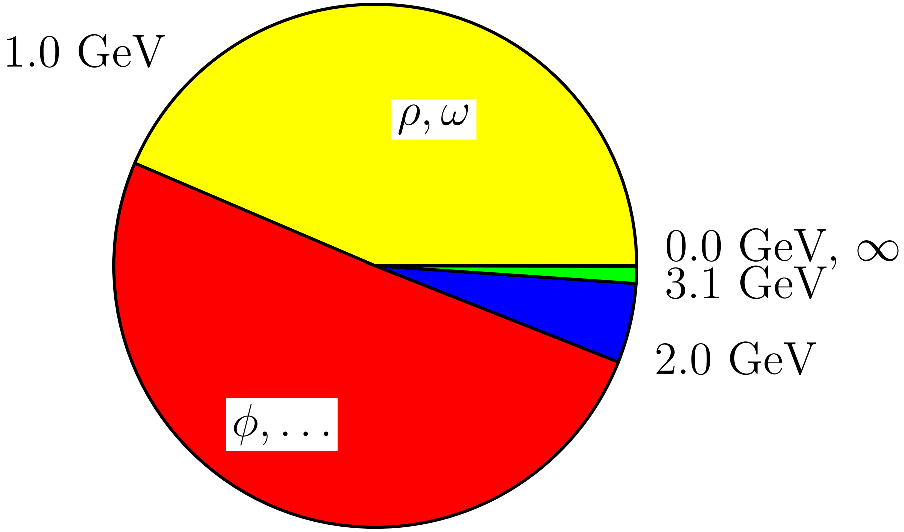
$\hat{K}(s)$ grows from 0.63 at $s = 4m_\pi^2$ to 1 at $s \to \infty$,

$1/s^2$ emphasizes low energies, particularly $e^+e^- \to \pi^+\pi^-$.

$a_\mu^{\rm had,LO} \sim 700 \cdot 10^{-10}$ ⇒ accuracy better than 1% needed

Contributions of Various Energy Ranges to $a_\mu^{\rm had,LO}$





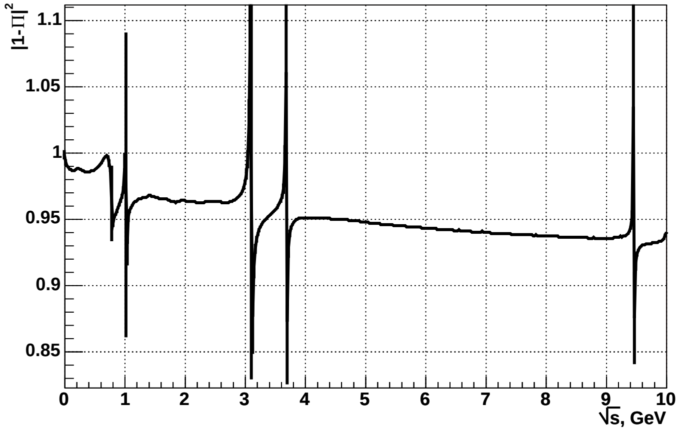
More than 72% of $a_\mu^{\rm had,LO}$ come from $e^+e^- \to \pi^+\pi^-$ and more than 90% from the energy range below 2 GeV

How is R(s) Measured?

- $\sqrt{s} < 2$ GeV – exclusive modes $(\pi^+\pi^-, \pi^+\pi^-\pi^0, \ldots, K\bar{K}, \ldots)$
- Possibly missing (small σ, undetected) final states
- Above 2 GeV – total R (all multihadronic events)
- Initial state radiation (ISR), vacuum polarization (VP), final state radiation (FSR): M. Drees, K. Hikasa, 1990
- Scan or radiative return

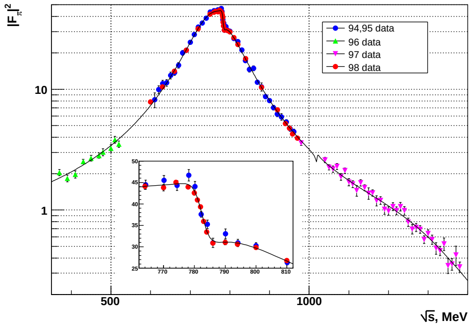
$$\sigma_{\rm dr} = \frac{N}{\int Ldt\ \varepsilon\ (1+\delta_{\rm ISR})}$$

$$\sigma_{\rm bare} = \sigma_{\rm dr}|1-\Pi(s)|^2(1+\delta_{\rm FSR})$$



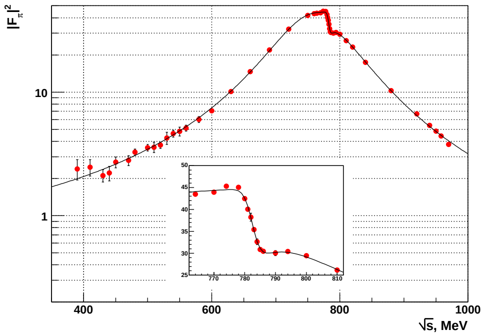
$|1-\Pi(s)|^2$

$e^+e^- \to \pi^+\pi^-$ (CMD-2, SND and KLOE)



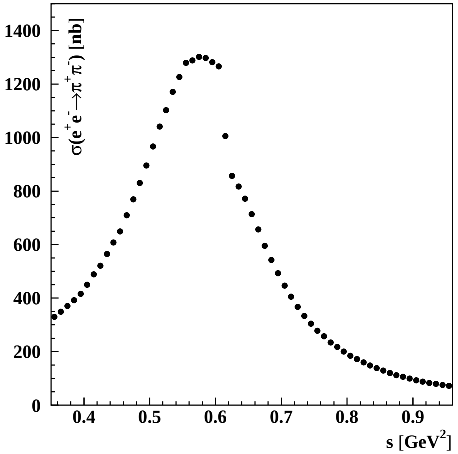
CMD-2, scan: $\sim 9 \cdot 10^5$ ev.

2E, MeV	σ, %
370-520	0.7
600-970	0.6-0.8
1040-1380	1.3-4.2



SND, scan: $\sim 8 \cdot 10^5$ ev.

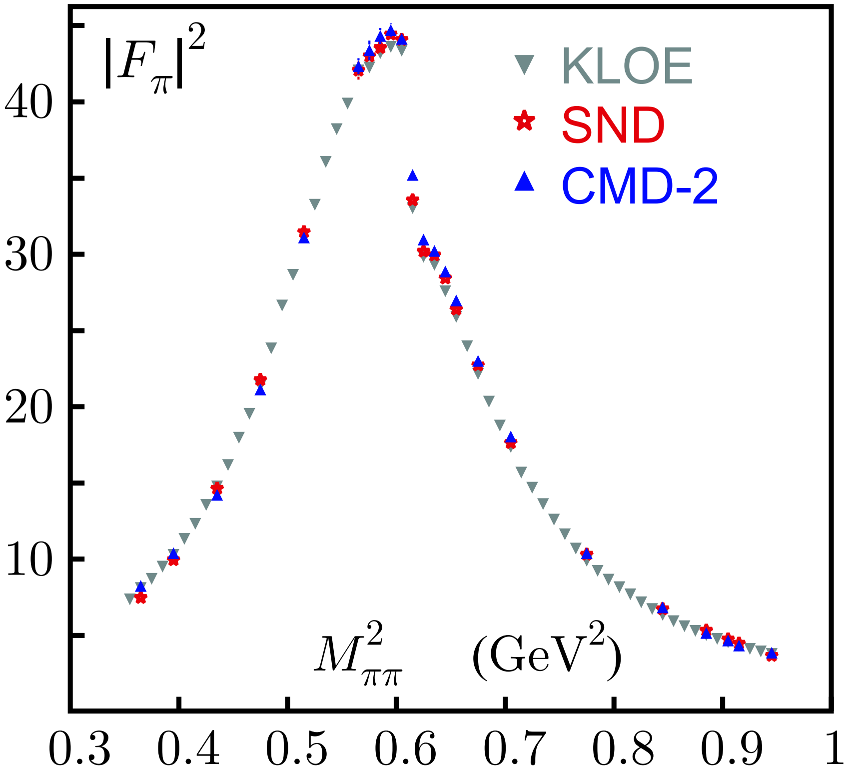
2E, MeV	σ, %
390-420	3.2
430-970	1.3

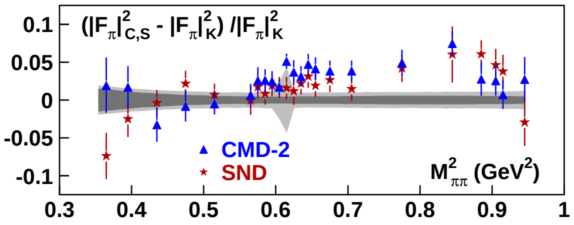


KLOE, ISR: $\sim 3.1 \cdot 10^6$ ev.

(590-970) MeV - 0.8-0.9%

Comparison of KLOE with Novosibirsk



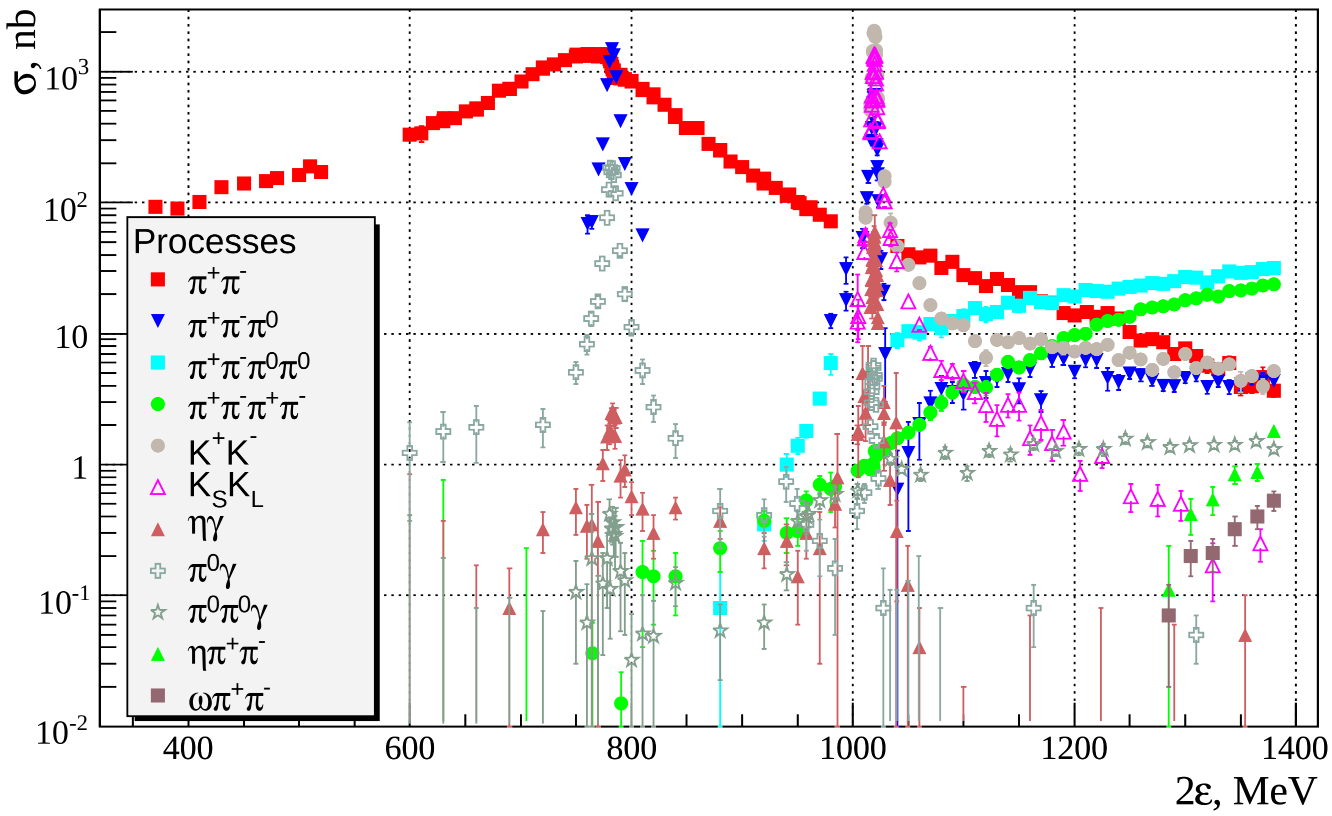


For [0.650,0.958]GeV

Group	$\Delta a_\mu^{\pi\pi} \cdot 10^{10}$
SND	361.0 ± 5.1
CMD-2	361.5 ± 3.4
KLOE	356.7 ± 3.1

Preliminary points of BaBar are somewhat higher

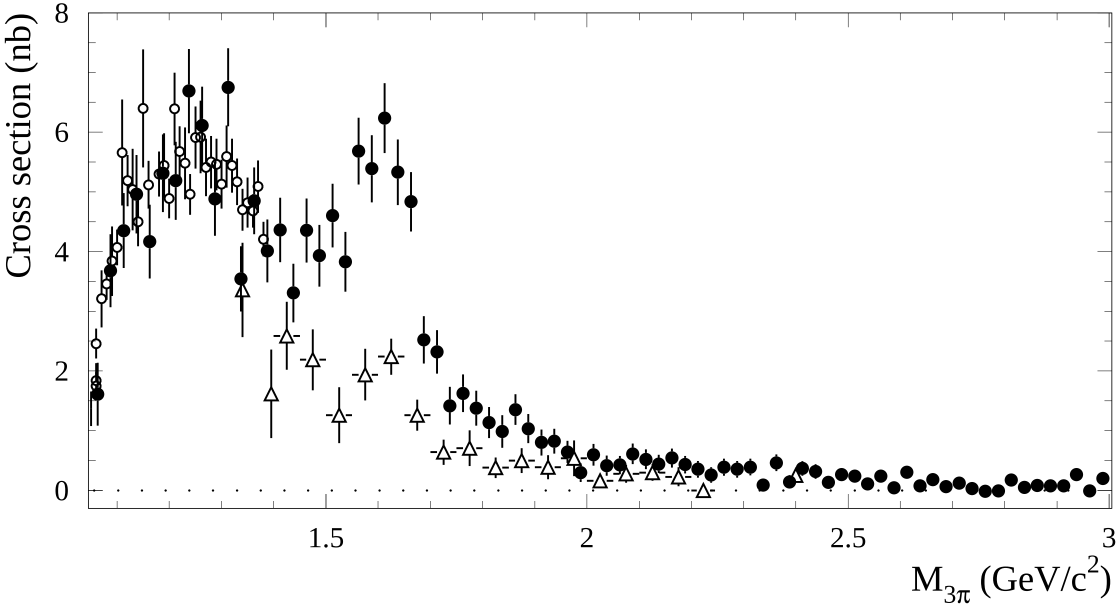
Hadronic Cross Sections at CMD-2



Until recent ISR results from BaBar, the energy range below 1.4 GeV was studied by CMD-2 and SND in Novosibirsk with consistent results

$e^+e^- \to \pi^+\pi^-\pi^0$

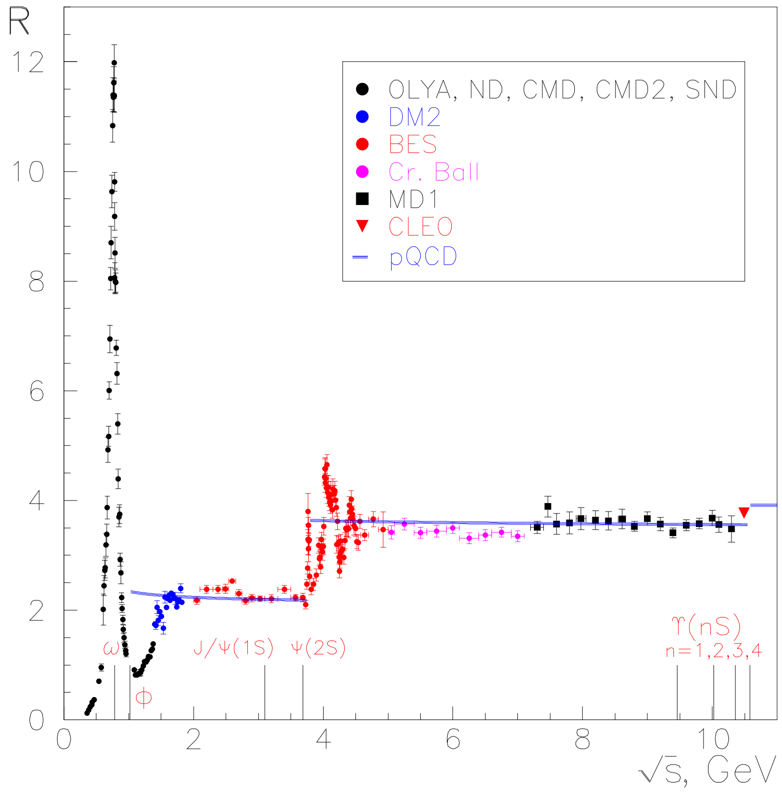
New ISR results from BaBar not always agree with the old datasets



Good agreement with SND. BaBar's points much higher than at DM2

Source	Before BaBar	All + BaBar	All-DM2+BaBar
$\delta a_\mu, 10^{-10}$	$2.45 \pm 0.26 \pm 0.03$	$2.79 \pm 0.19 \pm 0.01$	$3.25 \pm 0.09 \pm 0.01$

R Measurements at $\sqrt{s} < 10$ GeV



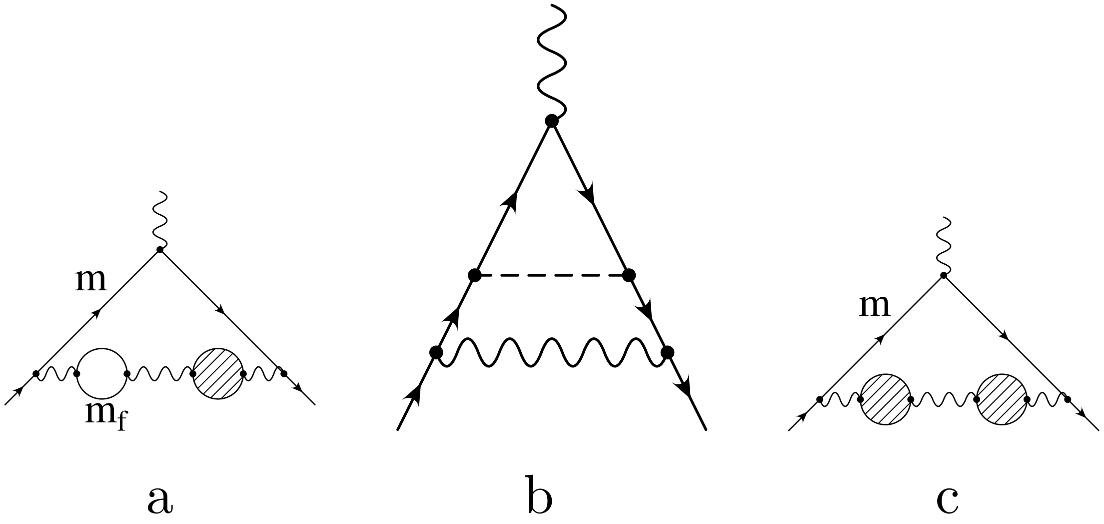
Excellent agreement with the pQCD predictions!

e^+e^- Data Based Calculation of $a_\mu^{\mathrm{had,LO}}$ (DEHZ-2006)

$\sqrt{s}$, GeV	$a_\mu^{\mathrm{had,LO}}$, 10^{-10}	$\delta a_\mu^{\mathrm{had,LO}}$, %
2π	$504.6 \pm 3.1 \pm 1.0$	73.0
ω	$38.0 \pm 1.0 \pm 0.3$	5.5
ϕ	$35.7 \pm 0.8 \pm 0.2$	5.2
0.6 – 1.8	$54.2 \pm 1.9 \pm 0.4$	7.8
1.8 – 5.0	$41.1 \pm 0.6 \pm 0.0$	6.0
$J/\psi, \psi'$	$7.4 \pm 0.4 \pm 0.0$	1.1
> 5.0	$9.9 \pm 0.2 \pm 0.0$	1.4
Total	$690.9 \pm 3.9_{\mathrm{exp}} \pm 1.9_{\mathrm{rad}} \pm 0.7_{\mathrm{QCD}}$	100.0

Higher accuracy of e^+e^- data: the $a_\mu^{\mathrm{had,LO}}$ error is 4.4 (0.63%)
compared to 15.3 of EJ, 1995 and 7.2 of DEHZ, 2003!

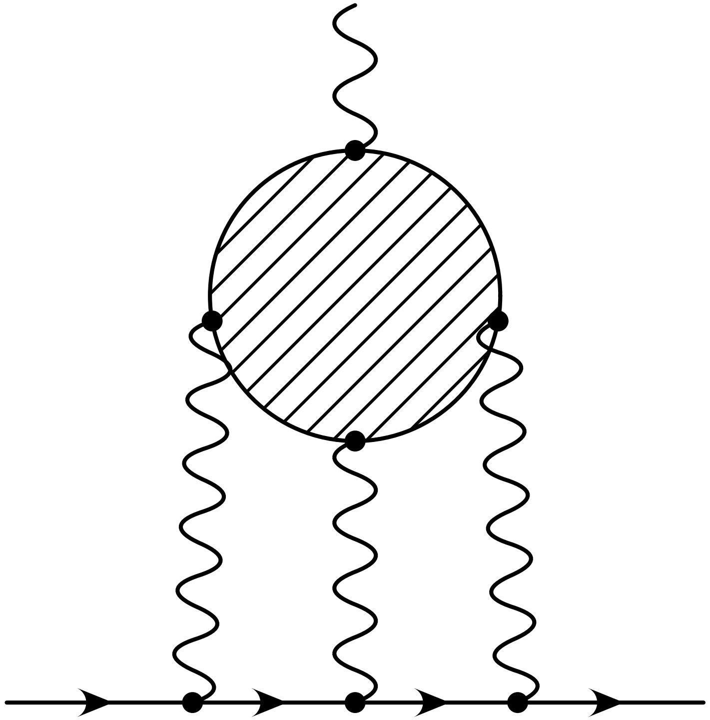
Higher-Order Hadronic Contributions $a_\mu^{\rm had,HO}$



The contributions of all 3 graphs can be calculated in terms of the $\int R(s)G(s)ds/s^{2(3)}$, where $G(s)$ is a smooth function of s, so that the low energy range again dominates the integral. Several calculations agree. The accepted value is (B. Krause, 1997; K. Hagiwara et al., 2003):

$$a_\mu^{\rm had,HO} = (-9.8 \pm 0.1) \cdot 10^{-10}.$$

Light-by-Light Scattering – I



Various approaches used:

- Vector Dominance and Chiral models
- Data on $\gamma\gamma^* \to \pi^0, \eta, \eta'$ (single-tag)
- QCD small-distance constraints
- Effective field theory

The sign of the dominant PS term wrong until 2002!

Light-by-Light Scattering – II

Authors	Year	$a_\mu^{\text{had,LBL}}, 10^{-10}$
J. Bijnens, E. Pallante, J. Prades	1996 (2002)	83 ± 32
M. Hayakawa, T. Kinoshita	1998 (2002)	89.6 ± 15.4
M. Knecht, A. Nyffeler	2002	80 ± 40
K. Melnikov, A. Vainshtein	2003	136 ± 25
M. Davier, W. Marciano	2004	120 ± 35
J. Bijnens, J. Prades	2006	110 ± 40
J. Prades, E. de Rafael, A. Vainshtein	2008	105 ± 26
F. Jegerlehner, A. Nyffeler	2009	116 ± 39

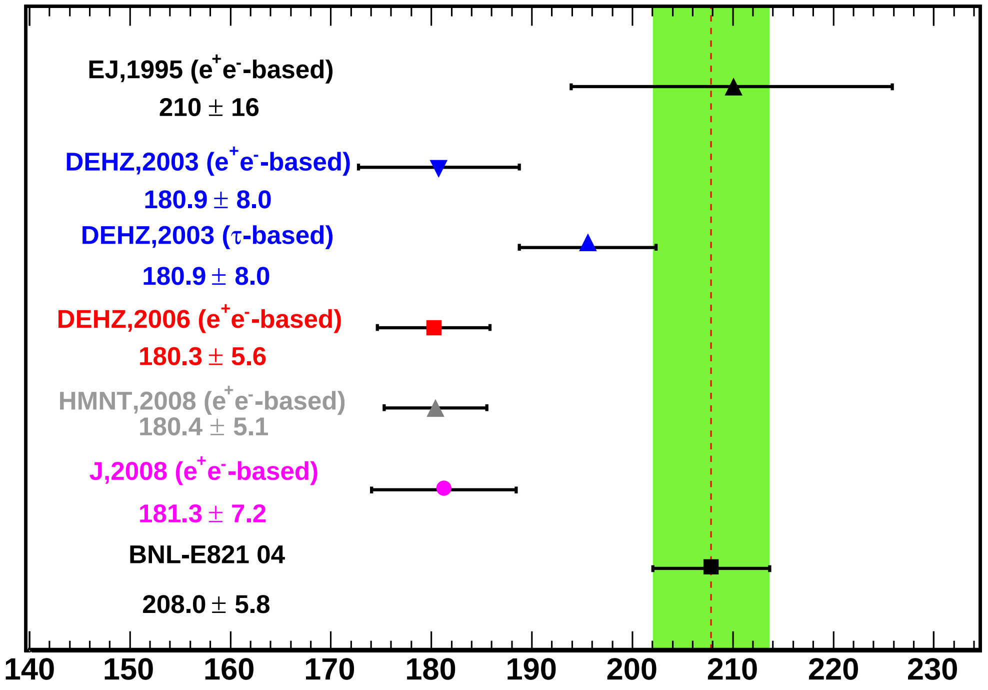
Theory vs Experiment – I

Contribution	$a_\mu, 10^{-10}$
Experiment	11659208.0 ± 6.3
QED	11658471.8 ± 0.016
Electroweak	$15.4 \pm 0.1 \pm 0.2$
Hadronic	693.1 ± 5.6
Theory	11659180.3 ± 5.6
Exp.–Theory	$27.7 \pm 8.4\ (3.3\sigma)$

The difference between experiment and theory is 3.3σ!

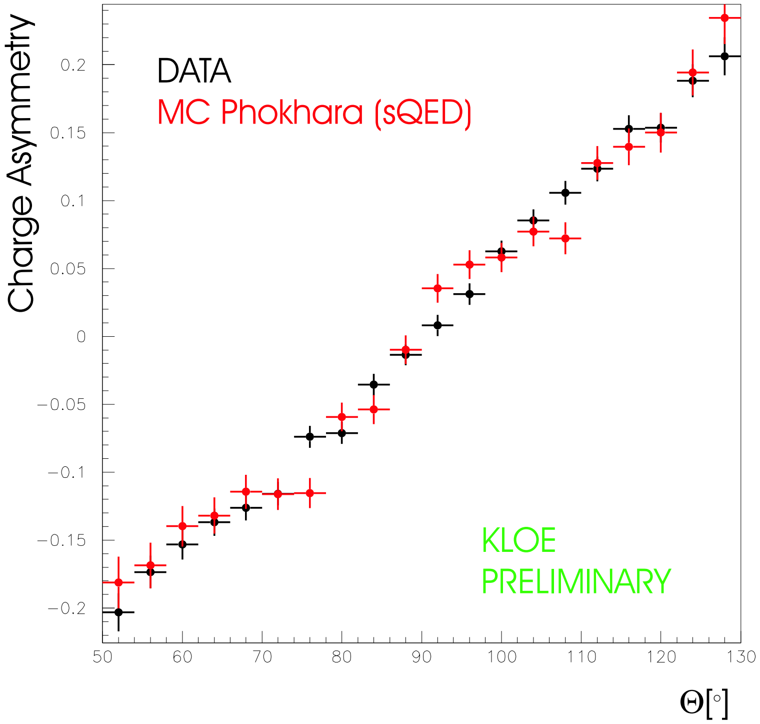
In all estimations the difference exceeds 3σ

Theory vs Experiment – II



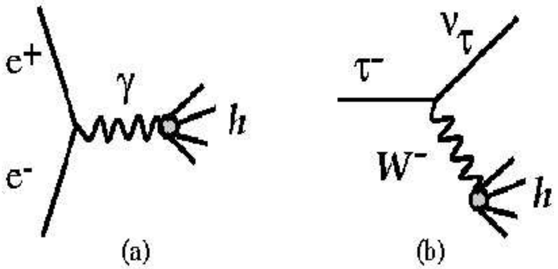
How Real Is $a_\mu^{\rm had}$ Accuracy?

- Missing states: neutrals; $\pi^+\pi^- n\pi^0, K\bar{K}n\pi$ - isospin
- New states from BaBar, double counting
- Radiative corrections (FSR): Charge asymmetry at KLOE 3k $e^+e^- \to \pi^+\pi^-\gamma$ evts at CMD-2
- Correlations
- Averaging
- Light-by-light term
- Double counting (LO and HO)



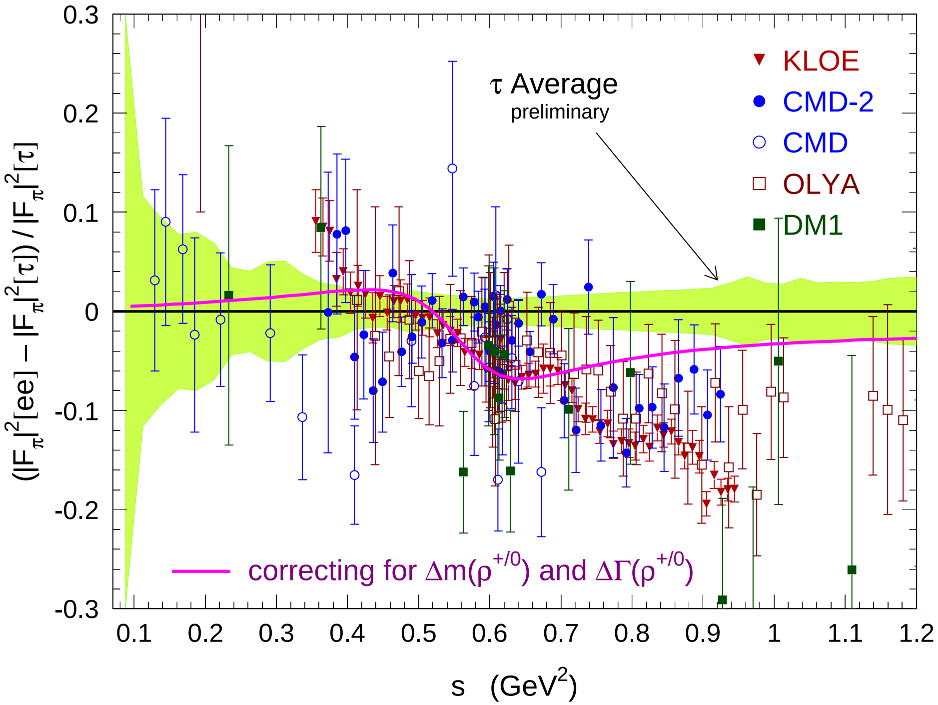
Charge asymmetry in $e^+e^- \to \pi^+\pi^-\gamma$ sQED is OK $(0.6 < M^2_{\pi\pi} < 0.7 \text{ GeV}^2)$

CVC. $e^+e^- \rightarrow X^0$ and $\tau^- \rightarrow \nu_\tau X^-$



Allowed $I^G J^P = 1^+1^-$:

$X^- = \pi^-\pi^0, (4\pi)^-, \omega\pi^-,$
$\eta\pi^-\pi^0, K^-K^0, (6\pi)^-, \ldots$

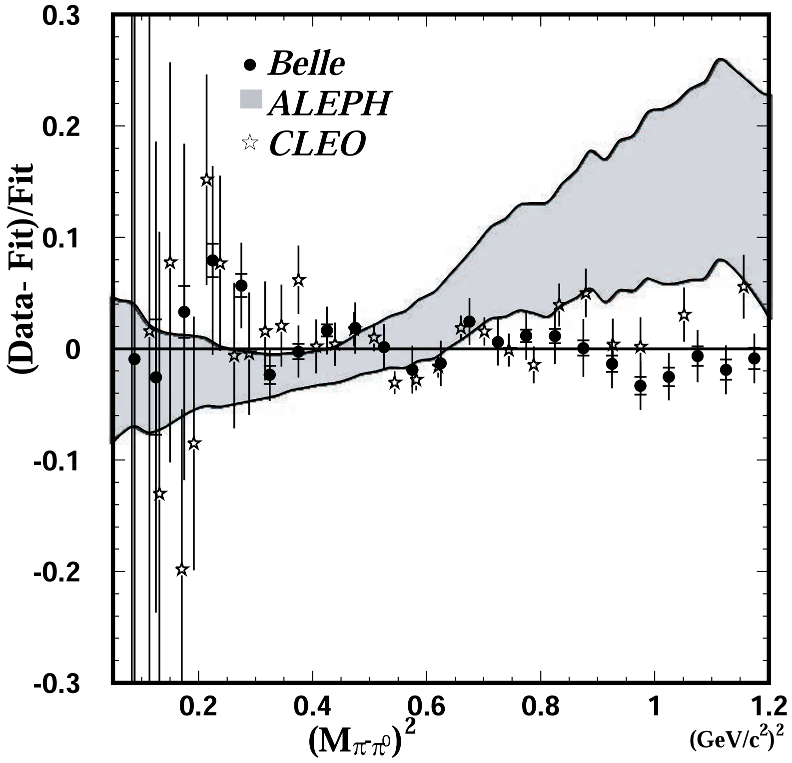


Large SU(2) breaking corrections from theory, V.Cirigliano et al., 2002

$M(\Gamma)_{\rho^0} \neq M(\Gamma)_{\rho^\pm}$ helps,

M.Davier, 2003; S.Ghozzi, F.Jegerlehner, 2004

New data on $\tau^- \to \pi^-\pi^0\nu_\tau$ from Belle

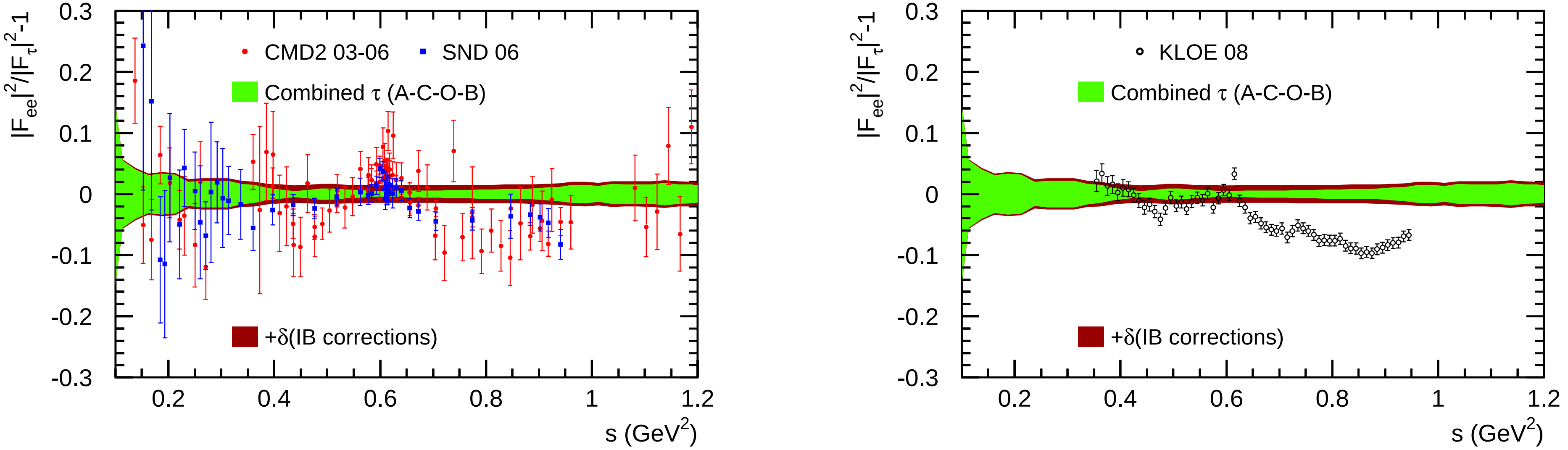


$\mathcal{B}_{\text{Belle}} = (25.24 \pm 0.01 \pm 0.39)\%$ $\mathcal{B}_{\text{ALEPH}} = (25.49 \pm 0.10 \pm 0.09)\%$

The contributions to a_μ^{had} are also compatible due to compensation at tails

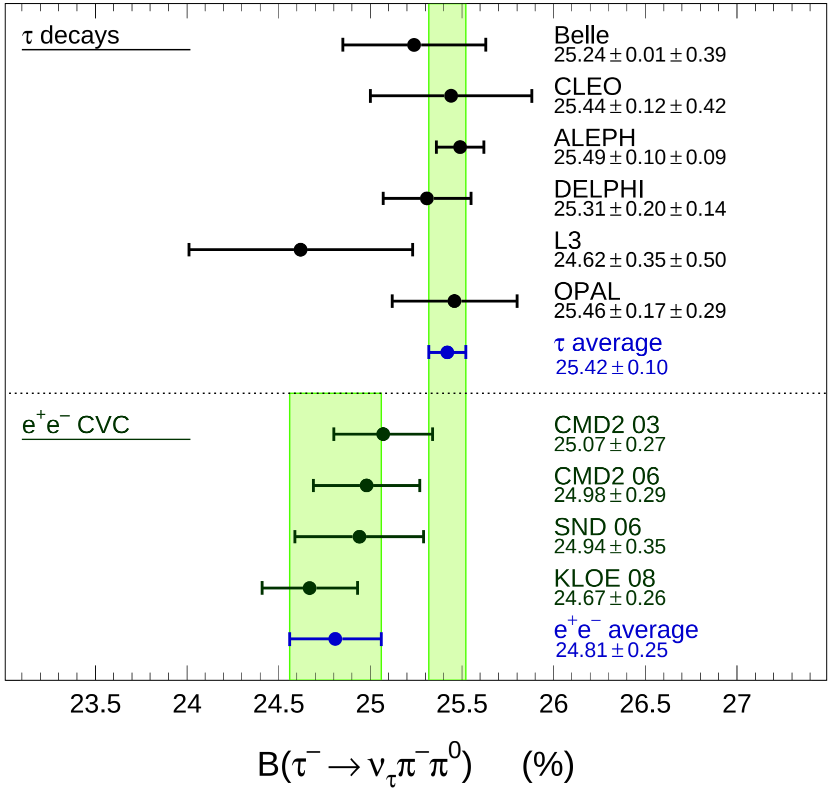
New developments in τ vs. e^+e^- – I

M. Davier et al., arXiv:0906.5443, applied the reconsidered IB corrections to the averaged 2π spectral functions from ALEPH/OPAL/CLEO/Belle.



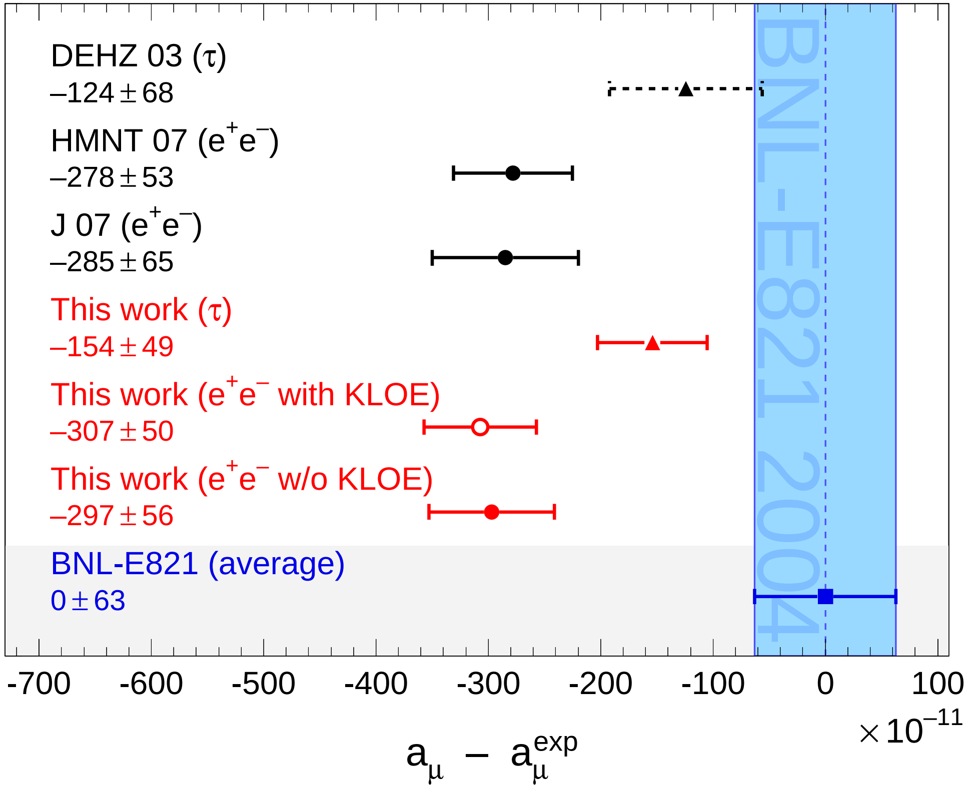
The discrepancy between the τ and e^+e^- is smaller but still there.

New developments in τ vs. e^+e^- – II



$\mathcal{B}_\tau - \mathcal{B}_{ee} = (0.61 \pm 0.27)\%$ or 2.3σ compared to 4.5σ before.

New comparison of experiment and theory for a_μ



Reestimation of $a_\mu^{\rm had}$ together with the updated estimate of the LBL term make the difference larger: 3.6σ

Future of $a_\mu^{\rm had}$ – I

What can be done from the e^+e^- side?

More ISR analysis from KLOE, BaBar, Belle; better R below 4.3 GeV from CLEO-c: 4.4 → 2.8

Experiments at VEPP-2000 with 2 detectors up to $\sqrt{s}$=2 GeV with $L_{\rm max} = 10^{32}$ cm^{-2}s^{-1}, 10^{31} cm^{-2}s^{-1} achieved at the ϕ

A similar machine (DAΦNE-II) is discussed in Frascati,
the $\tau - c$ factory in Beijing is taking data.

By 2012: 2.8 → 2.2, the total error of 4.6 limited by the LBL term (4.0)

Future of $(g_\mu - 2)/2$ – II

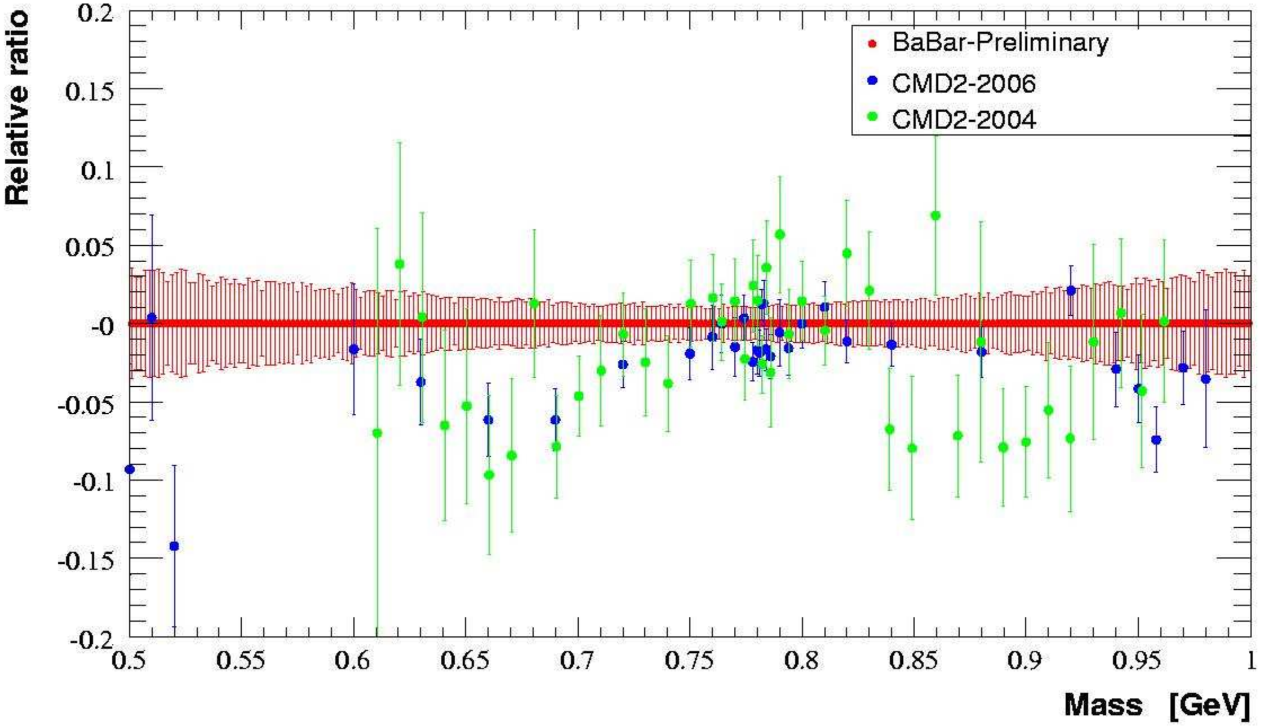
1. New $(g_\mu - 2)/2$ experiment at FLAB expects the accuracy of $0.1\text{ppm}_{\text{stat}}$ and $0.1\text{ppm}_{\text{syst}}$ or 1.5×10^{-10}
2. Such accuracy for $a^{\text{had,LO}}$ corresponds to 0.2%, hardly ever achievable for the absolute measurement of $\sigma(e^+e^- \to \text{ hadrons})$
3. a_μ^{had} calculation from 1st principles (QCD, Lattice).
   QCD instanton model (A. Dorokhov, 2003),
   Recently from the Lattice: $a_\mu^{\text{had}} = (545 \pm 65) \cdot 10^{-10} \Rightarrow (667 \pm 20) \cdot 10^{-10}$
   (C. Aubin and T. Blum, 2005),
   attempts to estimate a_μ^{lbl} (M. Hayakawa et al., 2005).

Conclusions

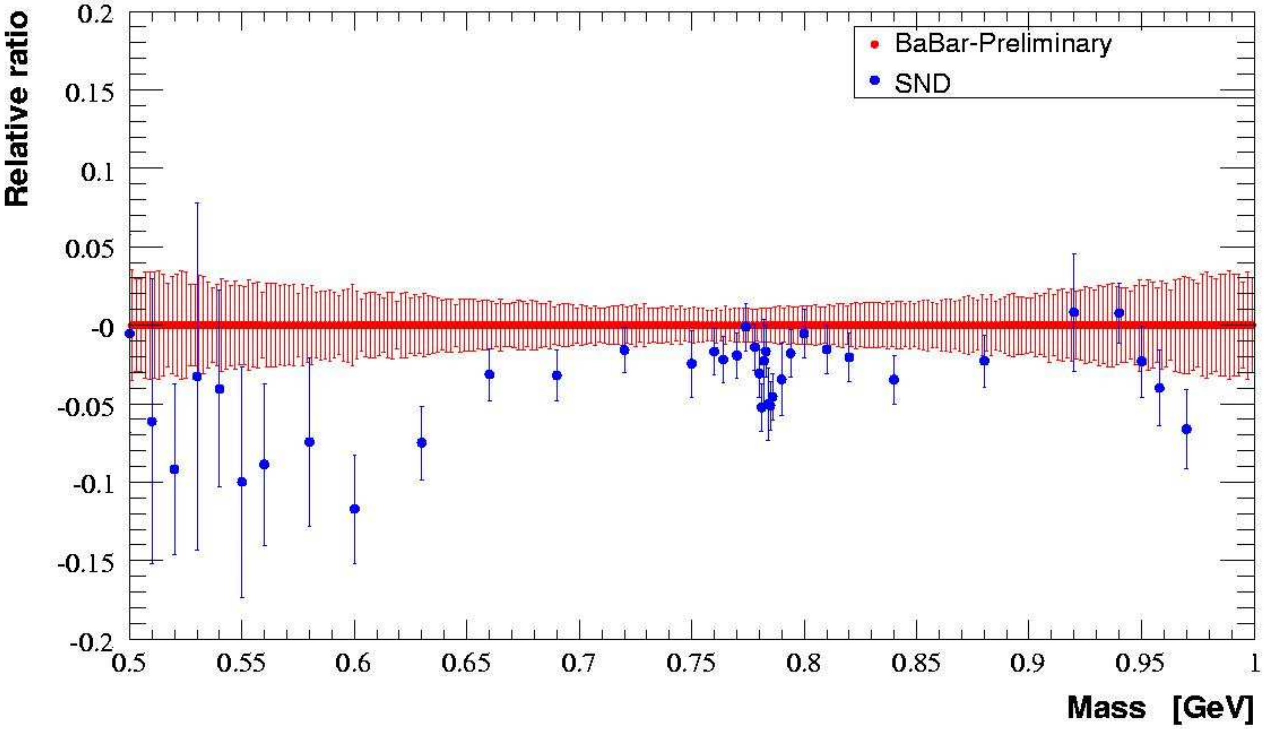
- BNL success stimulated significant progress of experiment and theory
- QED and EW terms are in good shape
- Improved e^+e^- data $\Rightarrow$ smaller $\delta a_\mu^{\mathrm{had,LO}}$ matching experiment
- τ data could further improve the accuracy, but CVC relations for e^+e^- and τ do not hold yet
- Further improvement in $a_\mu^{\mathrm{had,LO}}$ by a factor of 2 will be possible after VEPP-2000, DAΦNE-II, CESRc and $(c-\tau)$ factory + ISR at DAΦNE and B-factories
- Light-by-light term will soon limit the accuracy
- More theory input needed
- $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$ differs from 0 by more than 3 σ $\Rightarrow$ A hint to New Physics?

Back-up Slides

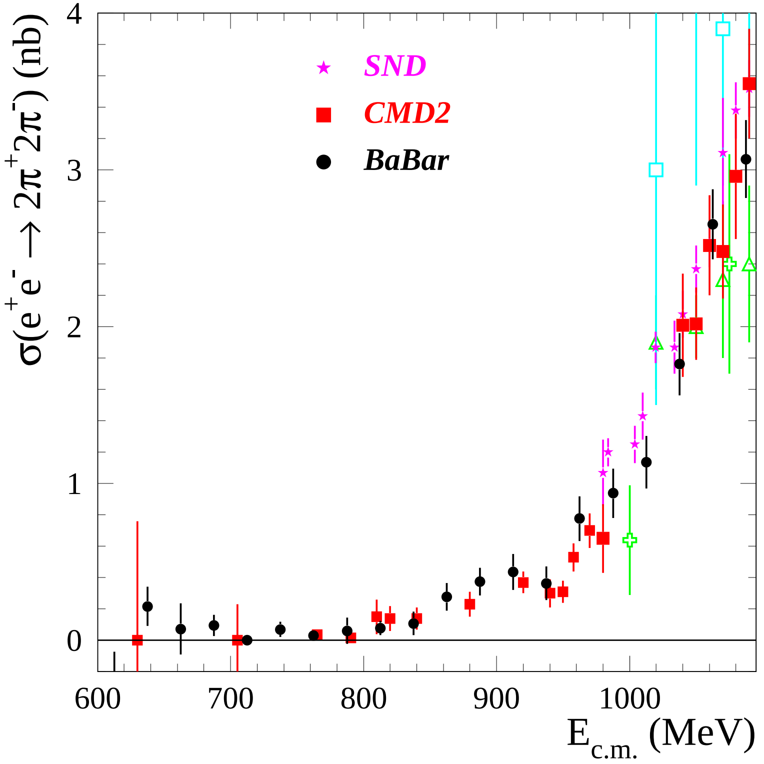
2π Cross Section from BaBar vs. CMD-2

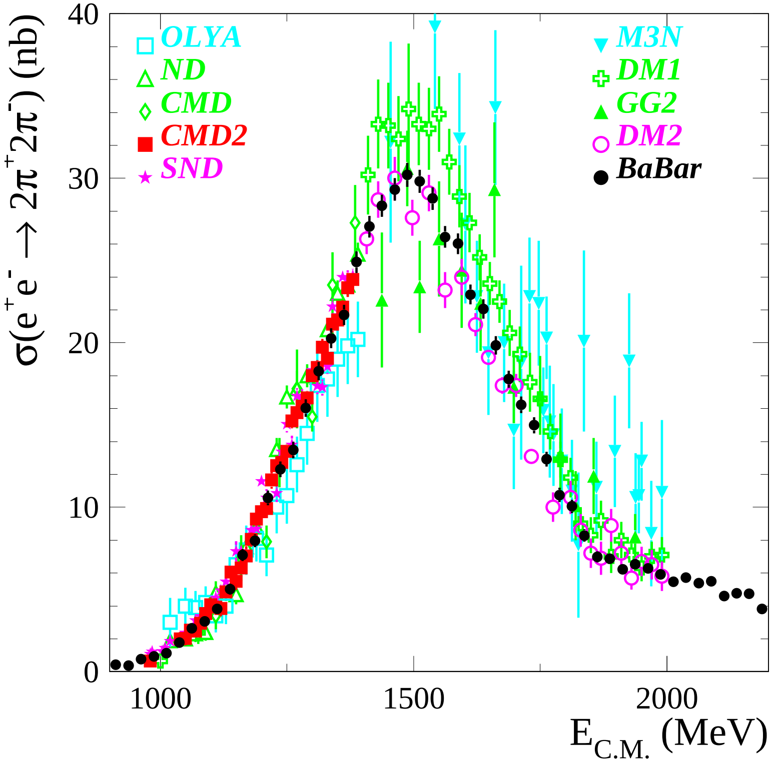


2π Cross Section from BaBar vs. SND



$e^+e^- \longrightarrow 2\pi^+2\pi^-$

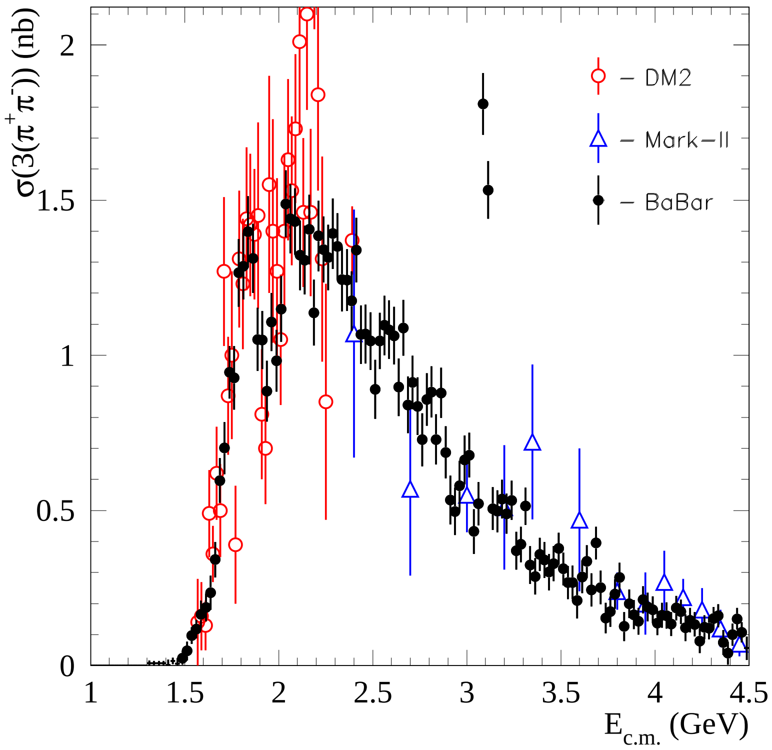


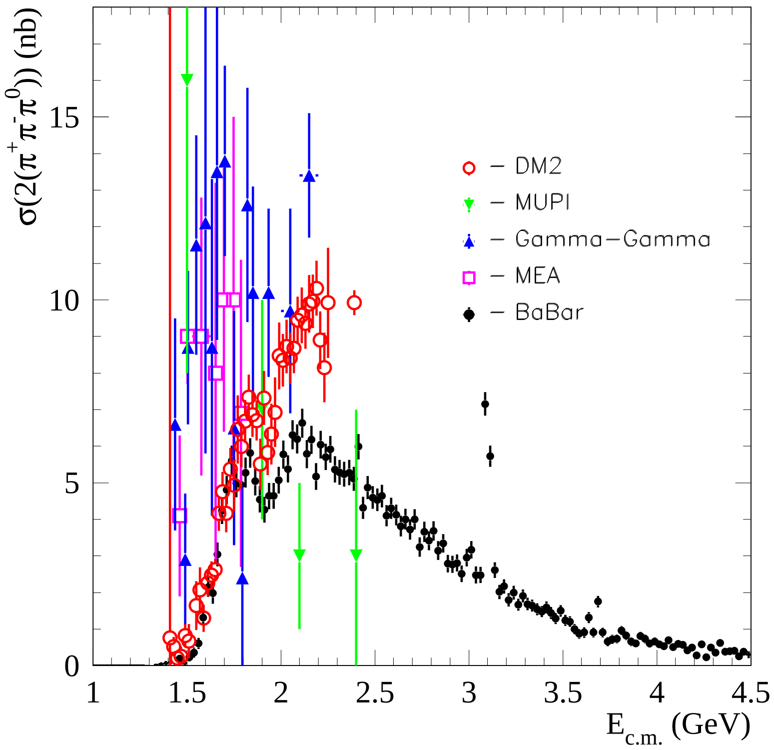


Good agreement with CMD-2/SND.

Error of continuum below 2 GeV improved.

$e^+e^- \longrightarrow 6\pi$





δa_μ changed from 0.10 ± 0.10 to 0.108 ± 0.016 and from 1.42 ± 0.30 to 0.890 ± 0.093

Significant improvement compared to the previous data!

Calculations of $a_\mu^{\mathrm{had,LO}}$

Authors	Year	$a_\mu^{\mathrm{had,LO}}$, 10^{-10}
C.Bouchiat, L.Michel	1961	$\simeq 648$
M.Gourdin, E. de Rafael	1969	650 ± 50
A.Bramon et al.	1972	680 ± 90
J.Calmet et al.	1977	702 ± 80
T.Kinoshita et al.	1985	707 ± 18
S.Eidelman, F.Jegerlehner	1995	702 ± 15
R.Alemany et al.	1998	701.1 ± 9.4
M.Davier, A.Höcker	1998	692.4 ± 6.2

Light-by-Light Scattering – II

- A. Pivovarov, 2001: $\sim 14.3 \cdot 10^{-10}$
  u, d, s with m_q, heavy quarks with standard masses
- J.F. de Trocóniz, F.J. Ynduráin, 2001: $(9.2 \pm 2.0) \cdot 10^{-10}$
  Constituent quarks + π^0
- A. Dorokhov, 2005: $(10.6 \pm 1.0) \cdot 10^{-10}$
  Instanton liquid model
- J. Erler, G. Toledo Sánchez,2006: $< 15.9 \cdot 10^{-10}$
  Parton model

Is there a way to consistently relate experimental data on $\gamma\gamma^* \to X^0$ with $J^{PC} = 0^{-+}, 0^{++}, 1^{++}, 2^{-+}, 2^{++}$ to the corresponding contributions to $a_\mu^{\rm had,LBL}$?

Theory vs Experiment – II

Calculation	$a_\mu^{\rm exp}, 10^{-10}$	$a_\mu^{\rm th}, 10^{-10}$	$a_\mu^{\rm exp} - a_\mu^{\rm th}, 10^{-10}$ (σ)
EJ, 1995	11659230 ± 84	11659210 ± 16	20 ± 86 $(--)$
DEHZ, 2003	11659203 ± 8	11659180.9 ± 8.0	22.1 ± 11.3 (1.9)
DEHZ, 2006	11659208.0 ± 6.3	11659180.3 ± 5.6	27.7 ± 8.4 (3.3)
HMNT, 2008	11659208.0 ± 6.3	11659180.4 ± 5.1	27.6 ± 8.1 (3.4)
J, 2008	11659208.0 ± 6.3	11659181.3 ± 7.2	26.7 ± 9.6 (2.8)

Explaining the Discrepancy: Errors or New Physics?

M.Passera, W.J.Marciano, A.Sirlin, 2008

- The lbl term is wrong ⇒ Should move it by 8-10σ
- If assume errors in $\sigma(s)$ and increase it, $\Delta\alpha^{(5)}_{\rm had}(M_Z)$ also increases ⇒ M_H^{UB} decreases restricting a narrow allowed region 114 GeV $< M_H <$ 154 GeV
- Using τ data also increases $\Delta\alpha^{(5)}_{\rm had}(M_Z)$ and leads to $M_H <$ 133 GeV
- To bridge Δa_μ, $\sigma(s)$ should be increased by 4% from threshold to infinity. Then $M_H <$ 70 GeV. If $\sigma(s)$ is increased locally, $M_H <$ 130 GeV.
- All scenarios look rather unlikely ⇒ New Physics?

Branchings of $\tau^- \to X^- \nu_\tau$ Decay,%

Hadronic State X	Experiment, 2002	CVC Prediction	$\mathcal{B}_{\rm exp} - \mathcal{B}_{\rm CVC}$
$\pi^-\pi^0$	25.31 ± 0.18	24.76 ± 0.25	0.55 ± 0.31
$\pi^- 3\pi^0$	1.08 ± 0.10	1.07 ± 0.05	0.01 ± 0.11
$2\pi^-\pi^+\pi^0$	4.19 ± 0.23	3.84 ± 0.17	0.35 ± 0.29
$\omega\pi^-$	1.94 ± 0.07	1.82 ± 0.07	0.12 ± 0.10
Total	31.59 ± 0.31	30.28 ± 0.34	1.31 ± 0.46

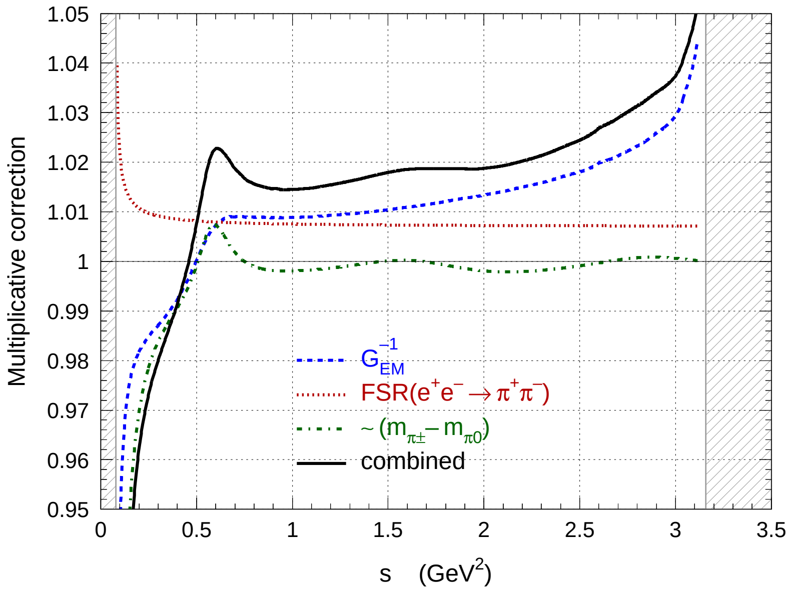
With more accurate data some deviations have been observed.

Corrections to the τ Spectral Functions

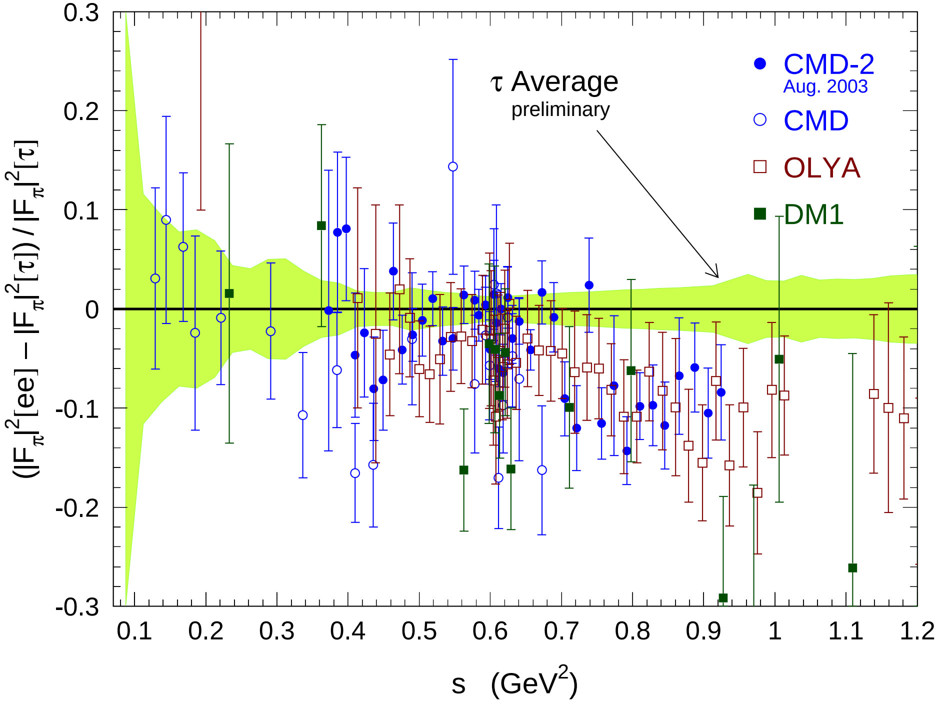
- $\mathrm{S_{EW}} = 1.0233 \pm 0.0006$
- Real photons, loops
- FSR
- $\mathrm{m}_{\pi^\pm} \neq \mathrm{m}_{\pi^0}$
  (phase space, Γ_ρ)
- $\mathrm{m}_{\rho^\pm} \neq \mathrm{m}_{\rho^0}$
- $\rho - \omega$ interference
- Radiative decays
  $(\pi\pi\gamma, \pi(\eta)\gamma, l^+l^-)$
- $\mathrm{m}_u \neq \mathrm{m}_d$
  and 2 class currents

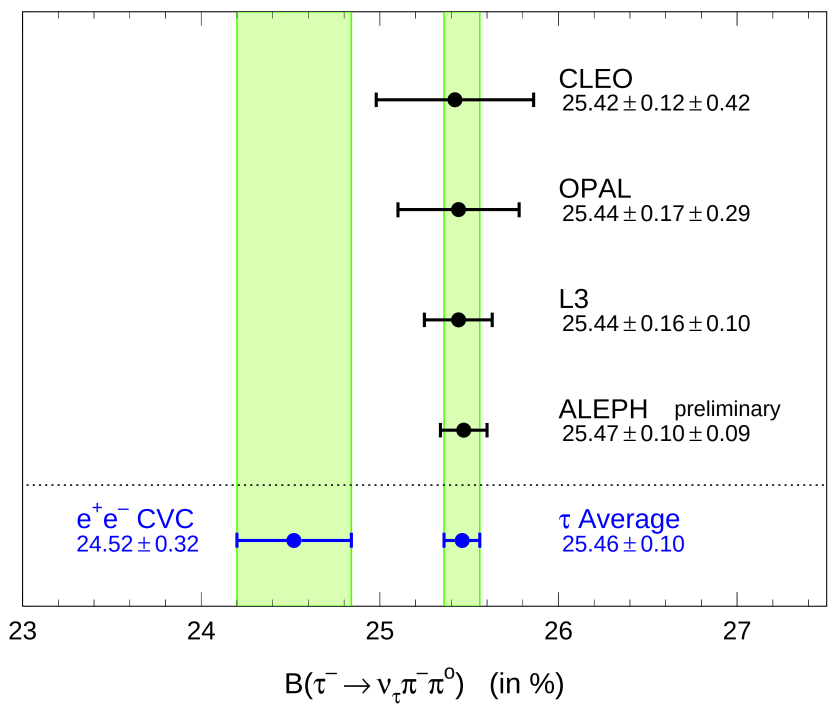
V. Cirigliano, G. Ecker, H. Neufeld, 2002

M. Davier, S. Eidelman, A. Höcker, Z. Zhang, 2002



CVC in the 2π Channel. e^+e^- vs. τ

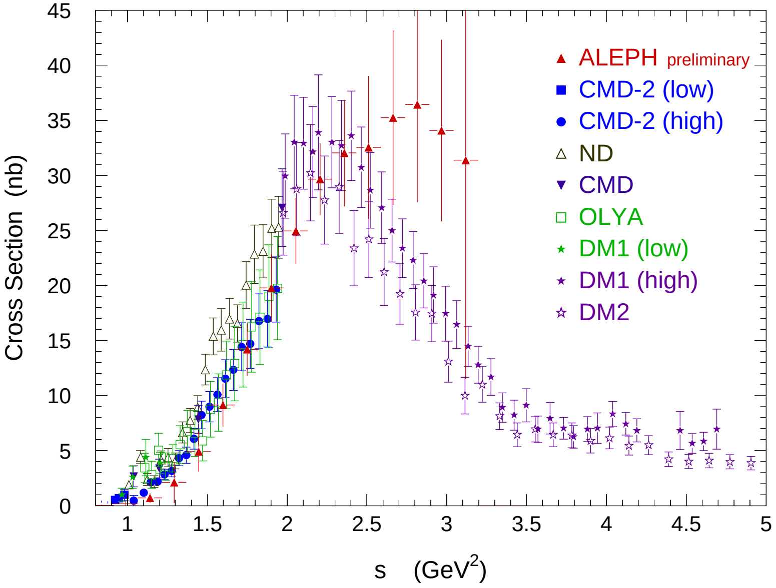




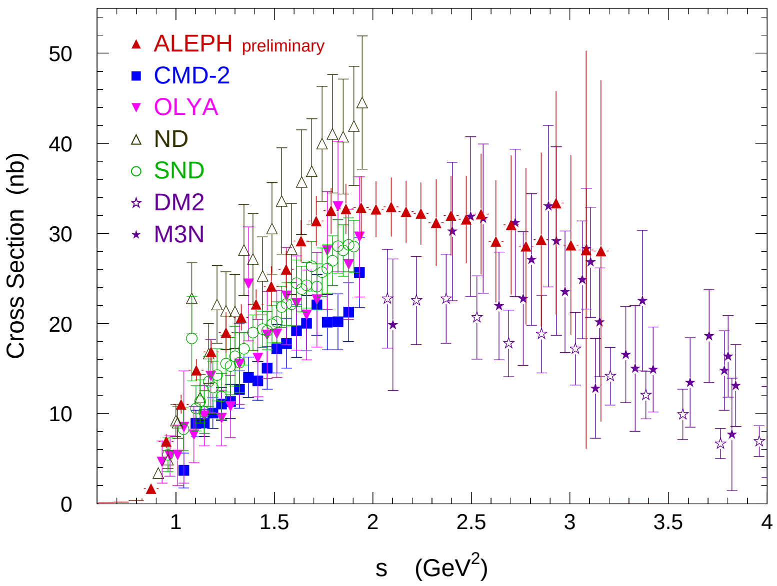
The branching from all groups is systematically higher than the CVC prediction: $\mathcal{B}_\tau - \mathcal{B}_{\rm ee} = (0.94 \pm 0.32)\%$!

CVC in the 4π Channel. e^+e^- vs. τ

$2\pi^+2\pi^-$



$\pi^+\pi^-2\pi^0$



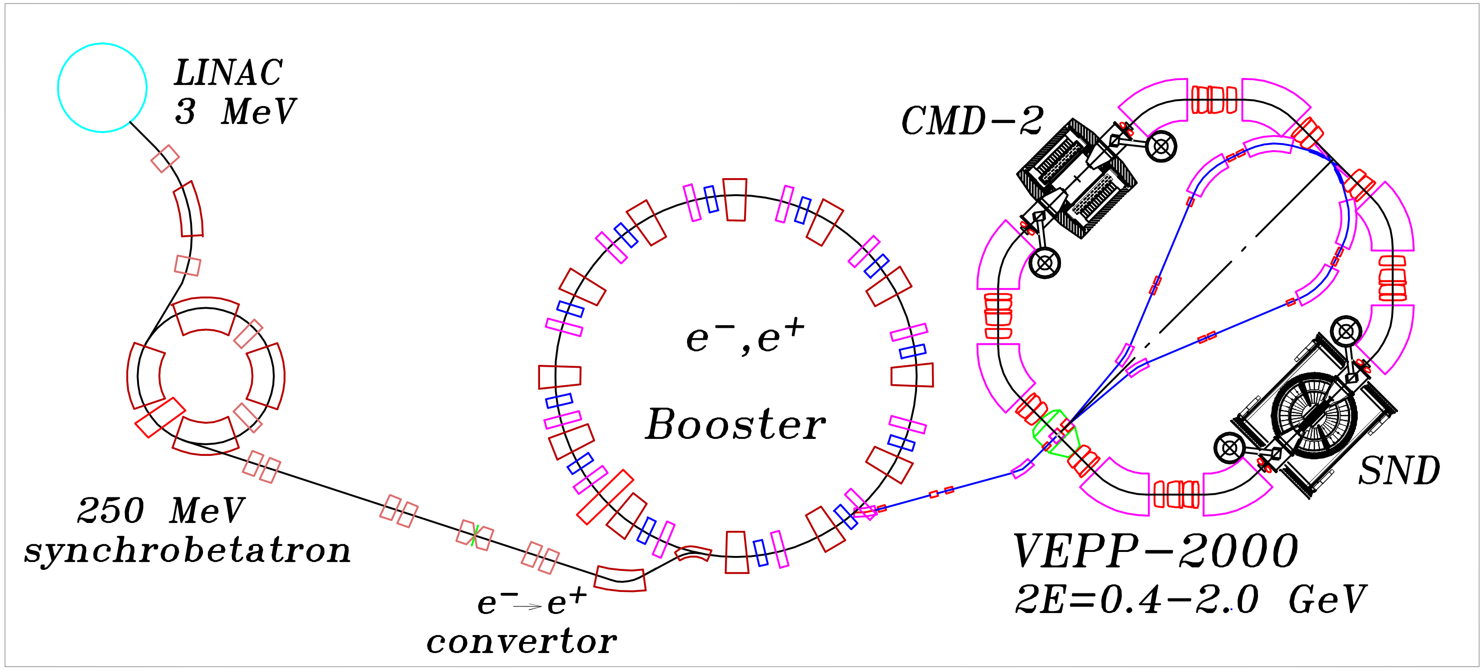
$\mathcal{B}(\tau), \%$	1.01 ± 0.08	4.54 ± 0.13
$\mathcal{B}(\text{CVC}), \%$	1.09 ± 0.08	3.63 ± 0.21
$\Delta\mathcal{B}, \%$	-0.08 ± 0.11	$+0.91 \pm 0.25$

Why are e^+e^- and τ Spectral Functions Different?

- Problems with data: underestimated systematics, normalization, rad. corr.
- Problems with SU(2) breaking corrections; Is ChPT reliable? The uncertainty of corrections may be large (K.Maltman, 2005)
- Non (V-A) contribution to e/w interactions (M.Chizhov, 2003) inspired by problems in $\pi^+ \to e^+\nu_{\rm e}\gamma$ (E.Frlez et al., 2003)
- Effect of charged Higgs propagator in τ decay
- $m_{\rho^\pm} > m_{\rho^0}$ by a few MeV (S.Ghozzi and F.Jegerlehner, 2003, M.Davier, 2003). Current experiments indicate equality within a few MeV.

VEPP-2000

Layout of the VEPP−2000 complex



The design luminosity $\mathcal{L} = 10^{32}$ cm^{-2}s^{-1}, with $\int Ldt \approx 1 - 2$ fb^{-1} during 3–5 years $\Delta a_\mu^{\rm had}/a_\mu^{\rm had}$ can be improved by a factor of 2!