Lepton Flavor Violation in Models with A_4 Flavour Symmetry

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based on
FHLM1 = F.F., Claudia Hagedorn, Yin Lin and Luca Merlo hep-ph/0807.3160
FHLM2 = F.F., Claudia Hagedorn, Yin Lin and Luca Merlo hep-ph/0808.0812
FHLM3 = F.F., Claudia Hagedorn, Yin Lin and Luca Merlo in preparation

Lepton mixing angles

[Fogli, talk at Neutrinos in Venice 2008]

$$\sin^2\vartheta_{13} < 3.2\times10^{-2} \qquad \vartheta_{13} < 10.3^0 \qquad \sin^2\vartheta_{13}^{TB} = 0$$

$$\sin^2\vartheta_{23} = 0.45\ {}^{+0.16}_{-0.09} \qquad \vartheta_{23} = \left(42.1^{+9.2}_{-5.3}\right)^0 \qquad \sin^2\vartheta_{23}^{TB} = \frac{1}{2}$$

$$\sin^2\vartheta_{12} = 0.326^{+0.05}_{-0.04} \qquad \vartheta_{12} = \left(34.8^{+3.0}_{-2.5}\right)^0 \qquad \sin^2\vartheta_{12}^{TB} = \frac{1}{3}$$

[2σ errors (95% C.L.)]

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tri-Bimaximal mixing

[Harrison, Perkins and Scott]

Mixing Pattern <-> Discrete Symmetries

in the basis where charged leptons are diagonal:

2-3 exchange symmetry (Z_2 generated by U)

[Fukuyama&Nishura 9702253
Ma&Raidal 0102255, Lam 0104166
Harrison&Scott 0210197]

$$m_\nu = \begin{pmatrix} x & y & y \\ y & w & z \\ y & z & w \end{pmatrix} \Leftrightarrow \begin{matrix} \vartheta_{13} = 0 \\ \vartheta_{23} = \frac{\pi}{4} \end{matrix}$$

$$U^T m_\nu U = m_\nu \qquad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$Z_2 x Z_2$ generated by two elements S and U

$$m_\nu = \begin{pmatrix} x & y & y \\ y & x+y-z & z \\ y & z & x+y-z \end{pmatrix} \Leftrightarrow \text{TB mixing}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$Z_2 x Z_2'$ generated by two elements S′ and U

$$m_\nu = \begin{pmatrix} x & y & y \\ y & x-z & z \\ y & z & x-z \end{pmatrix} \Leftrightarrow \begin{matrix} \vartheta_{13} = 0 \\ \vartheta_{23} = \frac{\pi}{4} \end{matrix} \quad \vartheta_{12} = \frac{\pi}{4}$$

to be corrected

$$S' = \frac{1}{2}\begin{pmatrix} 0 & -\sqrt{2} & -\sqrt{2} \\ -\sqrt{2} & 1 & -1 \\ -\sqrt{2} & -1 & 1 \end{pmatrix}$$

Bimaximal mixing

these are symmetries of ν mass matrix in the flavor basis

to keep charged leptons diagonal, extensions are needed

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{i\frac{2\pi}{3}}$$

$$T' = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -i & 0 \\ 0 & 0 & i \end{pmatrix}$$

$T^{+} (m_e^{+} m_e)\, T = (m_e^{+} m_e)$

↕ $(m_e^{+} m_e)$ diagonal

[same for T ']

- U and T generate S_3

  $[U^2 = T^3 = (UT)^2 = 1]$ → $\vartheta_{13} = 0 \quad \vartheta_{23} = \frac{\pi}{4}$

  [Grimus&Lavoura 0305046, 050415,...]

- S and T generate A_4

  $[S^2 = T^3 = (ST)^3 = 1]$ → TB mixing

  [Ma, Rajasekaran 2001; Ma 0409075,...]

  [S,U and T generate S_4]

- S' and T' generate S_4

  $[S'^2 = T'^4 = (S'T')^3 = 1]$ → Bimaximal mixing

  [Hagedorn, Lindner, Mohapatra 0602244 Ma 0508231,...]

  the full group should break down in two different directions for neutrinos and charged leptons (vacuum alignment)

a minimal model for TB mixing: A_4

[Altarelli & F. 0504165 & 0512103
Altarelli, F, Lin 0610165]

A_4 is the subgroup of SO(3) leaving a regular tetrahedron invariant

- mechanism to generate TB mixing from A_4

[He, Keum, Volkas 0601001
Lam 0708.3665 + 0804.2622]

$G_f = A_4 \times Z_3 \times U(1)_{FN}$

keeps separate ϕ_T and ϕ_S at the LO

explains why $m_e << m_\mu << m_\tau$

$\langle \varphi_T \rangle$ → Z_3 -> $(m_e^+ m_e)$ diagonal

$\langle \varphi_S \rangle, \ldots$ → Z_2 -> $U_{TB}^T m_\nu U_{TB} = (m_\nu)_{diag}$

[additional Z_2 can arise as an accidental symmetry]

- $$\frac{\langle \varphi_T \rangle}{\Lambda} \approx \frac{\langle \varphi_S \rangle}{\Lambda} \approx u \approx (0.005 \div 0.05)$$ [Λ = cutoff]

- TB at lowest order in u

$\vartheta_{13} = O(u)$ from subleading corrections

additional tests of flavor symmetries

evidence for lepton flavor conversion

direct	$\nu_e \to \nu_\mu, \nu_\tau$	sol
indirect	$\nu_\mu \to \nu_\tau$	atm

should show up in other processes if the scale of new physics $M \approx 1\ \mathrm{TeV}$

$$L_{eff} = i \frac{e}{M^2} e_i^c h_d \left(\sigma^{\mu\nu} F_{\mu\nu}\right) Z_{ij}^{dip}\, l_j + \frac{4\text{ - fermion}}{M^2} + \ldots$$

Z_{ij}^{dip} describes lepton EDM, MDM, $l_i \to l_j\, \gamma$

$$BR(\mu \to e\gamma) < 1.2 \times 10^{-11} \quad \to \quad Z_{\mu e}^{dip} < 10^{-8} \times \left[\frac{M(\mathrm{TeV})}{1\ \mathrm{TeV}}\right]^2$$

4-fermion:
μ->eee,...
μ->e in nuclei

if we insist on having M≈1TeV, what suppresses the rate?
[Antusch, King, Malinsky, Ross 0807.5047]

- A_4 provides a small symmetry breaking parameter: u ≈ 0.01 which is the degree of suppression predicted by A_4 in $Z^{dip}_{\mu e}$?

- introduce the normalized BR

$$R_{ij} = \frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)}$$

- if the underlying theory is symmetric under $G_f=A_4 \times \ldots$ all the new operators are invariant under G_f and the physical quantities are functions of the VEVs $u=\langle\varphi\rangle/\Lambda$

$$\left[Z^{dip}\right]_{ij} = z_{ij}^0 + z_{ij}^1 u + z_{ij}^2 u^2 + \ldots$$

the VEVs $\langle\varphi\rangle$ are the only source of SB and they control at the same time lepton masses and all operators depending on lepton fields.

→ list all G_f-invariant operators contributing to Z_{ij}

- Two different behaviors found in $G_f=A_4 \times Z_3 \times U(1)_{FN}$ [FHLM1+FHLM2]

- generic

$$R_{ij} = \frac{48\pi^3\alpha_{em}}{G_F^2 M^4}\left|w_{ij} u\right|^2$$

[w_{ij} are O(1) coefficients]

from μ->eγ

$$M > 30\left(\frac{u}{0.01}\right)^{1/2} \quad \text{TeV}$$

- special (some operators omitted from the full list) ⟷ [SUSY?]

$$R_{ij} = \frac{48\pi^3\alpha_{em}}{G_F^2 M^4}\left[\left|w_{ij}^{(1)} u^2\right|^2 + \frac{m_j^2}{m_i^2}\left|w_{ij}^{(2)} u\right|^2\right]$$

$$M > 3\left(\frac{u}{0.01}\right)^{1/2} \quad \text{TeV}$$

[M=(4π/g) x particle mass in a weaky interacting theory]

Explicit 1-loop computation in A_4-SUSY model + soft terms

- we assume that SUSY is broken at energies above Λ, via interactions constrained by $G_f=A_4 \times Z_3 \times U(1)_{FN}$
- soft terms originate from G_f-invariant operators via the spontaneous breaking of G_f

(preliminary) results

$$\hat{m}^2_{LL} = \begin{pmatrix} n & n_{12}\,u^2 & n_{13}\,u^2 \\ n_{12}\,u^2 & n & n_{23}\,u^2 \\ n_{13}\,u^2 & n_{23}\,u^2 & n \end{pmatrix} m^2_{SUSY} + \ldots$$

- non-universal soft-terms at the cutoff scale Λ
- the most general ones compatible with $G_f=A_4 \times Z_3 \times U(1)_{FN}$
- effects of running from Λ down to the weak scale can be absorbed in this parametrization

$$\hat{m}^2_{RR} = \begin{pmatrix} n^c_1 & 2c(n^c_1 - n^c_2)\frac{m_e}{m_\mu}u & 2c(n^c_1 - n^c_3)\frac{m_e}{m_\tau}u \\ 2c(n^c_1 - n^c_2)\frac{m_e}{m_\mu}u & n^c_2 & 2c(n^c_2 - n^c_3)\frac{m_\mu}{m_\tau}u \\ 2c(n^c_1 - n^c_3)\frac{m_e}{m_\tau}u & 2c(n^c_2 - n^c_3)\frac{m_\mu}{m_\tau}u & n^c_3 \end{pmatrix} m^2_{SUSY} + \ldots$$

$$\hat{m}^2_{RL} = \begin{pmatrix} n'_1 m_e & n'_{12} m_e u & n'_{13} m_e u \\ n'_{21} m_\mu u^2 & n'_2 m_\mu & n'_{23} m_\mu u \\ n'_{31} m_\tau u^2 & n'_{32} m_\tau\, u^2 & n'_3 m_\tau \end{pmatrix} m_{SUSY} - \mu \tan\beta\, \hat{m}_l + \ldots$$

mass insertions

$$(\delta_{ij})_{LL} \propto u^2 \qquad (\delta_{ij})_{RR} \propto \frac{m_j}{m_i} u$$

$$(\delta_{ij})_{LR} \propto \frac{m_i}{m_{SUSY}} u^2 \qquad (\delta_{ij})_{RL} \propto \frac{m_j}{m_{SUSY}} u$$

$$R_{ij} = \frac{48\pi^3\alpha_{em}}{G_F^2 M^4}\left[\left|w_{ij}^{(1)}u^2\right|^2 + \frac{m_j^2}{m_i^2}\left|w_{ij}^{(2)}u\right|^2\right] \qquad M \equiv \frac{4\pi}{g} m_{SUSY}$$

$w^{(1,2)}_{ij}$ are known O(1) functions of SUSY parameters

SUSY version of $G_f=A_4 \times Z_3 \times U(1)_{FN}$, with most general soft SUSY breaking terms gives rise to "special" behavior of BR($l_i \to l_j \gamma$)

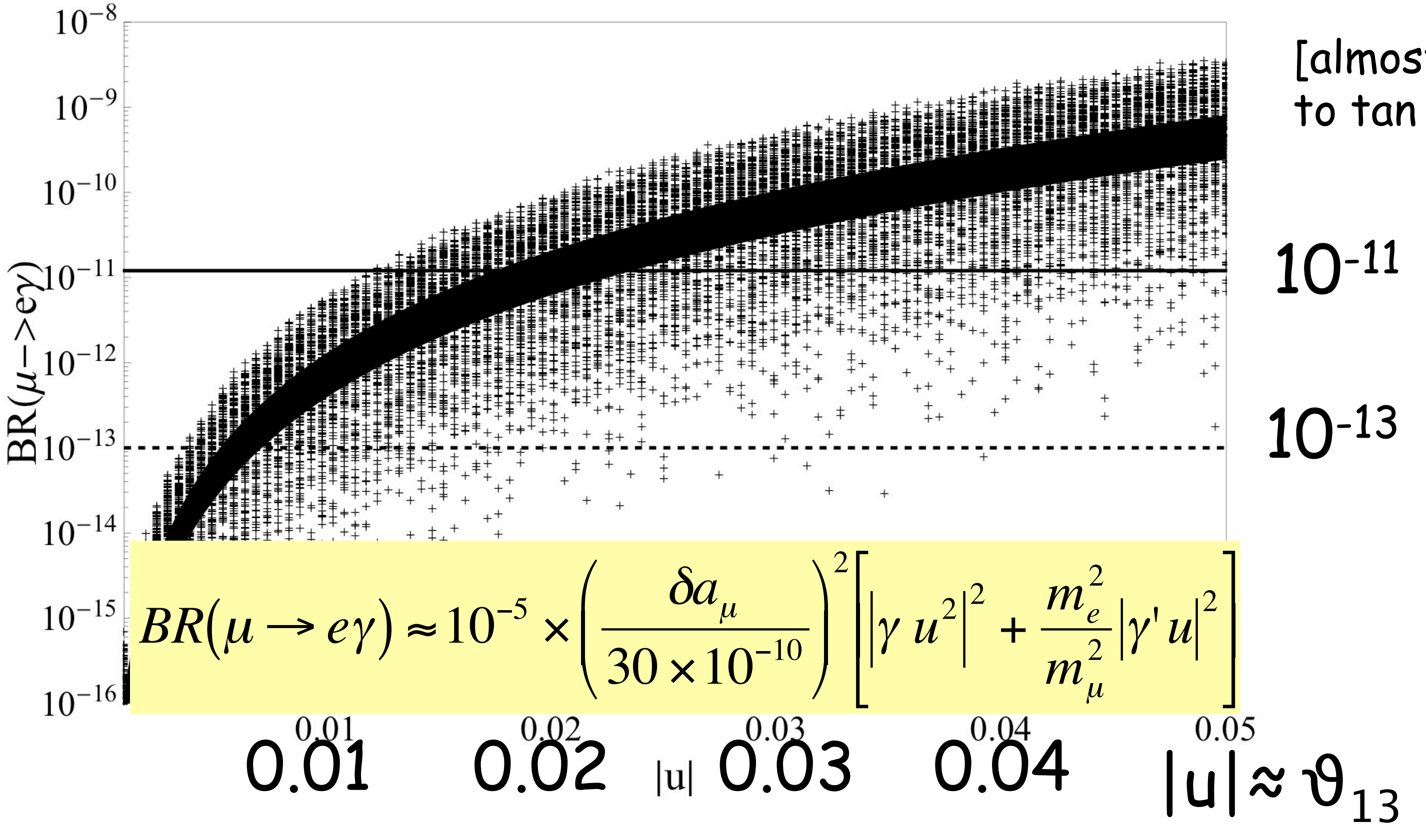
[FHLM3]

- up to O(1) coefficients $R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$ independently from $u \approx \vartheta_{13}$

- m_{SUSY} close to 100 GeV allowed for u≈0.01
  this range of slepton masses is relevant for a SUSY explanation of the discrepancy in muon (g-2)

$$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} \approx +300(80)\times 10^{-11}$$

$$\delta_{SUSY} a_\mu \approx \frac{g^2}{192\pi^2}\frac{m_\mu^2}{m_{SUSY}^2}\left[(5+\tan^2\vartheta_W)\tan\beta - \frac{1}{2}\left(1+7\tan^2\vartheta_W\right)\right]$$



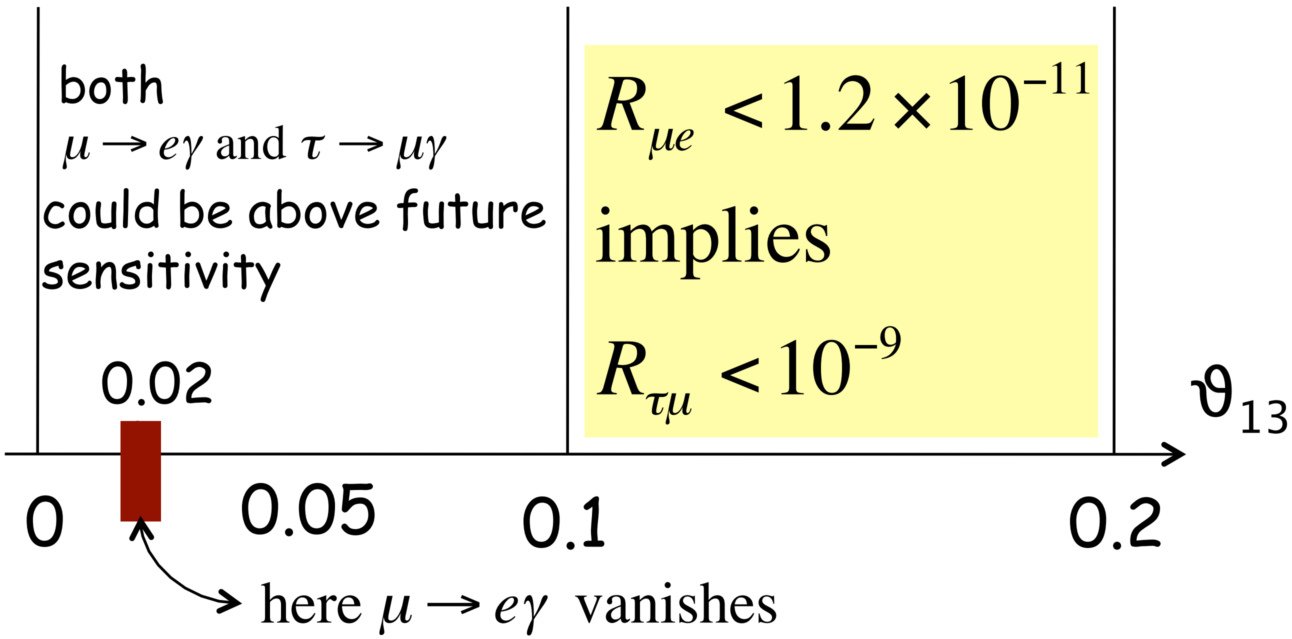
$$BR(\mu\to e\gamma)\approx 10^{-5}\times\left(\frac{\delta a_\mu}{30\times 10^{-10}}\right)^2\left[\left|\gamma\, u^2\right|^2+\frac{m_e^2}{m_\mu^2}\left|\gamma' u\right|^2\right]$$

[almost insensitive to tan β]

in MFV: $G_F = SU(3)_{e^c} \times SU(3)_l$ $\langle \varphi \rangle = y_e\ , z$ [those of dim-5 operator]

$$\left(\frac{R_{\mu e}}{R_{\tau\mu}}\right) \approx \left|\frac{2}{3} r \pm \sqrt{2}\sin\vartheta_{13} e^{i\delta}\right|^2 < 1 \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$$

[Cirigliano, Grinstein, Isidori, Wise 2005]



conclusion

- additional tests of A_4 models from LFV
  generic prediction

$$R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$$ independently from ϑ_{13} (cfr MFV)

$\tau \to \mu\gamma$ $\quad \tau \to e\gamma$ below expected future sensitivity

- in the generic, non-SUSY, case

$$R_{ij} = \frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)} \propto \left(\frac{u}{M^2} \right)^2$$

$0.005 < u$ requires M above 20 TeV

no match with M fitting $(g-2)_\mu$

- in the SUSY, case

$$R_{ij} = \frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)} \propto \left(\frac{u^2}{M^2} \right)^2$$

M can be much smaller, in the range of interest for $(g-2)_\mu$

O(1) coefficient

$$BR(\mu \to e\gamma) \approx 10^{-5} \times \left(\frac{\delta a_\mu}{30 \times 10^{-10}} \right)^2 [\gamma \vartheta_{13}]^4$$

other slides

$G_f = A_4 \times Z_3 \times U(1)_{FN}$

$$l = (3, \omega, 0)$$
$$e^c = (1, \omega^2, +2)$$
$$\mu^c = (1'', \omega^2, +1)$$
$$\tau^c = (1', \omega^2, 0)$$

can also be extended to the quark sector

[F, Hagedorn, Lin, Merlo 0702194,
Altarelli,F, Hagedorn 08020090]

$$\varphi \equiv \begin{cases} \varphi_T / \Lambda = (3,1,0) \\ \varphi_S / \Lambda = (3,\omega,0) \\ \xi / \Lambda = (1,\omega,0) \\ \vartheta / \Lambda = (1,1,-1) \end{cases}$$

symmetry breaking sector

vacuum alignment

$$\langle \varphi_T \rangle / \Lambda = (u,0,0) + O(u^2)$$
$$\langle \varphi_S \rangle / \Lambda \propto (u,u,u) + O(u^2)$$
$$\langle \xi \rangle / \Lambda \propto u + O(u^2)$$
$$\langle \vartheta \rangle / \Lambda \equiv t$$

tau Yukawa coupling < 4π

$$0.001 < u < \lambda^2$$
$$t \approx \lambda^2$$

corrections to TB mixing

$\lambda = 0.22$ Cabibbo angle

$$y_e(\langle \varphi \rangle) = \begin{pmatrix} c_e t^2 u & 0 & 0 \\ 0 & c_\mu t u & 0 \\ 0 & 0 & c_\tau u \end{pmatrix} + O(u^2)$$

$$\vartheta_{13} = O(u)$$

from subleading corrections

$\mathcal{M}_{ij}$ (i≠j) from two sources

- NLO corrections to ϕ_T $\langle \varphi_T \rangle / \Lambda = (u,0,0) + O(u^2)$
- double flavon insertions of the type $\xi^+ \varphi_S,\ \xi \varphi_S^+$ [other combinations vanish]

in a SUSY version of the model, with SUSY softly broken, a chirality flip requires an insertion of ϕ_T , at the LO in the SUSY breaking parameters. Example:

$$\int d^2\theta_{SUSY}\ e^c h_d \left(\frac{\varphi_T}{\Lambda} l \right) \qquad \int d^2\theta_{SUSY}\ e^c h_d \left(\frac{\varphi_T}{\Lambda} l \right) \theta^2_{SUSY}\ m_{SUSY}$$

other insertions can give rise to a chirality flip, but are suppressed by powers of (m_{SUSY}/Λ)

$$\frac{1}{\Lambda} \int d^2\theta_{SUSY} d^2\overline{\theta}_{SUSY}\ e^c h_d \left(\frac{\xi^+ \varphi_S}{\Lambda^2} l \right) \theta^2_{SUSY} \overline{\theta}^2_{SUSY}\ m^2_{SUSY}$$

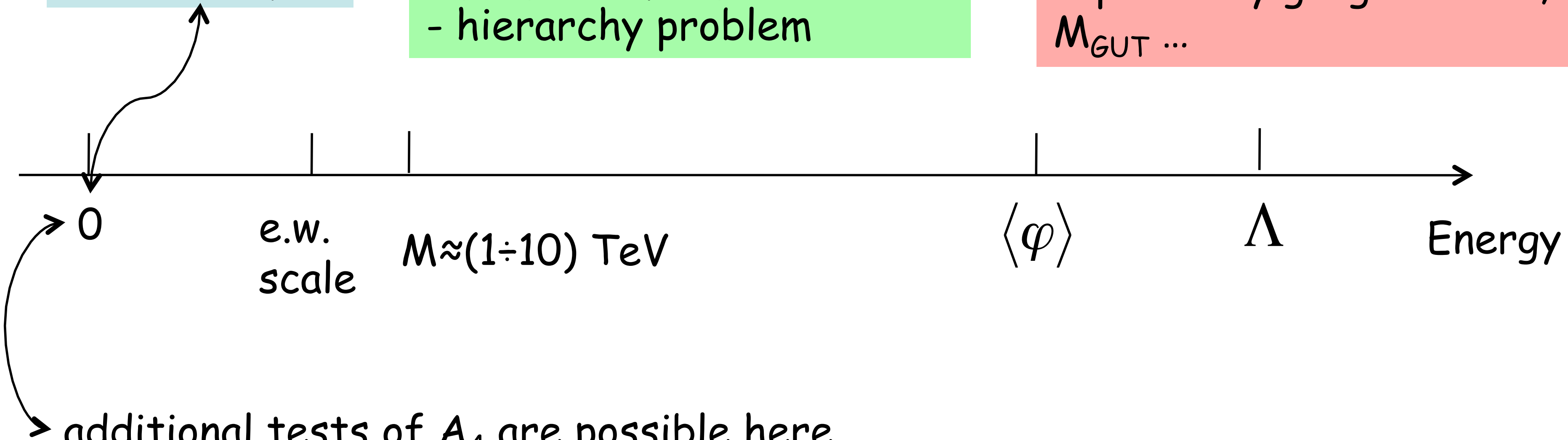
if the only sources of chirality flip are fermion and sfermion (LR) masses, then there is no contribution to $\mathcal{M}_{ij}$ (i≠j) from $\xi^+ \varphi_S,\ \xi \varphi_S^+$ [at LO in m_{SUSY}] and the main effect comes from ϕ_T alone
[we take this as a definition of SUSY case in the present context]

additional assumption:
there is new physics at a scale $M \approx (1 \div 10)$ TeV << $\langle\phi\rangle$ << Λ

ν masses
ν oscillations
$0\nu\beta\beta$ decay ?

- (g-2) discrepancy
- dark matter
- gauge coupling unification
- hierarchy problem

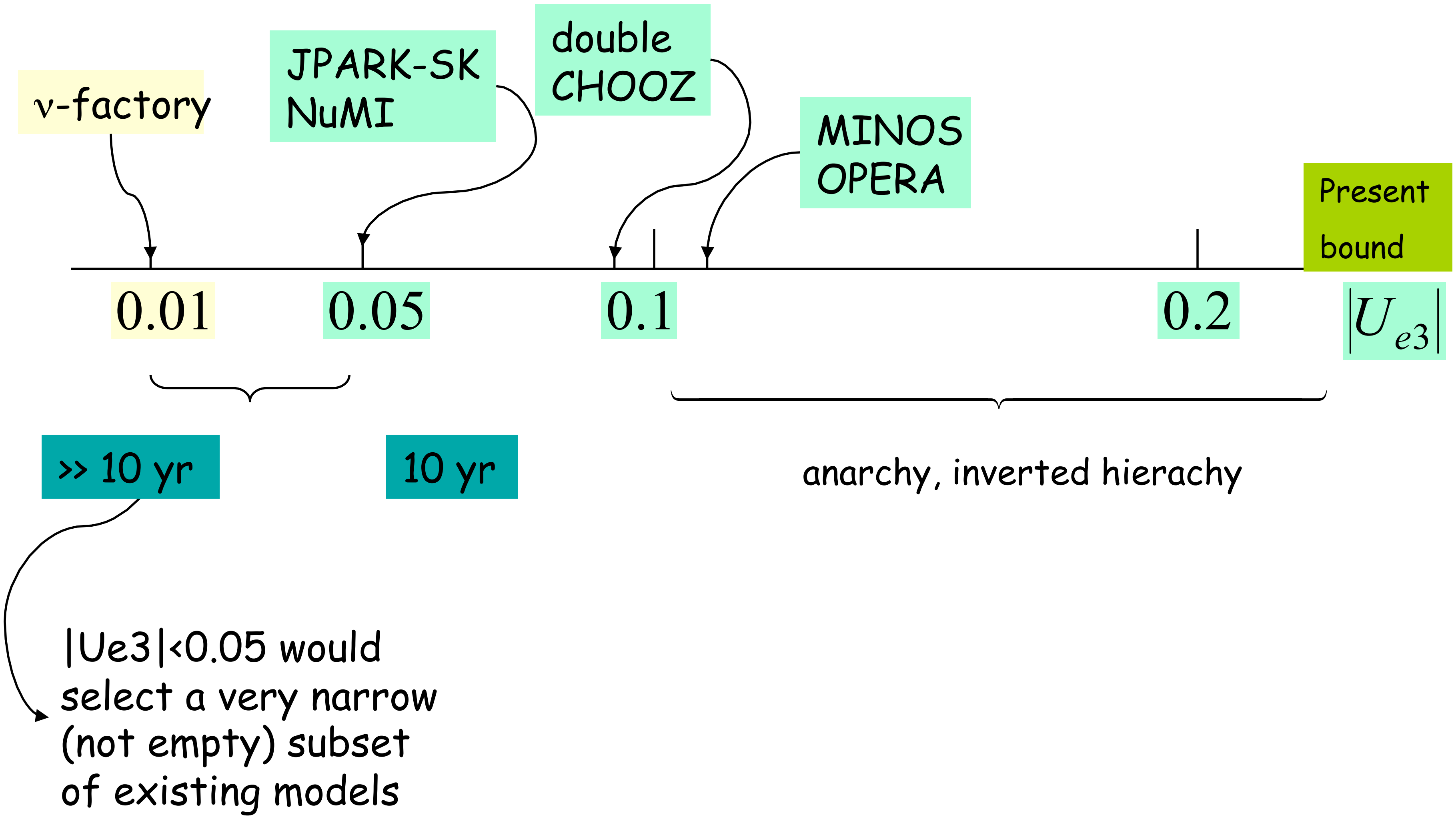
extended gauge symmetry?
additional d.o.f.
like ν^c,
superheavy gauge bosons,
M_{GUT} ...



additional tests of A_4 are possible here

the energy region close to M will be explored by LHC soon

- Most of plausible range for Ue3 explored in 10 yr from now



	current precision	future < 10 yr
Δm_{12}^2	$(8.0 \pm 0.3)\times 10^{-5}\ \mathrm{eV}^2\ \ [\approx 4\%]$	few percent [KamLAND]
$\lvert\Delta m_{23}^2\rvert$	$(2.5 \pm 0.3)\times 10^{-3}\ \mathrm{eV}^2\ \ [\approx 12\%]$	$0.15\times 10^{-3}\ \mathrm{eV}^2$ LBL conventional beams $0.05\times 10^{-3}\ \mathrm{eV}^2\ [\approx 2\%]$ superbeams
ϑ_{12}	$\tan^2\vartheta_{12} = 0.45^{+0.09}_{-0.08}$ $\vartheta_{12} = 33^0 \pm 2^0$	$\delta\tan^2\vartheta_{12} \approx 2\delta\sin^2\vartheta_{12}$ ν_e scattering rate of pp neutrinos to 1% down by about a factor 2: challenging
ϑ_{13}	$< 0.23\ (13^0)$ 90% C.L.	0.10 rad LBL, ChoozII 0.05 rad superbeams
ϑ_{23}	$\sin^2\vartheta_{23} = 0.52^{+0.07}_{-0.08}$ $\vartheta_{12} = 46^{0}{}^{+4^0}_{-5^0}$	$\delta\sin^2\vartheta_{23} \approx \delta\vartheta_{23}$ down by about a factor 2 superbeams
sign Δm_{23}^2	---	> 10 yr
δ	---	> 10 yr

A_4 and leptogenesis

[Jenkins and Manohar 2008]

Lepton asymmetry parameters ϵ_i from out-of equilibrium, CP and (B-L) violating decays of heavy right-handed neutrinos ν_i

$$\varepsilon_i \propto \sum_{j \neq i} \mathrm{Im}\left[\left(Y_\nu Y_\nu^+\right)_{ij}\right]^2 f\left(\frac{M_j^2}{M_i^2}\right)$$

see-saw realization of A_4 model

$$L = y(\nu^c l)h_u + x_a \xi(\nu^c \nu^c) + x_b(\varphi_S \nu^c \nu^c) + \ldots$$

ν^c triplet of A_4

both normal [nh] and inverted [ih] hierarchy are allowed

$$\left(Y_\nu Y_\nu^+\right) = |y|^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

real ⇒ $\epsilon_i = 0$ at the LO

$\epsilon_i \neq 0$ from the NLO corrections

$$\varepsilon_3 \approx \frac{u^2}{32\pi} \qquad \text{[nh]}$$

$$\varepsilon_3 \approx \frac{u^2}{32\pi r} \qquad \text{[ih]} \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{30}$$

$\epsilon_i \geq 10^{-6}$ to produce an acceptable baryon asymmetry

$$u \geq \begin{cases} 0.01 & \text{[nh]} \\ 0.002 & \text{[ih]} \end{cases}$$