QCD factorization beyond leading twist in exclusive processes: $\rho T$-meson production

Samuel Wallon

Laboratoire de Physique Théorique
Université Paris Sud
Orsay

The 2009 Europhysics Conference on High Energy Physics
Krakow, July 17th 2009

in collaboration with
I. V. Anikin (JINR, Dubna), D. Yu. Ivanov (SIM, Novossibirsk), B. Pire (CPhT, Palaiseau) and L. Szymanowski (SINS, Warsaw)
Since a decade, there have been much developments in hard exclusive processes.
- form factors, Distribution Amplitudes $\rightarrow$ Generalized Distribution Amplitudes
- DVCS $\rightarrow$ Generalized Parton Distributions, Transition Distribution Amplitudes
- The key tool is the collinear factorization
Introduction: Collinear factorization

Exclusive processes at high energy in QCD: extensions from DIS

- **DIS**: inclusive process $\rightarrow$ forward amplitude ($t = 0$)

  Structure Function
  
  $\equiv$ Coefficient Function (hard) $\otimes$ Parton Distribution Function (soft)

- **DVCS**: exclusive process $\rightarrow$ non forward amplitude ($-t \ll s = W^2$)

  Amplitude
  
  $\equiv$ Coefficient Function (hard) $\otimes$ Generalized Parton Distribution (soft)
Introduction: **Collinear factorization**

**Extensions from GPD**

- **Meson production:** $\gamma$ replaced by $\rho$, $\pi$, $\cdots$

\[
\text{Amplitude} = \text{GPD (soft)} \otimes \text{CF (hard)} \otimes \text{Distribution Amplitude (soft)}
\]

- **Crossed process:** $s \ll -t$

\[
\text{Amplitude} = \text{Coefficient Function (hard)} \otimes \text{Generalized Distribution Amplitude (soft)}
\]
starting from usual DVCS, one allows initial hadron $\neq$ final hadron example:

\[
\begin{align*}
\gamma^* & \quad \text{(Pert.)} & \gamma & \quad \text{(Pert.)} \\
Q^2 & \quad \text{hadron} & x & \quad \text{hadron} & x' & \quad \text{hadron} \\
& \quad \text{(GPD)} & & & & \\
t & \rightarrow & u \\
\end{align*}
\]

which can be further extended by replacing the outgoing $\gamma$ by any hadronic state

\[
\text{Amplitude} = \text{Transition Distribution Amplitude (soft)} \otimes \text{CF (hard)} \otimes \text{DA (soft)}
\]
Experimental tests are possible in **fixed target** experiments

- $e^\pm p, \mu^\pm p$: HERA (HERMES), JLab, COMPASS...

as well as in **colliders**, mainly for medium $s$

- $e^\pm p$ colliders: HERA (H1, ZEUS)
- $e^+e^-$ colliders: LEP, Belle, BaBar, BEPC

**Collinear factorization** has been proven only for specific cases:

- e.g.: $\rho_T$ production cannot directly be factorized (appearance of end point singularities)

$\Rightarrow$ improvement needed for a consistent approach of exclusive processes
At the same time, at large $s$, the interest for phenomenological tests of hard Pomeron and related resummed approaches has become pretty wide:

- **inclusive** tests (total cross-section) and semi-inclusive tests (diffraction, forward jets, ...)
- **exclusive** tests (meson production)

These tests concern all type of collider experiments:

- $e^\pm p$: HERA: (H1, ZEUS)
- $p\bar{p}$ and $pp$: TEVATRON (CDF, D0); LHC (CMS, ATLAS, ALICE)
- $e^+e^-$: (LEP, ILC)

These high energy exclusive processes in the perturbative Regge limit may provide new ideas when dealing with collinear factorization
Our studies attempt to describe exclusive processes involving the production of ρ-mesons in diffraction-type experiment. We choose $t = t_{min}$ for simplicity.

- $\gamma^*(q) + \gamma^*(q') \to \rho_T(p_1) + \rho(p_2)$ process in $e^+ e^- \to e^+ e^- \rho_T(p_1) + \rho(p_2)$ with double tagged lepton at ILC
- $\gamma^*(q) + P \to \rho_T(p_1) + P$ at HERA

This process was studied by H1 and ZEUS

- the total cross-section strongly decreases with $Q^2$
- dramatic increase with $W^2 = s \gamma^* P$
  (transition from soft to hard regime governed by $Q^2$)

(from X. Janssen (H1), DIS 2008)
Polarization effects in $\gamma^* P \rightarrow \rho P$ at HERA

- one can experimentally measure all spin density matrix elements
- at $t = t_{min}$ one can experimentally distinguish
  \[
  \begin{cases}
  \gamma_L^* \rightarrow \rho_L : & \text{dominates (twist 2 dominance)} \\
  \gamma_T^* \rightarrow \rho_T : & \text{sizable (twist 3)}
  \end{cases}
  \]

- S-channel helicity conservation:
  \[
  \begin{cases}
  \gamma_L^* \rightarrow \rho_L & (\equiv T_{00}) \\
  \gamma_T^* \rightarrow \rho_T,
  \end{cases}
  \]
  Dominate with respect to all other transitions.
  Experimentally, $\gamma_T^* \rightarrow \rho_T$ is dominated by $\gamma_T^*(-) \rightarrow \rho_T(-)$ and $\gamma_T^*(+) \rightarrow \rho_T(+) (\equiv T_{11})$

(from X. Janssen (H1), DIS 2008)
The processes with vector particle such as $\rho$—meson probe deeper into the fine features of QCD.
It deserves theoretical develeppement to describe HERA data in its special kinematical range:

- large $s_{\gamma^*P} \Rightarrow$ small-$x$ effects expected, within $k_t$-factorization
- large $Q^2 \Rightarrow$ hard scale $\Rightarrow$ perturbative approach and collinear factorization
  $\Rightarrow$ the $\rho$ can be described through its chiral even Distribution Amplitudes

\[
\begin{cases}
\rho_L & \text{twist 2} \\
\rho_T & \text{twist 3}
\end{cases}
\]

The main ingredient is the $\gamma^* \rightarrow \rho$ impact factor

- For $\rho_T$, special care is needed: a pure 2-body description would violate gauge invariance.
- We show that:
  - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
  - Our treatment is free of end-point singularities and does not violates the QCD factorization
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in $t$ channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.
\( \gamma^* \gamma^* \rightarrow \rho \rho \) as an example

- Use Sudakov decomposition \( k = \alpha p_1 + \beta p_2 + k_\perp \) \((p_1^2 = p_2^2 = 0, \ 2p_1 \cdot p_2 = s)\)
- write \( d^4k = \frac{s}{2} d\alpha d\beta d^2k_\perp \)
- \( t \)-channel gluons with non-sense polarizations \((\epsilon_{NS}^{up} = \frac{2}{s} p_2, \ \epsilon_{NS}^{down} = \frac{2}{s} p_1)\) dominate at large \( s \)

\[ \begin{align*}
\gamma^*(q_1) & \quad \alpha \ll \alpha_{\text{quarks}} \quad \rho(p_1 + r_\perp) \quad \Rightarrow \text{set } \alpha = 0 \text{ and } \int d\beta \\
\gamma^*(q_2) & \quad \beta \ll \beta_{\text{quarks}} \quad \rho(p_2 - r_\perp) \quad \Rightarrow \text{set } \beta = 0 \text{ and } \int d\alpha \\
\end{align*} \]

(illustration for 2-body case)
Impact factor for exclusive processes

$k_T$ factorization

\[ \mathcal{M} = i s \int \frac{d^2 k}{(2\pi)^2 k^2 (r - k)^2} \Phi^{\gamma^* (q_1) \to \rho (p_1^\rho)} (k, r - k) \Phi^{\gamma^* (q_2) \to \rho (p_2^\rho)} (-k, -r + k) \]

The $\gamma^*_{L,T} (q) g(k_1) \to \rho_{L,T} g(k_2)$ impact factor is normalized as

\[ \Phi^{\gamma^* \to \rho (k^2)} = e^{\gamma^*_\mu} \frac{1}{2 s} \int \frac{d\kappa}{2\pi} \text{Disc}_{\kappa} \mathcal{S}^{\gamma^*_\mu \to \rho} \mu (k^2), \]

with $\kappa = (q + k)^2 = \beta s - Q^2 - k^2$
Gauge invariance

- **QCD gauge invariance** (probes are colorless)
  \[ \Rightarrow \text{impact factor should vanish when } k \to 0 \text{ or } r - k \to 0 \]

- In the following we will restrict ourself to the case \( t = t_{\text{min}} \), i.e. to \( r = 0 \)

\[
\begin{align*}
q & \quad \Phi \\
\quad k_1 & = k \\
\quad k_2 & \\
\rho & \\
\end{align*}
\]

\[
k_1 = \frac{k + Q^2 + k^2}{s} p_2 + k_\perp
\]

\[
k_2 = \frac{k + k^2}{s} p_2 + k_\perp,
\]

\[
k_1^2 = k_2^2 = -k^2
\]

This kinematics takes into account skewedness effects along \( p_2 \)

\[
\begin{cases}
0 & \Rightarrow 0 \quad \text{(twist 2)} \\
(+) \text{ or } (-) & \Rightarrow (+) \text{ or } (-) \quad \text{(twist 3)}
\end{cases}
\]

- At twist 3 level (for \( \gamma^* \to \rho_T \) transition), gauge invariance is a non trivial statement which requires 2 and 3 body correlators
Collinear factorization
Light-Cone Collinear approach

- The impact factor can be written as

\[ \Phi = \int d^4 l \cdots \text{tr}[H(l \cdots) \quad S(l \cdots)] \]

  hard part           soft part

- At the 2-body level:

\[ S_{q\bar{q}}(l) = \int d^4 z \ e^{-i l \cdot z} \langle \rho(p) | \psi(0) \bar{\psi}(z) | 0 \rangle, \]

- \( H \) and \( S \) are related by \( \int d^4 l \) and by the summation over spinor indices
Collinear factorization
Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization (1)

- Use Sudakov decomposition in the form \( p = p_1, \ n = 2p_2/s \Rightarrow p \cdot n = 1 \)

\[
l_\mu = yp_\mu + l_\mu \perp + (l \cdot p) n_\mu, \quad y = l \cdot n
\]

scaling: \( 1 \quad 1/Q \quad 1/Q^2 \)

- decompose \( H(k) \) around the \( p \) direction:

\[
H(l) = H(yp) + \left. \frac{\partial H(l)}{\partial l_\alpha} \right|_{l=yp} (l - yp)_\alpha + \ldots \quad \text{with} \quad (l - yp)_\alpha \approx l_\alpha \perp
\]

- In Fourier space, the twist 3 term \( l_\alpha \perp \) turns into a derivative of the soft term

\[
\Rightarrow \text{one will deal with } \int d^4z \ e^{-il \cdot z} \langle \rho(p)|\psi(0) i \partial_{\alpha \perp} \bar{\psi}(z)|0\rangle
\]
1 - Momentum factorization (2)

- write

\[ d^4l \longrightarrow d^4l \, \delta(y - l \cdot n) \, dy \]

- \( \int d^4l \, \delta(y - l \cdot n) \) is then absorbed in the soft term:

\[
(\tilde{S}_{q\bar{q}}, \partial_\perp \tilde{S}_{q\bar{q}}) \quad \equiv \quad \int d^4l \, \delta(y - l \cdot n) \int d^4z \, e^{-il \cdot z} \langle \rho(p)|\psi(0)\rangle \langle 1, i \, \partial_\perp \rangle \bar{\psi}(z)|0\rangle
\]

\[
(\delta(y - l \cdot n) = \int \frac{d\lambda}{2\pi} e^{-i\lambda(y \cdot n)} \Rightarrow) \quad = \quad \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \int d^4z \, \delta^{(4)}(z - \lambda n) \langle \rho(p)|\psi(0)\rangle \langle 1, i \, \partial_\perp \rangle \bar{\psi}(z)|0\rangle
\]

\[
= \quad \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p)|\psi(0)\rangle \langle 1, i \, \partial_\perp \rangle \bar{\psi}(\lambda n)|0\rangle
\]

- \( \int dy \) performs the longitudinal momentum factorization
2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0) \bar{\psi}(z)$ and $\psi(0) i \\partial_\perp \bar{\psi}(z)$:

- $\Phi$ has now the simple factorized form:

$$\Phi = \int dx \left\{ \text{tr} [H_{qq}(x p) \Gamma] S_{qq}^\Gamma(x) + \text{tr} [\partial_\perp H_{qq}(x p) \Gamma] \partial_\perp S_{qq}^\Gamma(x) \right\}$$

$$\Gamma = \gamma^\mu \text{ and } \gamma^\mu \gamma^5 \text{ matrices}$$

$$S_{qq}^\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle \rho(p)|\bar{\psi}(\lambda n) \Gamma \psi(0)|0\rangle$$

$$\partial_\perp S_{qq}^\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle \rho(p)|\bar{\psi}(\lambda n) \Gamma i \quad \partial_\perp \psi(0)|0\rangle$$

- choose axial gauge condition for gluons, i.e. $n \cdot A = 0 \Rightarrow$ no Wilson line
Collinear factorization
Light-Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  ⇒ no need for Taylor expansion

- Momentum factorization goes in the same way as for 2-body case

- Spinorial (and color) factorization is similar:
Collinear factorization
Parametrization of vacuum–to–rho-meson matrix elements (DAs): 2-body correlators

2-body non-local correlators

- **vector correlator**

\[ \langle \rho(p) | \bar{\psi}(z) \gamma_\mu \psi(0) | 0 \rangle \equiv m_\rho f_\rho \left[ \varphi_1(y) (e^* \cdot n) p_\mu + \varphi_3(y) e^{*T}_\mu \right] \]

- **axial correlator**

\[ \langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(0) | 0 \rangle \equiv m_\rho f_\rho i \varphi_A(y) \varepsilon_{\mu \lambda \beta \delta} e^{*T}_\lambda p_\beta n_\delta \]

- **vector correlator with transverse derivative**

\[ \langle \rho(p) | \bar{\psi}(z) \gamma_\mu i \partial_\alpha \psi(0) | 0 \rangle \equiv m_\rho f_\rho \varphi^{T}_1(y) p_\mu e^{*T}_\alpha \]

- **axial correlator with transverse derivative**

\[ \langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu i \partial_\alpha \psi(0) | 0 \rangle \equiv m_\rho f_\rho i \varphi^{T}_A(y) p_\mu \varepsilon_{\alpha \lambda \beta \delta} e^{*T}_\lambda p_\beta n_\delta, \]

where \( y (\bar{y} \equiv 1 - y) = \) momentum fraction along \( p \equiv p_1 \) of the quark (antiquark) and

\[ \mathcal{F} \equiv \int_0^1 dy \exp [i y p \cdot z], \text{ with } z = \lambda n \]

\[ \Rightarrow 5 \text{ 2-body DAs} \]
3-body non-local correlators

• vector correlator

\[ \langle \rho(p) | \bar{\psi}(z_1) \gamma_\mu g A^T_\alpha (z_2) \psi(0) | 0 \rangle \equiv F_2 \ \rho \ f_3^V \ B(y_1, y_2) \ p_\mu \ e^{*T}_\alpha, \]

• axial correlator

\[ \langle \rho(p) | \bar{\psi}(z_1) \gamma_5 \gamma_\mu g A^T_\alpha (z_2) \psi(0) | 0 \rangle \equiv m_\rho \ f_3^A \ i \ D(y_1, y_2) \ p_\mu \ \epsilon_{ \alpha \beta \delta} \ e^{*T}_\lambda \ p_\beta \ n_\delta, \]

where \( y_1, \bar{y}_2, y_2 - y_1 = \) quark, antiquark, gluon momentum fraction

and \( F_2 \ \frac{1}{0} \int dy_1 \ \frac{1}{0} \int dy_2 \ \exp [i \ y_1 \ p \cdot z_1 + i(y_2 - y_1) \ p \cdot z_2], \) with \( z_{1,2} = \lambda n \)

\[ \Rightarrow 2 \ 3\text{-body DAs} \]
From C-conjugation on the previous correlators, one gets:

**2-body correlators:**

\[
\begin{align*}
\varphi_1(y) & = \varphi_1(1-y) \\
\varphi_3(y) & = \varphi_3(1-y) \\
\varphi_A(y) & = -\varphi_A(1-y) \\
\varphi_T^1(y) & = -\varphi_T^1(1-y) \\
\varphi_T^A(y) & = \varphi_T^A(1-y)
\end{align*}
\]

**3-body correlators:**

\[
\begin{align*}
B(y_1, y_2) & = -B(1-y_2, 1-y_1) \\
D(y_1, y_2) & = D(1-y_2, 1-y_1)
\end{align*}
\]
Equations of motion

- **Dirac equation leads to**

  \[ \langle i(\not{p} (0)\psi(0))_\alpha \bar{\psi}_\beta(z) \rangle = 0 \quad (i \not{D}_\mu = i \not{\partial}_\mu + g A_\mu) \]

- **Apply the Fierz decomposition to the above 2 and 3-body correlators**

  \[ - \langle \psi(x) \bar{\psi}(z) \rangle = \frac{1}{4} \langle \bar{\psi}(z) \gamma_\mu \psi(x) \rangle \gamma_\mu + \frac{1}{4} \langle \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(x) \rangle \gamma_\mu \gamma_5. \]

  \[ \Rightarrow \text{2 Equations of motion:} \]

  \[ \bar{y}_1 \varphi_3(y_1) + \bar{y}_1 \varphi_A(y_1) + \varphi_T^1(y_1) + \varphi_A^T(y_1) \]

  \[ + \int dy_2 \left[ \xi_3^V B(y_1, y_2) + \xi_3^A D(y_1, y_2) \right] = 0 \quad \text{and} \quad (\bar{y}_1 \leftrightarrow y_1) \]

- **In WW approximation: genuine twist 3 = 0 i.e. \( B = D = 0 \)**

  \[ \begin{align*}
  \varphi_A^T(y) &= \frac{1}{2} [(y - \bar{y}) \varphi_A^{WW}(y) - \varphi_A^{WW}(y)] \\
  \varphi_1^T(y) &= \frac{1}{2} [(y - \bar{y}) \varphi_3^{WW}(y) - \varphi_A^{WW}(y)]
  \end{align*} \]
A minimal set of DAs

- The non-perturbative correlators cannot be obtained from perturbative QCD (!)
- one should reduce them to a minimal set before using any model
- this can be achieved by using an additional condition:
  independency of the full amplitude with respect to the light-cone direction \( n \)

\[ \Rightarrow \text{we prove that 3 independent Distribution Amplitudes are needed:} \]

\[ \phi_1(y) \leftarrow 2 \text{ body twist 2 correlator} \]

\[ B(y_1, y_2) \leftarrow 3 \text{ body genuine twist 3 vector correlator} \]

\[ D(y_1, y_2) \leftarrow 3 \text{ body genuine twist 3 axial correlator} \]
$n$–independence in practice

- $\rho_T$ polarization: $e^*_\mu^T = e^*_\mu - p_\mu \cdot e^* \cdot n$ keeping $n \cdot p = 1$

- for the full factorized amplitude:
  
  $$A = H \otimes S \quad \frac{dA}{dn^\mu} = 0,$$

  where 
  $$\frac{d}{dn^\mu} = \frac{\partial}{\partial n^\mu} + e^*_\mu \frac{\partial}{\partial (e^* \cdot n)}$$

- rewrite hard terms in one single form, of 2-body type: use Ward identities

  Example: hard 3-body $\rightarrow$ hard 2-body

  $$\text{tr} \left[ H_3 \rho (y_1, y_2) p^\rho \right] B(y_1, y_2) = \frac{1}{y_1 - y_2} \left( \text{tr} \left[ H_2 (y_1) \right] - \text{tr} \left[ H_2 (y_2) \right] \right) B(y_1, y_2),$$

- thus, symbolically,

  $$\frac{dS}{dn^\mu} = 0$$
Constraints from \( n \)-independence

- **vector correlators**
  
  \[
  \frac{d}{dy_1} \varphi_1^T(y_1) = -\varphi_1(y_1) + \varphi_3(y_1)
  \]

  \[
  -\zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1} \left( B(y_1, y_2) + B(y_2, y_1) \right)
  \]

- **axial correlators**
  
  \[
  \frac{d}{dy_1} \varphi_A^T(y_1) = \varphi_A(y_1) - \zeta_3^A \int_0^1 \frac{dy_2}{y_2 - y_1} \left( D(y_1, y_2) + D(y_2, y_1) \right)
  \]

- twist 2
  - kinematical twist 3 (WW)
  - genuine twist 3
  - genuine + kinematical twist 3
Collinear factorization
A set of independent non-perturbative correlators

Solution

- the set of 4 equations (2 EOM + 2 $n$-independence relations) can be solved analytically
- $7 \rightarrow 3$ independent DAs

Twist 2
- kinematical twist 3 ($WW$)
- genuine twist 3
- genuine + kinematical twist 3
2-body diagrams

- without derivative

- practical trick for computing $\partial_\perp H$: use the Ward identity

\[
\frac{\partial}{p_\mu} \cdot p = p \cdot \gamma^\mu \cdot p
\]

where

\[
p = \frac{1}{m - p - i\epsilon}
\]
3-body diagrams

- "abelian" type

- "non-abelian" type
Recall: $\gamma^*_L \rightarrow \rho_L$ impact factor

$$\Phi_{\gamma^*_L \rightarrow \rho_L}(k^2) = \frac{2e g^2 f_\rho}{Q} \frac{\delta^{ab}}{2 N_c} \int dy \varphi_1(y) \frac{k^2}{y \bar{y} Q^2 + k^2}$$

pure twist 2 scaling (from $\rho$-factorization point of view)
Computation and results
Results: $\gamma_T^* \rightarrow \rho_T$ impact factor

$\gamma_T^* \rightarrow \rho_T$ impact factor:

Spin Non-Flip/Flip separation appears

$$
\Phi_{\gamma_T^* \rightarrow \rho_T}(k^2) = \Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T}(k^2) T_{n.f.} + \Phi_{f.}^{\gamma_T^* \rightarrow \rho_T}(k^2) T_f.
$$

where

$$
T_{n.f.} = -(e_\gamma \cdot e^*) \quad \text{and} \quad T_f = \frac{(e_\gamma \cdot k)(e^* k)}{k^2} + \frac{(e_\gamma \cdot e^*)}{2}
$$

non-flip transitions $\left\{ \begin{array}{c} + \rightarrow + \\ - \rightarrow - \end{array} \right.$

flip transitions $\left\{ \begin{array}{c} + \rightarrow - \\ - \rightarrow + \end{array} \right.$
pure twist 3 scaling (from ρ-factorization point of view)

\[ \Phi_{n.f.}^{\gamma^*_T \rightarrow \rho T} (k^2) = - \frac{e g^2 m_\rho f_\rho}{2\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \left\{ -2 \int dy_1 \frac{\left( k^2 + 2 Q^2 y_1 (1 - y_1) \right) k^2}{y_1 (1 - y_1) \left( k^2 + Q^2 y_1 (1 - y_1) \right)^2} \left[ (2y_1 - 1) \varphi_1^T(y_1) + \varphi_A^T(y_1) \right] \\
+ 2 \int dy_1 dy_2 \left[ \zeta_3^V B(y_1, y_2) - \zeta_3^A D(y_1, y_2) \right] \frac{y_1 (1 - y_1) k^2}{k^2 + Q^2 y_1 (1 - y_1)} \left[ \frac{2 - N_c/C_F) Q^2}{k^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} \right] \\
- \frac{N_c}{C_F} \frac{Q^2}{y_2 k^2 + Q^2 y_1 (y_2 - y_1)} \right\} - 2 \int dy_1 dy_2 \left[ \zeta_3^V B(y_1, y_2) + \zeta_3^A D(y_1, y_2) \right] \left[ \frac{2 + N_c/C_F}{y_1} \right] \\
+ \frac{y_1 Q^2}{k^2 + Q^2 y_1 (1 - y_1)} \left( \frac{2 - N_c/C_F) y_1 k^2}{k^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} - 2 \right) \\
+ \frac{N_c}{C_F} \frac{(y_1 - y_2) (1 - y_2)}{1 - y_1} \frac{Q^2}{k^2 (1 - y_1) + Q^2 (y_2 - y_1) (1 - y_2)} \right\} \]

and

\[ \Phi_f^{\gamma^*_T \rightarrow \rho T} (k^2) = - \frac{e g^2 m_\rho f_\rho}{2\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \left\{ 4 \int dy_1 \frac{k^2 Q^2}{(k^2 + Q^2 y_1 (1 - y_1))^2} \left[ \varphi_A^T(y_1) - (2y_1 - 1) \varphi_1^T(y_1) \right] \\
- 4 \int dy_1 dy_2 \frac{y_1 k^2}{k^2 + Q^2 y_1 (1 - y_1)} \left[ \zeta_3^A D(y_1, y_2) (-y_1 + y_2 - 1) + \zeta_3^V B(y_1, y_2) (y_1 + y_2 - 1) \right] \\
\times \left[ \frac{(2 - N_c/C_F) Q^2}{k^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} - \frac{N_c}{C_F} \frac{Q^2}{y_2 k^2 + Q^2 y_1 (y_2 - y_1)} \right] \right\} \]
Computation and results
Results: $\gamma_T^* \rightarrow \rho_T$ impact factor

**WW limit**

- **WW limit:** keep only twist 2 + kinematical twist 3 terms (i.e. $B = D = 0$)

- The only remaining contributions come from the two-body correlators

- **non-flip** transition

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T} (k^2) = -\frac{e m_\rho f_\rho}{2 \sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \left\{ \left( y - \bar{y} \right) \varphi_{TWW}^T (y) + 2 y \bar{y} \varphi_{WWW}^T (y) + \varphi_{AWWW}^T (y) \right\}$$

  \[ \begin{array}{l}
  \frac{1}{y \bar{y}} \left( k^2 + 2 Q^2 y \bar{y} \right) \left( (y - \bar{y}) \varphi_{TWW}^T (y) + \varphi_{AWWW}^T (y) \right) \\
  \left( y^2 + Q^2 y (1 - y)^2 \right)
  \end{array} \]

  which simplifies, using equation of motion:

$$\int dy \left[ (y - \bar{y}) \varphi_{TWW}^T (y) + 2 y \bar{y} \varphi_{WWW}^T (y) + \varphi_{AWWW}^T (y) \right] = 0$$

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T} (k^2) = \frac{e m_\rho f_\rho}{\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \frac{2 k^2 \left( k^2 + 2 Q^2 y \bar{y} \right)}{y \bar{y} \left( k^2 + Q^2 y \bar{y} \right)^2} \left[ (2 y - 1) \varphi_{TWW}^T (y) + \varphi_{AWWW}^T (y) \right].$$

- **flip** transition:

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T} (k^2) = -\frac{e m_\rho f_\rho}{\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \frac{2 k^2 Q^2}{\left( k^2 + Q^2 y \bar{y} \right)^2} \left[ (1 - 2 y) \varphi_{TWW}^T (y) + \varphi_{AWWW}^T (y) \right].$$
The obtained results are gauge invariant:

$$\Phi^{\gamma^*_T \to \rho_T} \to 0 \quad \text{when} \quad k \to 0$$

- this is straightforward in the WW limit
- at the full twist 3 order:
  - the $C_F$ part of the abelian 3-body contribution cancels the 2-body contribution after using the equation of motion
  - the $N_c$ part of the abelian 3-body contribution cancels the 3-body non-abelian contribution
- thus $\gamma^*_T \to \rho_T$ impact factor is gauge-invariant only provided the 2 and 3-body contributions have been taken into account in a consistent way
Our results are free of end-point singularities, in both WW approximation and full twist-3 order calculation:

- the flip contribution obviously does not have any end-point singularity because of the $k^2$ which regulates them

- the potential end-point singularity for the non-flip contribution is spurious since $\varphi_A^T(y)$, $\varphi_1^T(y)$ vanishes at $y = 0, 1$ as well as $B(y_1, y_2)$ and $D(y_1, y_2)$. 
We have performed a full up to twist 3 computation of the $\gamma^* \rightarrow \rho$ impact factor, in the $t = t_{min}$ limit.

Our result respects gauge invariance. This is achieved only after including 2 and 3 body correlators.

It is free of end-point singularities
(this should be contrasted with standard collinear treatment, at moderate $s$, where $k_T$-factorization is NOT applicable: see Mankiewicz-Piller).

Phenomenological applications will be done in the near future.

In this talk we relied on the Light-Cone Collinear approach
(Ellis + Furmanski + Petronzio; Efremov + Teryaev; Anikin + Teryaev),
which is non-covariant, but very efficient for practical computations.

This Light-Cone Collinear approach is systematic, and can be extended to any process, including higher twist effects (but does not preclude potential end-point singularities)
Comparison with a fully covariant approach by Ball+Braun et al:
The dictionary between the two approaches within a full twist 3 treatment is now established:

\[
B(y_1, y_2) = -\frac{V(y_1, 1-y_2, y_2-y_1)}{y_2-y_1},
\]

\[
D(y_1, y_2) = -\frac{A(y_1, 1-y_2, y_2-y_1)}{y_2-y_1}.
\]

\[
\varphi_1(y) = f_\rho m_\rho \phi_{||}(y),
\]

\[
\varphi_3(y) = f_\rho m_\rho g^{(v)}(y),
\]

\[
\varphi_A(y) = -\frac{1}{4} f_\rho m_\rho \frac{\partial g^{(a)}(y)}{\partial y}.
\]

We also performed calculations of the same impact factor within the covariant approach by Ball+Braun et al: calculations proceed in quite different way: eg. no \(\varphi_{1,A}^T\) – DAs but Wilson line effects are important!! We got a full agreement with our approach.