Threshold resummation for the LHC: all order colour structure and application to squark production

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(Based on M.Beneke, P.Falgari, CS, arXiv:0907.1443 [hep-ph] and work in progress)

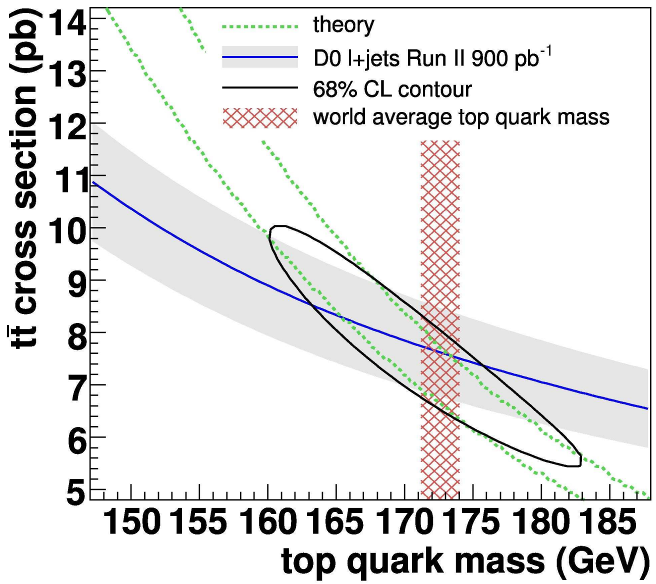
Pair production of heavy coloured particles at Tevatron/LHC

$$N(K_1)N'(K_2) \to H(p_1)H'(p_2) + X$$

- N, N': pp, $p\bar{p}$; HH': top-quark, squark, gluino... pairs

Precise knowledge of cross sections:

- sensitivity on mass,
- exclusion bounds,
- model discrimination,...



NLO corrections:

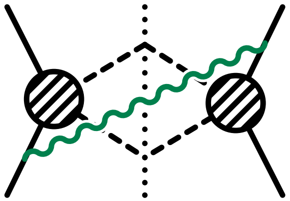
enhanced for

$$\beta = \sqrt{1 - \frac{(M_H + M_{H'})^2}{\hat{s}}} \to 0$$

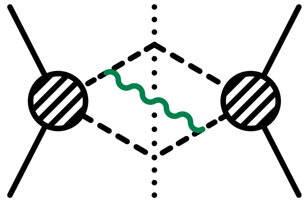
$$\hat{\sigma}^{(1)}_{pp' \to HH'} = \hat{\sigma}^{(0)}_{pp' \to HH'}\ \alpha_s \left[\underbrace{a \log^2(8\beta^2) + b \log(8\beta^2)}_{\text{“threshold logarithms”}} + \underbrace{c\frac{1}{\beta}}_{\text{“Coulomb singularity”}} + \dots \right]$$

Soft-Coulomb factorization

Soft corrections: (Resummation: Sterman 87; Catani, Trentadue 89, ...)



$\Rightarrow \alpha_s \ \log^2(8\beta^2)$



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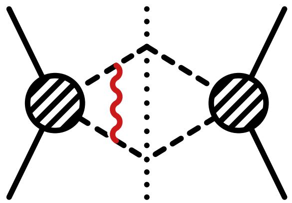
Coulomb gluon corrections (Fadin, Khoze 87; Peskin, Strassler 90, NRQCD,...)



$$\Rightarrow \alpha_s \ \frac{1}{\beta}$$

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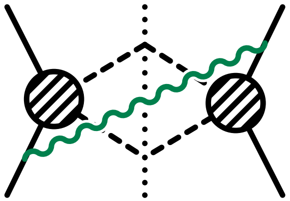

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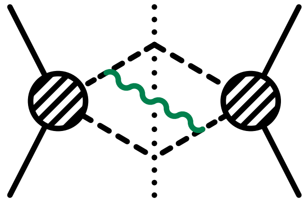

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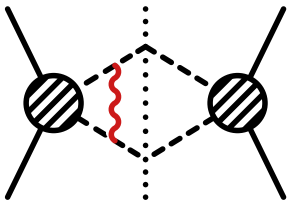


$$\Rightarrow \alpha_s \ \frac{1}{\beta}$$

Soft-Coulomb factorization for $\beta \to 0$ (Beneke, Falgari, CS, 09)

$$\hat{\sigma}_{pp'}(\hat{s}, \mu) = \sum_{i,i'} H_{ii'}(M, \mu) \int d\omega \sum_{R_\alpha} \underbrace{J_{R_\alpha}(E - \tfrac{\omega}{2})}_{\text{Coulomb gluons}} \ \underbrace{W_{ii'}^{R_\alpha}(\omega, \mu)}_{\text{soft gluons}} \qquad (E = \sqrt{s} - (m_H + m'_H))$$

- $H_{ii'}$ and $W_{ii'}^{R_\alpha}$ colour matrices
  in basis $c^{(i)}_{\{a\}}$ of $pp' \to HH'$ scattering
- R_α: irreducible colour representation of HH' system

Diagonalization of one-loop soft function for top/ squark/gluino production (Kidonakis/Sterman 97; Kulesza/Moytka 08)

Physical picture: (Bonciani et.al. 98)

radiation off total colour charge of final state system

Construction of diagonal basis to all-orders (Beneke, Falgari, CS 09)

- Decompose initial and final state systems into irreps:

$$r \otimes r' = \sum_\alpha r_\alpha \,, \qquad R \otimes R' = \sum_{R_\alpha} R_\alpha$$

- pairs of equivalent initial- and final state representations:

e.g. $8 \otimes 8 \to 3 \otimes \bar{3}:$ $\qquad P_i \in \{(1,1),\ (8_S,8),\ (8_A,8)\}$

- Clebsch-Gordon coefficients e.g. $8 \otimes 8 \to 8_A$, $3 \otimes \bar{3} \to 8$:

$$C^{(8_A)}_{\alpha a_1 a_2} = \frac{i}{\sqrt{3}} f^{a_2 \alpha a_1} \,, \qquad C^{(8)}_{\alpha a_1 a_2} = \sqrt{2} T^{\alpha}_{a_2 a_1}$$

- construct basis tensors:

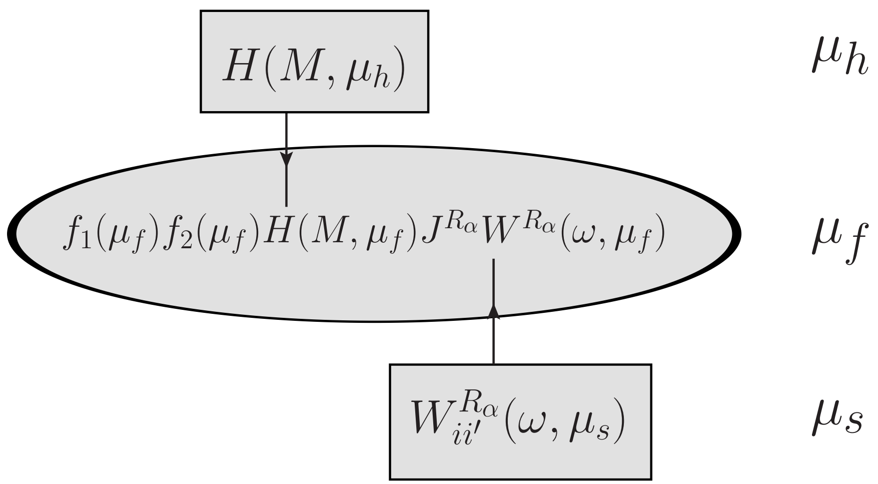
$$c^{(i)}_{\{a\}} = \frac{1}{\sqrt{\mathrm{dim}(r_\alpha)}} C^{r_\alpha}_{\alpha a_1 a_2} C^{R_\beta *}_{\alpha a_3 a_4} \qquad \text{e.g. } c^{(3)}_{\{a\}} = \frac{i}{\sqrt{12}} f^{a_2 \alpha a_1} T^{\alpha}_{a_3 a_4}$$

Evolution of soft function: (as for Drell-Yan: Korchemsky, Marchesini 92)

$$\frac{d}{d\log\mu} W_i^{R_\alpha}(z^0,\mu) = \left(2(\Gamma^r_{\text{cusp}} + \Gamma^{r'}_{\text{cusp}})\log\left(\frac{iz_0\mu e^{\gamma_E}}{2}\right) - 2(\gamma_s^{H,R_\alpha} + \gamma_s^r + \gamma_s^{r'})\right) W_i^{R_\alpha}(z^0,\mu)$$

Resummation:

- Solution to RGE in momentum space (Becher, Neubert, Pecjak 07)
- evolve hard function from scale $\mu_h \sim Q \sim 4M$ to scale μ_f
- evolve soft function from scale μ_s to scale μ_f
  Choose μ_s to minimize hadronic $\Delta\sigma_{\text{soft}}$ (Becher, Neubert, Xu 07)



Ingredients needed for resummation:

$$\left.\begin{array}{r}\text{NLL: tree-level}\\ \text{NNLL: one-loop}\end{array}\right\} H_i W_i^{R_\alpha};\ \left.\begin{array}{c}\text{one-loop}\\ \text{two-loop}\end{array}\right\} \gamma_s^r,\ \gamma_s^{H,R_\alpha}\ ;\ \left.\begin{array}{c}\text{two-loop}\\ \text{three-loop}\end{array}\right\} \Gamma_{\text{cusp}}^r$$

Results for γ_s^r, Γ_{cusp}^r up to three loops (Moch, Vermaseren, Vogt 04/05)

Soft anomalous dimension (Beneke, Falgari, CS 09)

One loop: (agrees with Kidonakis,Sterman 97, Kulesza, Motyka 08)

$$\gamma_s^{(0),H,R_\alpha} = -2C_{R_\alpha}$$

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$$\gamma_s^{(0),H,R_\alpha} = -2C_{R_\alpha}$$

Two loops: (agrees with Czakon, Mitov, Sterman 09)

$$\gamma_s^{(1),H,R_\alpha} = -C_{R_\alpha} C_A \left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3\right) + \frac{20}{9} C_{R_\alpha} n_f$$

extracted using

- constraints from soft-collinear factorization (Becher, Neubert 09)
- two-loop HQET formfactor (Korchemsky Radyushkin 92, Kidonakis 09)

Two partonic processes (Simplified setup: equal squark masses, exclude stop.)

$$q_i \bar{q}_j \to \tilde{q}_i \overline{\tilde{q}_j} \;, \qquad gg \to \tilde{q}_i \overline{\tilde{q}_j}$$

Perform NLL resummation in momentum space: (Becher, Neubert 06):

$$\hat{\sigma}^{\mathrm{NLL}}_{pp'}(\hat{s},\mu_f) = \sum_i H_i^{(0)}(M,\mu_f) \sum_{R_\alpha=1,8} \int d\omega \, J_{R_\alpha}(E - \frac{\omega}{2}) \, W_i^{R_\alpha,\mathrm{NLL}}(\omega,\mu_f)$$

(Mellin-space NLL resummation: Kulesza, Motyka 09, partial NNLO: Langenfeld, Moch 09)

Application to squark-antisquark production

Two partonic processes (Simplified setup: equal squark masses, exclude stop.)

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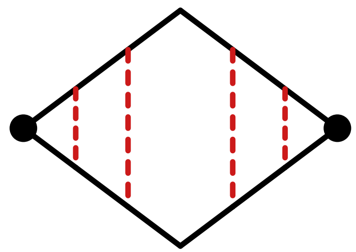
Combined soft and Coulomb resummation possible

Use Coulomb-Green function

with resummed single-gluon exchange $J(q) =$



Scale choice $\alpha_s(\mu_C)$ for Coulomb resummation?

momentum of Coulomb gluon: $|\vec{k}| \sim M\beta \sim M\alpha_s$

$$\Rightarrow \mu_C = \max\{m_{\tilde{q}}\beta, m_{\tilde{q}}\alpha_s(\mu_C)\}$$

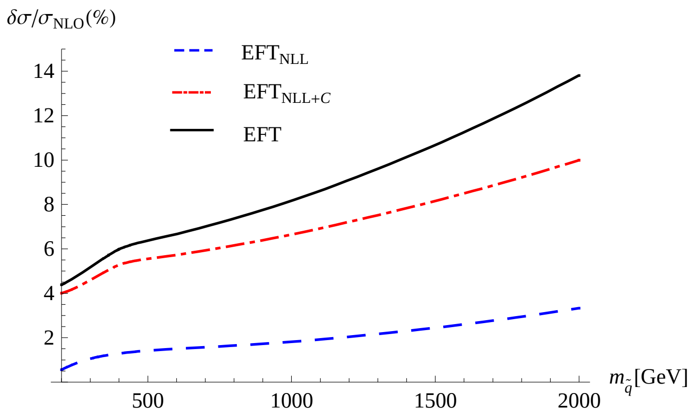
Matching to fixed order NLO results (Beenakker, Höpker, Spira, Zerwas 96, PROSPINO (Plehn et.al.), Langenfeld, Moch 09)

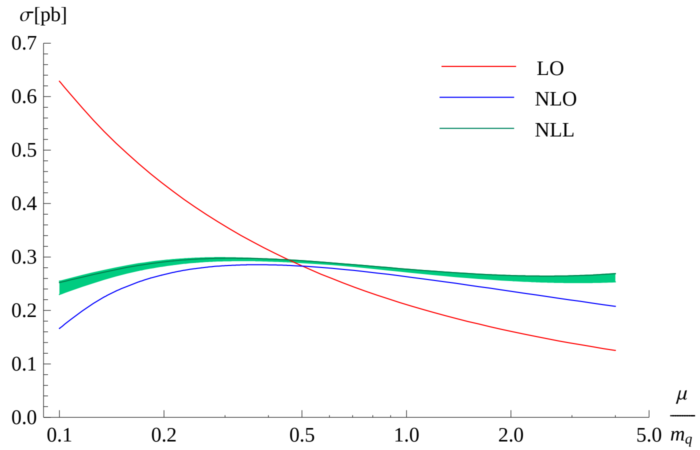
$$\hat{\sigma}^{\text{match}}_{pp'}(\hat{s},\mu_f) = \left[\hat{\sigma}^{\text{NLL}}_{pp'}(\hat{s},\mu_f) - \hat{\sigma}^{\text{NLL}}_{pp'}(\hat{s},\mu_f)|_{\text{NLO}}\right] + \hat{\sigma}^{\text{NLO}}_{pp'}(\hat{s},\mu_f)$$

NLL: soft gluon resummation

C: Coulomb resummation (Scale $\mu_C = \max\{m_{\tilde{q}}\beta, m_{\tilde{q}}\alpha_s(\mu_C)\}$)

EFT: NLL+C+mixed Single-Coulomb $\times$ resummed soft corrections (Beneke, Falgari, CS; PRELIMINARY!)





(LHC, 14 TeV, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$, MSTW08NLO) ($m_{\tilde{q}} = 1$ TeV, $0.5\mu_s^0 < \mu_s < 2\mu_s^0$)

Threshold resummation

- Threshold logarithms, Coulomb correction

Factorization and resummation

- Factorization of soft and Coulomb gluons
- Resummation from momentum space solution to RGEs

Colour structure of soft function

- diagonal basis to all orders
- two-loop soft anomalous dimension for arbitrary $SU(3)$ representations

Application to squark-antisquark production

- Soft and Coulomb resummation, mixed Soft+Coulomb corrections
- total corrections $4-14\%$ for $m_{\tilde{q}} = 300$ GeV-2 TeV