Top-Quark Production at Hadron Colliders

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Johannes Gutenberg Universität Mainz

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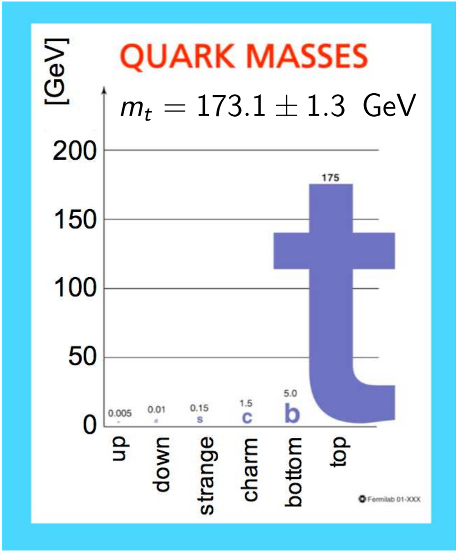
Outline

The Top Quark



A particle which tends to stick out...

- Up-type quark of the 3rd family
- According to the SM it is an elementary particle, but it is almost as heavy as a gold atom
- It decays very rapidly via EW interactions: $t \to bW$ ($\tau_t = 1/\Gamma_t \sim 5 \times 10^{-25} s$)
- The top quark decays before it can form hadronic bound states
- Because of its large mass, the top quark couples strongly to the electroweak breaking sector

The Tevatron and The LHC

Fermilab Tevatron

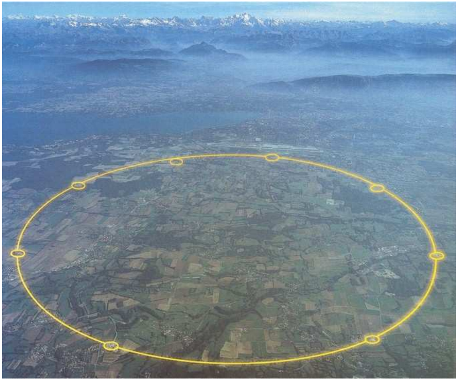
$p\bar{p} \qquad \sqrt{s} = 1.8 - 1.96\,\text{TeV}$

- Discovered at Tevatron in 1995
- So far it was observed only at the Tevatron (few thousands top quarks produced)
- The mass of the top-quark could be measured with a percent accuracy
- Production cross-sections and couplings are know with larger uncertainties

The Tevatron and The LHC

CERN LHC

$pp \qquad \sqrt{s} = 14\,\text{TeV}$



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With the large number of top quarks expected to be produced at the LHC, the study of the top-quark properties will become precision physics

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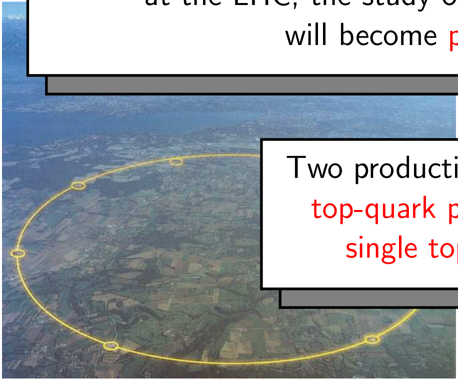

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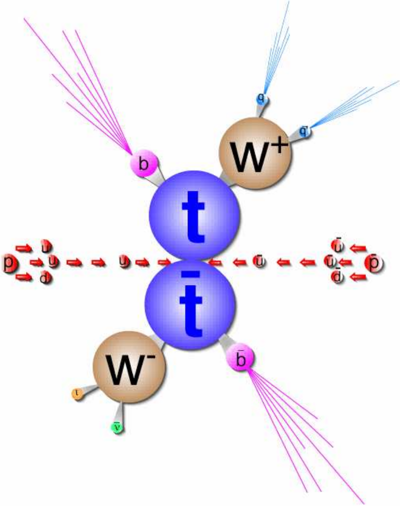
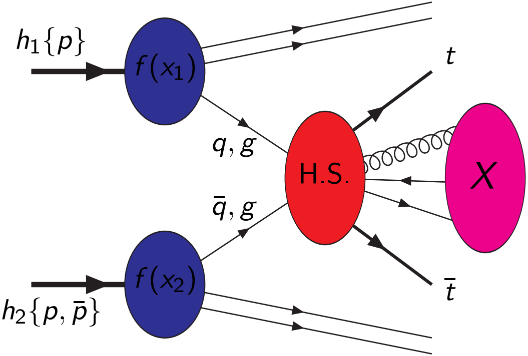
Two production mechanisms:
top-quark pair production
single top production



Top-Quark Pair Production

Top Quark Pair Production

Top-quark pair production is a hard scattering process which can be computed in perturbative QCD



$$\sigma^{t\bar{t}}_{h_1,h_2} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_i^{h_1}(x_1, \mu_F) f_j^{h_2}(x_2, \mu_F) \hat{\sigma}_{ij}\left(\hat{s}, m_t, \alpha_s(\mu_R), \mu_F, \mu_R\right)$$

$$s = (p_{h_1} + p_{h_2})^2 \;, \; \hat{s} = x_1 x_2 s$$

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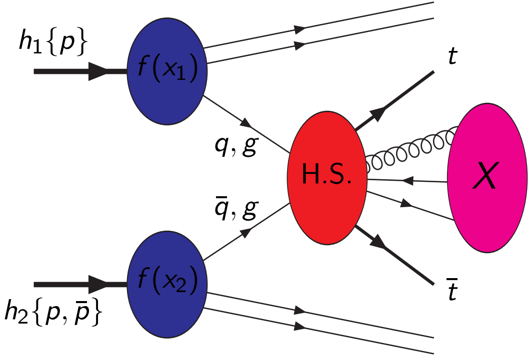
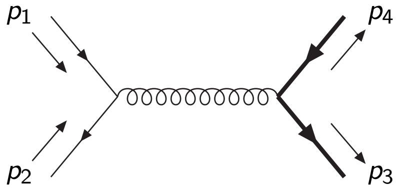


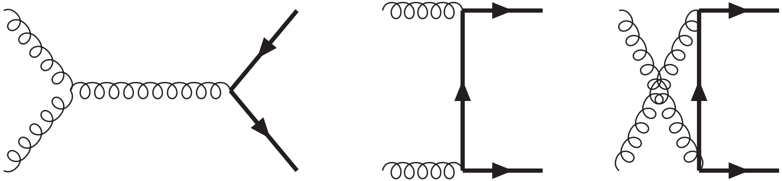
$$\sigma_{h_1,h_2}^{t\bar{t}}(s_{\text{had}}, m_t^2) = \sum_{ij} \int_{4m_t^2}^{s_{\text{had}}} d\hat{s} \; \underbrace{L_{ij}\left(\hat{s}, s_{\text{had}}, \mu_f^2\right)}_{\text{partonic luminosity}} \; \overbrace{\hat{\sigma}_{ij}(\hat{s}, m_t^2, \mu_f^2, \mu_r^2)}^{\text{partonic cross section}}$$

Tree Level QCD Partonic Processes

$$q(p_1) + \bar{q}(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



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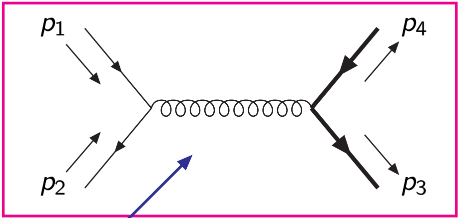

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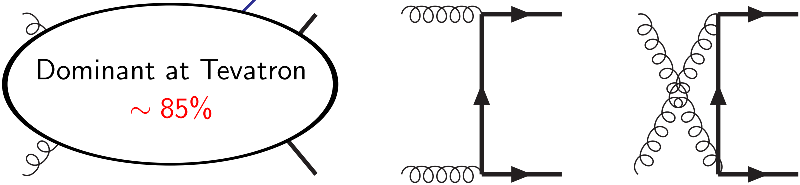

Dominant at Tevatron
$\sim 85\%$

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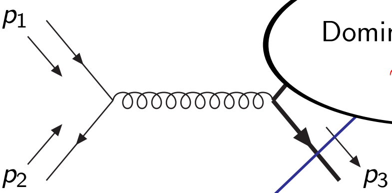
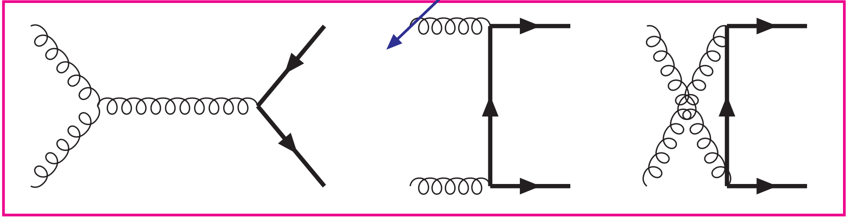


Dominant at LHC
$\sim 90\%$

$$g(p_1) + g(p_2) \longrightarrow t(p_3) + \bar{t}(p_4)$$



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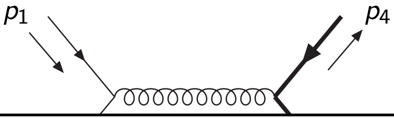
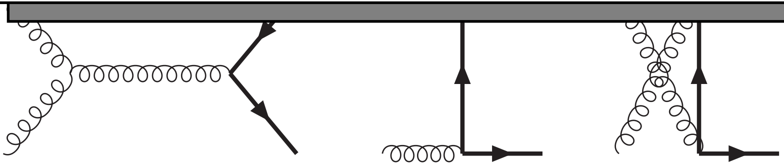


The NLO QCD corrections in both channels
(and to $qg \to t\bar{t}q$) have been known for a long time

Nason, Dawson, Ellis ('88-'90) Beenakker, Kuijf, van Neerven, Smith ('89) Beenakker *et al*('91) Mangano, Nason, Ridolfi ('92) Frixione *et al* ('95) Czakon and Mitov ('08)



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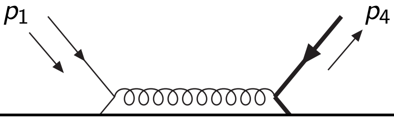
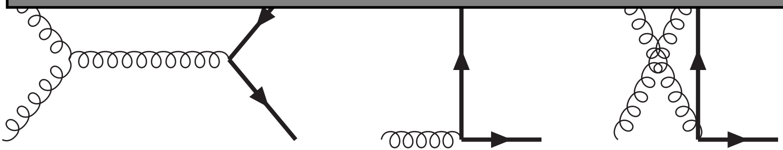


The mixed QCD-EW corrections in both channels are also known (they are smaller than current QCD uncertainties)

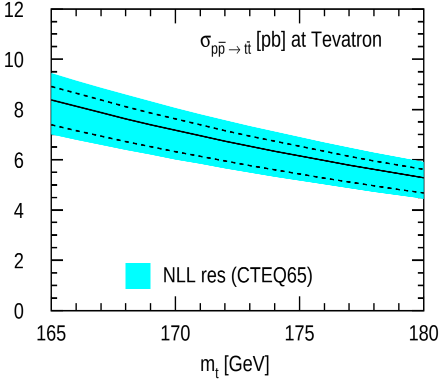
Beenakker *et al.* ('94) Bernreuther, Fuecker, and Si ('05-'08)
Kühn, Scharf, and Uwer ('05-'06)
Moretti, Nolten, and Ross ('06)

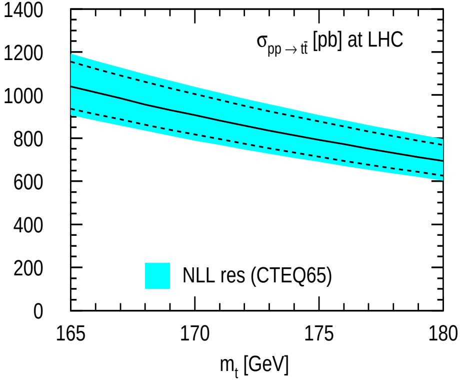


Uncertainty on the NLO Cross Section

The partonic cross sections involve terms like $\ln(1 - 4m^2/s)$ which become large near threshold and must be resummed

Kidonakis, Sterman ('97), Bonciani *et al.* ('98), Kidonakis *et al.* ('01), Kidonakis, Vogt ('03), Banfi, Laenen ('05), Cacciari *et al* ('08) Czakon *et al* ('09)





S. Moch and P. Uwer ('08)

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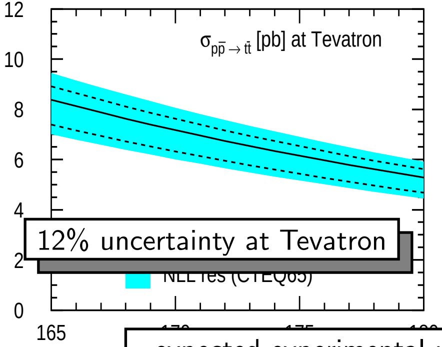
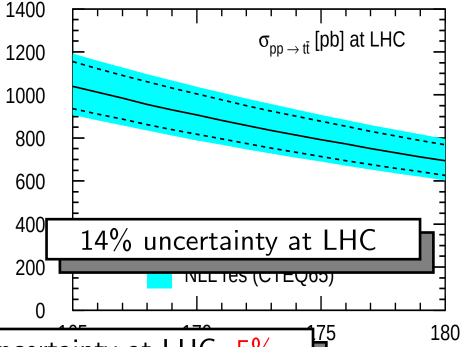


12% uncertainty at Tevatron



14% uncertainty at LHC

expected experimental uncertainty at LHC 5%

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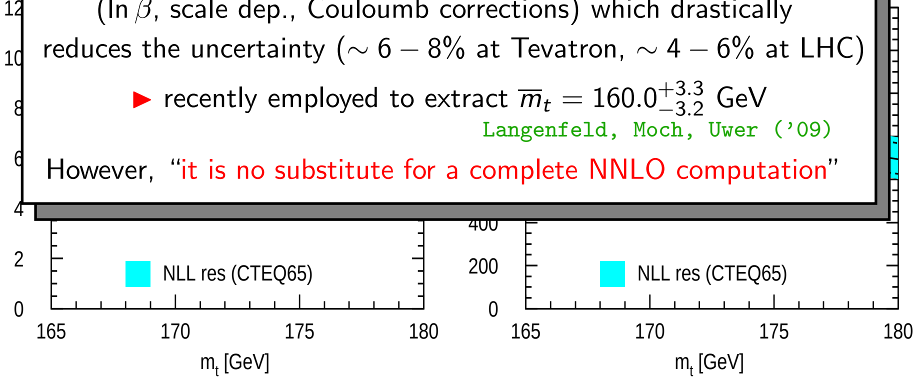
Moch and Uwer presented an approximated NNLO result ($\ln\beta$, scale dep., Couloumb corrections) which drastically reduces the uncertainty ($\sim 6-8\%$ at Tevatron, $\sim 4-6\%$ at LHC)

- recently employed to extract $\overline{m}_t = 160.0^{+3.3}_{-3.2}$ GeV

Langenfeld, Moch, Uwer ('09)

However, "it is no substitute for a complete NNLO computation"



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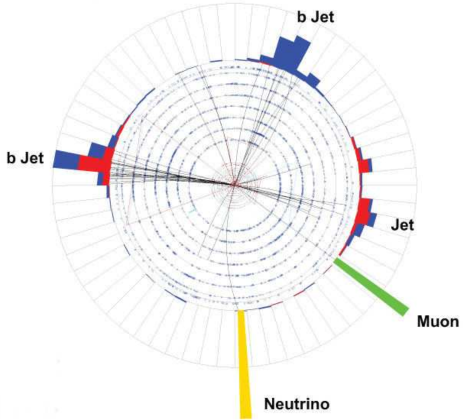

Single Top-Quark Production

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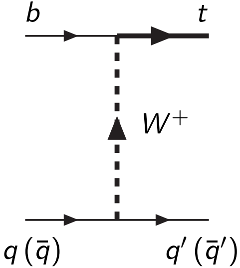
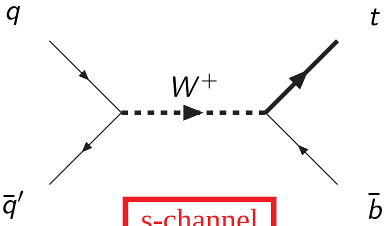
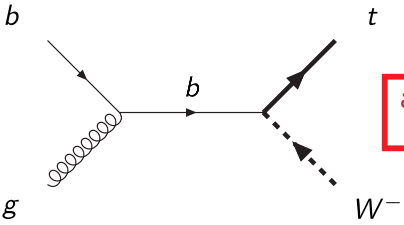


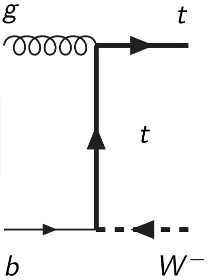
t-channel



s-channel



associated tW production



cross section	t-channel (pb)	s-channel(pb)	tW mode (pb)
$\sigma^{t}_{\text{Tevatron}}$	1.15 ± 0.07	0.54 ± 0.04	0.14 ± 0.03
σ^{t}_{LHC}	150 ± 6	7.8 ± 0.7	44 ± 5
$\sigma^{\bar{t}}_{\text{LHC}}$	92 ± 4	4.3 ± 0.3	44 ± 5

$m_t = 171.4 \pm 2.1$ Kidonakis ('06-'07)

The numbers include NLO QCD corrections (Harris *et al* ('02), Sullivan ('04-'05), Campbell *et al* ('04), Chao *et al* ('04 - '05), Smith and Willenbrock ('96), Chao *et al* ('04), Giele *et al* ('95), Zhu ('02))

Single Top Production: Predictions

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- The cross-section is proportional to $|V_{\text{tb}}|^2$
- $\sigma^t + \sigma^{\bar{t}}$ is 38% (48%) of $\sigma^{t\bar{t}}$ at the LHC (Tevatron)
  $\Longrightarrow$ 3×10^6 single top events at the LHC ($L = 10\,\text{fb}^{-1}$)
- The final states from single top events have large backgrounds
- CDF and D0 reported evidence for single-top production at the Tevatron, at the LHC it should be possible to disentangle the different modes

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> *t*-channel: dominant at both the Tevatron and the LHC
> at the LHC: $ub \to dt\,(\sim 74\%)$, $\bar{d}b \to \bar{u}t\,(\sim 12\%), \cdots$
> It should be measurable with a total error of about 10%

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s-channel: small at the LHC
a measurement uncertainty of 36% was estimated

- The ...
- $\sigma^t + \sigma^t$ is 38% (48%) of σ^{tt} at the LHC (Tevatron) $\Longrightarrow$ 3×10^6 single top events at the LHC ($L = 10\,\text{fb}^{-1}$)
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Associated tW production: large at the LHC

estimated uncertainties for the CS measurement are $\sim 25\%$

- $\sigma^t + \sigma^{\bar{t}}$ is 38% (48%) of $\sigma^{t\bar{t}}$ at the LHC (Tevatron) $\Longrightarrow$ 3×10^6 single top events at the LHC ($L = 10\,\text{fb}^{-1}$)
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- The c...
- $\sigma^t +$ ...

  $\Longrightarrow$ 3 ...

The theoretical uncertainties on the single top cross sections are under control

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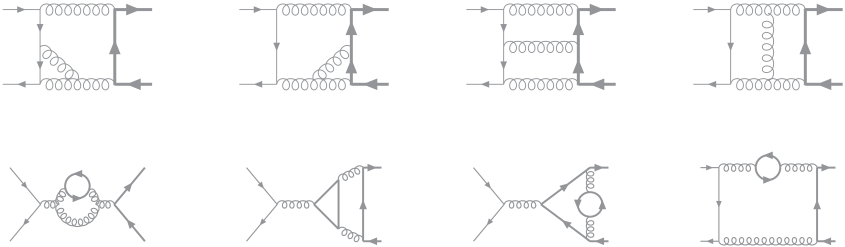
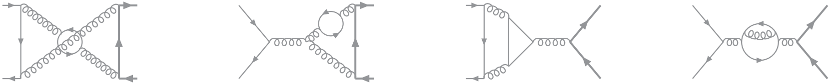


Top-Quark Pair Production at NNLO



NNLO Laundry List

The NNLO calculation of the top-quark pair hadroproduction requires several ingredient

- Virtual Corrections
  - two-loop matrix elements for $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
  - interference of one-loop diagrams
- Real Corrections
  - one-loop matrix elements for the hadronic production of $t\bar{t}$ + 1 parton
  - tree-level matrix elements for the hadronic production of $t\bar{t}$ + 2 partons

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`Körner et al. ('05,'08)`
`Anastasiou, Aybat ('08)`

`Dittmaier, Uwer, Wenzierl ('07,'08)`

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- In the $q\bar{q} \to t\bar{t}$ channel, `QGRAF` generates 218 two-loop diagrams (one massive flavor, one massless flavor)
- There are 789 two-loop diagrams in the $gg \to t\bar{t}$ channel
- All the diagrams were evaluated in the $s, |t|, |u| \gg m_t^2$

`Czakon, Mitov, Moch ('07,'08)`

- However, to study the top-quark pair production at the Tevatron and the LHC it is necessary to retain the exact dependence on m_t

Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

$$|\mathcal{M}|^2 (s, t, m, \varepsilon) = \frac{4\pi^2 \alpha_s^2}{N_c} \left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi} \right) \mathcal{A}_1 + \left(\frac{\alpha_s}{\pi} \right)^2 \mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3 \right) \right]$$

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$$\boxed{\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}}$$

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One-Loop $\times$ One-Loop

Körner, Merebashvili, Rogal ('05,'08)

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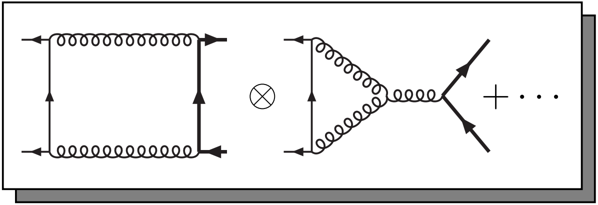

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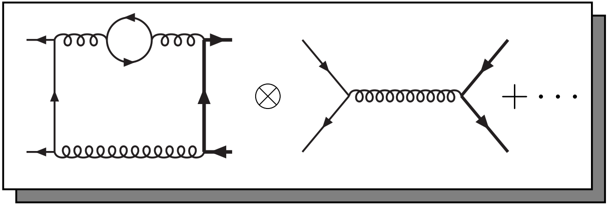
Two-Loop $\times$ Tree

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$



Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times 0)} &= N_c C_F\Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l\left(N_c D_l + \frac{E_l}{N_c}\right)\\ &\quad + N_h\left(N_c D_h + \frac{E_h}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\Big]\end{aligned}$$

10 different color coefficients

Two-Loop Corrections to $q\bar{q} \to t\bar{t}$

$$|\mathcal{M}|^2 (s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$\mathcal{A}_2^{(2\times0)}$ is known numerically (`Czakon '08`)
It was calculated with a method based on
Laporta algorithm + numerical solutions of the
differential equations satisfied by the Master Integrals

$$\begin{aligned}\mathcal{A}_2^{(2\times0)} &= N_c C_F\Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l\left(N_c D_l + \frac{E_l}{N_c}\right) \\ &\quad + N_h\left(N_c D_h + \frac{E_h}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\Big]\end{aligned}$$

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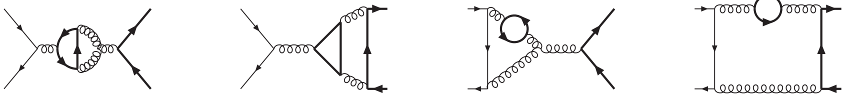
All the diagrams with a closed quark loop
(massive or massless) were calculated analytically
(Bonciani, AF, Gehrmann, Maître, Studerus '08)
Laporta algorithm + diff. eq. method + HPLs

$$\begin{aligned}\mathcal{A}_2^{(2\times 0)} &= N_c C_F\Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l\left(N_c D_l + \frac{E_l}{N_c}\right)\\ &+N_h\left(N_c D_h + \frac{E_h}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\Big]\end{aligned}$$



$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

The coefficient of the leading color structure in the squared matrix element was calculated analytically (Bonciani, AF, Gehrmann, Studerus '09)
It involves planar diagrams only

$$\begin{aligned}\mathcal{A}_2^{(2\times 0)} \quad &= \quad N_c C_F\Big[N_c^2 A + B + \frac{C}{N_c^2} + N_l\left(N_c D_l + \frac{E_l}{N_c}\right) \\ &\quad + N_h\left(N_c D_h + \frac{E_h}{N_c}\right) + N_l^2 F_l + N_l N_h F_{lh} + N_h^2 F_h\Big]\end{aligned}$$

Two-Loop Corrections to $gg \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

Two-Loop Corrections to $gg \to t\bar{t}$

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One-Loop × One-Loop

Anastasiou, Aybat ('08)
Körner, Kniehl, Merebashvili, Rogal ('08)

Two-Loop Corrections to $gg \to t\bar{t}$

$$|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]$$

$$\mathcal{A}_2 = \mathcal{A}_2^{(2\times 0)} + \mathcal{A}_2^{(1\times 1)}$$

$$\begin{aligned}\mathcal{A}_2^{(2\times 0)} = (N_c^2-1)\Bigg(& N_c^3 A + N_c B + \frac{1}{N_c} C + \frac{1}{N_c^3} D + N_c^2 N_l E_l + N_c^2 N_h E_h \\ & + N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h \\ & + N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh}\Bigg)\end{aligned}$$

16 color structures

$$
|\mathcal{M}|^2(s,t,m,\varepsilon) = \frac{4\pi^2\alpha_s^2}{N_c}\left[\mathcal{A}_0 + \left(\frac{\alpha_s}{\pi}\right)\mathcal{A}_1 + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{A}_2 + \mathcal{O}\left(\alpha_s^3\right)\right]
$$

$$
\mathcal{A}_2 = \mathcal{A}^{(2\times0)} + \mathcal{A}^{(1\times1)}
$$

The two-loop corrections to $gg \to t\bar{t}$ are only available for $s, |t|, |u| \gg m_t^2$ (Czakon, Mitov, Moch '08)

Exact calculation: not all the color coefficients can be expressed in terms of HPLs. Numerical evaluation?

$$
\mathcal{A}_2^{(2\times0)} \quad \ldots \quad N_c^2 N_h E_h
$$

$$
+N_l F_l + N_h F_h + \frac{N_l}{N_c^2} G_l + \frac{N_h}{N_c^2} G_h + N_c N_l^2 H_l + N_c N_h^2 H_h
$$

$$
+N_c N_l N_h H_{lh} + \frac{N_l^2}{N_c} I_l + \frac{N_h^2}{N_c} I_h + \frac{N_l N_h}{N_c} I_{lh}\Bigg)
$$

16 color structures

Summary & Conclusions

- Top-quark physics is one of the keys for the study of the EWSB. LHC will provide precise measurements for several top-quark related observables. To fully exploit the LHC potential, top-quark observables must be under theoretical control and include higher-order corrections
- A complete fixed-order NNLO QCD calculation for $\sigma^{t\bar{t}}$ is needed. Some of the necessary building block were studied by several groups
- The calculation of two-loop corrections is a crucial ingredient in the NNLO program. It presents serious technical challenges, especially in the gluon fusion channel (dominant at the LHC). For the latter, only results in the ultra-relativistic limit are available
- In this talk the main focus was on the production cross sections. There are many other interesting observables in top-quark physics: spin correlations, asymmetries, associated production of $\gamma, Z, H, \ldots$:

$\Longrightarrow$ W. Bernreuther arXiv:0805.1333

Backup Slides

Size of the NLO corrections

LHC	$\sigma^{t\bar{t}}$	σ^t $t-$channel	σ^t $s-$channel	σ^t associated tW
NLO QCD	$\sim +50\%^{\S}$	$\sim +5\%^{\flat}$	$\sim +44\%^{\perp}$	$\sim +10\%^{\top}$
EW	$\sim -0.5\%^{\ddagger}$	$< 1\%^{\natural}$		
MSSM	up to $\pm 5\%^{\P}$	$< 1\%^{\natural}$		

Tevatron	$\sigma^{t\bar{t}}$	σ^t $t-$channel	σ^t $s-$channel
NLO QCD	$\sim +25\%^{\S}$	$\sim +9\%^{\flat}$	$\sim +47\%^{\perp}$
EW	$\sim -1\%^{\ddagger}$	$< 1\%^{\natural}$	
MSSM	up to $\pm 5\%^{\P}$	$< 1\%^{\natural}$	

$\S$ see talk (W. Bernreuther `arXiv:0805.1333`)

$\P$ S. Berge *et al.* `hep-ph/0703016` (W. Bernreuther `arXiv:0805.1333`)

$\ddagger$ see talk (W. Bernreuther `arXiv:0805.1333`)

$\flat$ G. Bordes and B. van Eijk NPB **435** (1995), T.Stelzer *et al.* `hep-ph/9705398`, `hep-ph/9807340`(W. Bernreuther `arXiv:0805.1333`)

$\natural$ M.Beccaria *et al.* `hep-ph/0605108`, `arXiv:0802.1994`(W. Bernreuther `arXiv:0805.1333`)

$\perp$ M. C. Smith and S. Willenbrock `hep-ph/9604223` B. W. Harris *et al.* `hep-ph/0207055` (W. Bernreuther `arXiv:0805.1333`)

$\top$ W. T. Giele *et al.*`hep-ph/9511449`, S. Zhu PLB **524** (2002) (W. Bernreuther `arXiv:0805.1333`)

Complete Analytic NLO calculation

Recently, the NLO total cross section was evaluated analytically

M. Czakon, A. Mitov ('08)

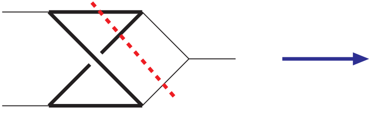
Laporta algorithm and differential equation method are employed also for the phase space integrals (by exploiting the optical theorem); cut propagators are treated by using

$$\delta\left(q^2+m^2\right)=\frac{1}{2\pi i}\left(\frac{1}{q^2+m^2-i\delta}-\frac{1}{q^2+m^2+i\delta}\right)$$

Anastasiou Melnikov ('02)

The $gg \to t\bar{t}X$ cross section cannot be completely written in terms of HPLs and their generalizations.

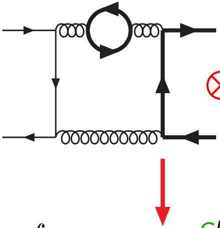
Integrals over elliptic functions



$$\begin{aligned} K(k) &= \int_0^1 dz \frac{1}{\sqrt{1-z^2}\sqrt{1-k^2z^2}} \\ E(k) &= \int_0^1 dz \frac{\sqrt{1-k^2z^2}}{\sqrt{1-z^2}} \end{aligned}$$

Method: The General Strategy

After interfering a two-loop graph with the Born amplitude one obtains a linear combinations of scalar integrals



$\otimes$ Born Amplitude

$$\int \mathfrak{D}^d k_1 \mathfrak{D}^d k_2 \frac{S_1^{n_1} \cdots S_q^{n_q}}{\mathcal{D}_1^{m_1} \cdots \mathcal{D}_t^{m_t}}$$

$k_i \rightarrow$ integration momenta

$p_i \rightarrow$ external momenta

$S \rightarrow$ scalar products $k_i \cdot k_j$ or $k_i \cdot p_k$

$\mathcal{D} \rightarrow$ propagators $[\sum c_i k_i + \sum d_j p_j]^2 \ (+m_t^2)$

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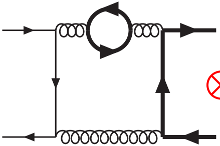

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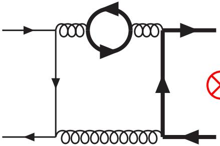

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Luckily, just a "small" number of these integrals are independent: the MIs

It is necessary to

- identify the MIs $\Longrightarrow$ Reduction through the Laporta Algorithm
- calculate the MIs $\Longrightarrow$ Differential Equation Method

The Laporta Algorithm

The set of denominators $\mathcal{D}_1, \cdots, \mathcal{D}_t$ defines a topology; for each topology

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- The scalar integrals are related via Integration By Parts identities (10 identities per integral for a two-loop four-point function)

$$\int \mathfrak{D}^d k_1 \mathcal{D}^d k_2 \frac{\partial}{\partial k_i^\mu}\left[v^\mu \frac{S_1^{n_1} \cdots S_q^{n_q}}{\mathcal{D}_1^{m_1} \cdots \mathcal{D}_t^{m_t}}\right] = 0 \quad v^\mu = k_1, k_2, p_1, p_2, p_3$$

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- Building the IBPs for growing powers of the propagators and scalar products the number of equations grows faster that the number of unknown: one finds a system of equations which is apparently over-constrained

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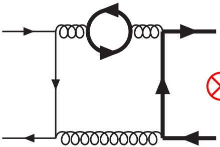
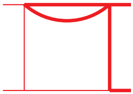
- Building the IBPs for growing powers of the propagators and scalar products the number of equations grows faster that the number of unknown: one finds a system of equations which is apparently over-constrained
- Solving the system of IBPs (in a problem with a small number of scales) one finds that only a few of the scalar integrals above (if any) are independent: the MIs.

Reduction at Work

The two-loop box diagrams entering in the calculation of the heavy-fermion loop corrections are reducible, i.e. they can be rewritten in terms of integrals belonging to the subtopologies only:



$\otimes$ Born Amplitude $\Longrightarrow \sum C$



$+$ Triangles

$+$ Bubbles $+$ Tadpoles

Calculation of the MIs:
Differential Equation Method

For each Master Integral belonging to a given topology
$F_l^{(q)} \to \{\mathcal{D}_1, \cdots, \mathcal{D}_q\}$

- Take the derivative of a given integral with respect to the external momenta p_i

$$p_j^\mu \frac{\partial}{\partial p_i^\mu} F_l^{(q)} = p_j^\mu \int \mathfrak{D}^d k_1 \mathfrak{D}^d k_2 \frac{\partial}{\partial p_i^\mu} \frac{S_1^{n_1} \cdots S_q^{n_q}}{\mathcal{D}_1^{m_1} \cdots \mathcal{D}_q^{m_q}}$$

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- The integrals are regularized, therefore we can apply the derivative to the integrand in the r. h. s. and use the IBPs to rewrite it as a linear combination of the MIs

$$p_j^\mu \int \mathfrak{D}^d k_1 \mathfrak{D}^d k_2 \frac{\partial}{\partial p_i^\mu} \frac{S_1^{n_1} \cdots S_q^{n_q}}{\mathcal{D}_1^{m_1} \cdots \mathcal{D}_q^{m_q}} = \sum c_i F_i^{(q)} + \sum_{r \neq q} \sum_j k_j F_j^{(r)}$$

Calculation of the MIs: Differential Equation Method

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- Rewrite the diff. eq. in terms of derivatives with respect to s and t

$$\frac{\partial}{\partial s} F_l^{(q)}(s,t) = \sum_j c_j(s,t) F_j^{(q)}(s,t) + \sum_{r \neq q} \sum_l k_l(s,t) F_l^{(r)}(s,t)$$

Calculation of the MIs: Differential Equation Method

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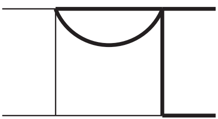
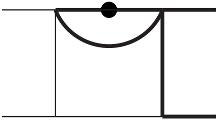
- Take the derivative of a given integral with respect to the external momenta p_i
- The integrals are regularized, therefore we can apply the derivative to the integrand in the r. h. s. and use the IBPs to rewrite it as a linear combination of the MIs
- Rewrite the diff. eq. in terms of derivatives with respect to s and t
- Fix somehow the initial condition(s) (ex. knowing the behavior of the integral at $s = 0$) and solve the system of DE(s)

Five Denominator MIs-II

$M_1 =$ 

$M_2 =$ 

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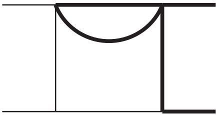
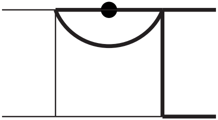
$M_1 =$ 

$M_2 =$ 

- the two MIs satisfy two independent first order differential equations

$$\frac{dM_i(s,t)}{dt} = C_i(s,t)M_i(s,t) + \Omega_i(s,t)$$

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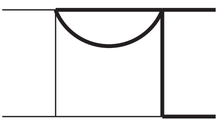
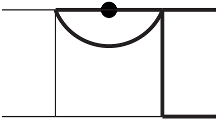
$M_1 =$  $M_2 =$ 

- the two MIs satisfy two independent first order differential equations

$$\frac{dM_i(s,t)}{dt} = C_i(s,t)M_i(s,t) + \Omega_i(s,t)$$

- One of the two needed initial conditions can be fixed by imposing the regularity of the integrals in $t = 0$
  The second integration constant can be fixed by calculating the integral in $t = 0$ with MB techniques

Charge Asymmetry

The charge asymmetry is the difference in production rate for top and antitop at fixed angle or rapidity

$$\underbrace{A(y) = \frac{N_t(y) - N_{\bar{t}}(y)}{N_t(y) - N_{\bar{t}}(y)}}_{\text{differential CA}} \qquad \underbrace{A = \frac{N_t(y \geq 0) - N_{\bar{t}}(y \geq 0)}{N_t(y \geq 0) - N_{\bar{t}}(y \geq 0)}}_{\text{integrated CA}} \qquad \left(N_i \equiv \frac{d\sigma^{t\bar{t}}}{dy_i} \right)$$

- Arising at order α_s^3 for $q\bar{q} \to t\bar{t}$ because of
  - I) interference of final state with initial state gluon radiation
  - II) interference of virtual box diagrams with the Born process
- Top quarks are preferentially emitted in the direction of the incoming quark at the partonic level (which translates to a preference in the direction of the incoming proton in $p\bar{p}$ collisions)
- Prediction for the Tevatron $A = 0.051(6)$. (At the LHC with no cuts $A = 0$)
- In QCD $N_{\bar{t}}(y) = N_t(-y)$, therefore $A = A_{\mathrm{FB}}^t$

Correlation of the t and $\bar{t}$ spins

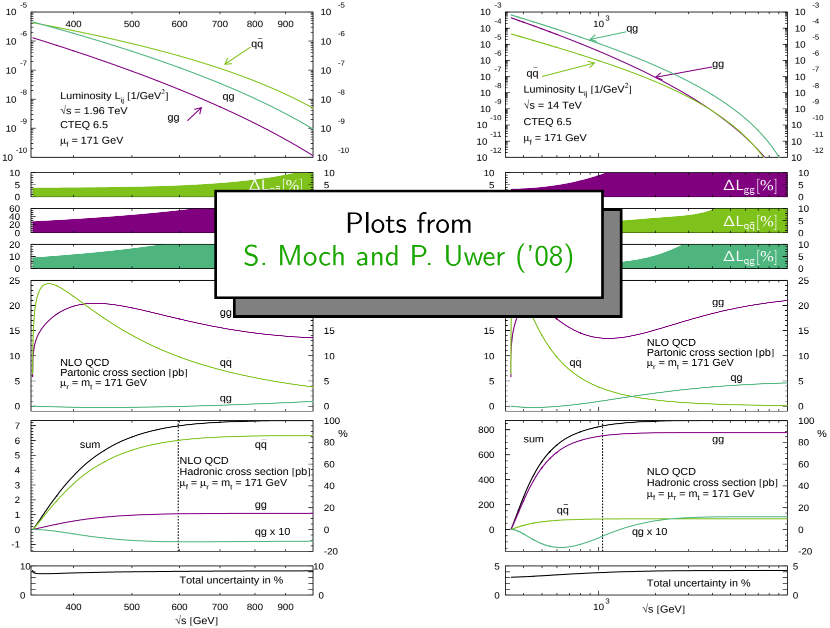
The correlation of the t and $\bar{t}$ spins with respect to the reference axes $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ is given by the expectation value

$$\mathcal{A} = \langle 4(\hat{\mathbf{a}} \cdot \hat{\mathbf{S}}_t)(\hat{\mathbf{b}} \cdot \hat{\mathbf{S}}_{\bar{t}})\rangle = \frac{N(\uparrow\uparrow) + N(\downarrow\downarrow) - N(\uparrow\downarrow) - N(\downarrow\uparrow)}{N(\uparrow\uparrow) + N(\downarrow\downarrow) + N(\uparrow\downarrow) + N(\downarrow\uparrow)}$$

helicity basis: $\hat{\mathbf{a}} = \mathbf{k}_t$, $\hat{\mathbf{b}} = \mathbf{k}_{\bar{t}}$; beam basis $\hat{\mathbf{a}} = \hat{\mathbf{b}} =$ beam axis (lab frame)

- At the Tevatron: $q\bar{q}$ channel dominant; top-pairs produced near threshold. The initial state has $S = J = 1$, the spins of t $\bar{t}$ are 100% correlated in the beam basis (both parallel or both antiparallel to the beam)
- Gluon channel near threshold: the total spin along the beam axis in the initial state is 0; therefore the spins of t and $\bar{t}$ point in opposite directions (along the beam axis)
- In the ultra-relativistic limit, the spins of t and $\bar{t}$ are 100% anti-correlated in the helicity basis (both for $q\bar{q}$ and gg)

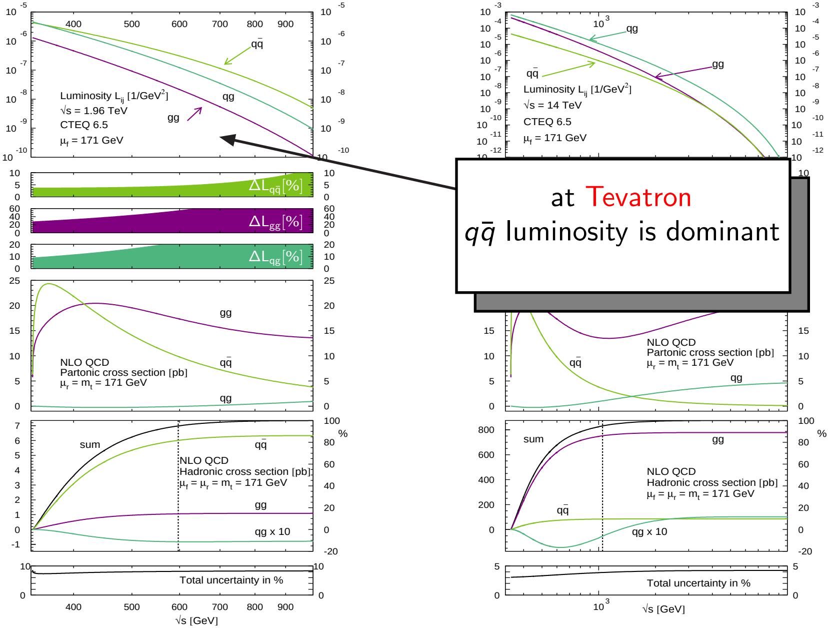
Luminosity and Partonic CS at NLO



Plots from

S. Moch and P. Uwer ('08)

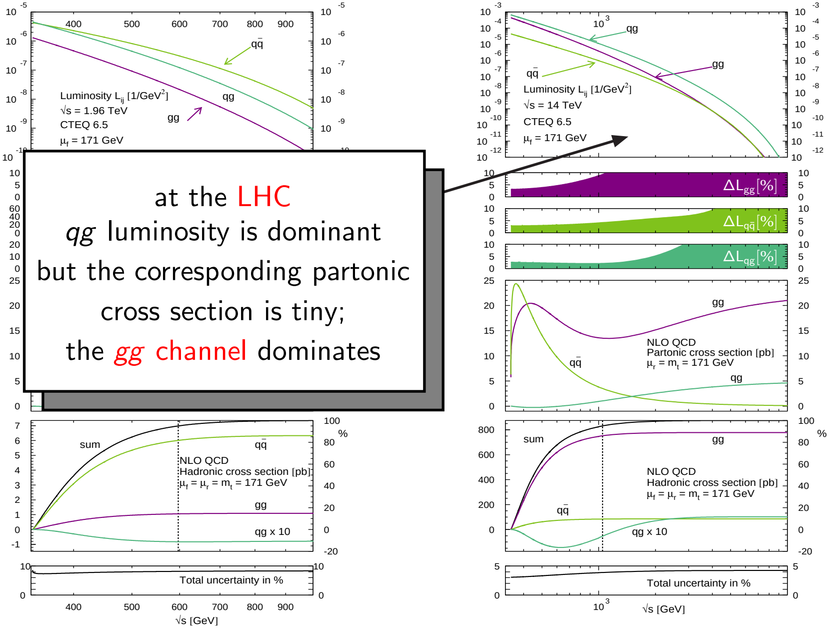
Luminosity and Partonic CS at NLO



at Tevatron

$q\bar{q}$ luminosity is dominant

Luminosity and Partonic CS at NLO



at the LHC

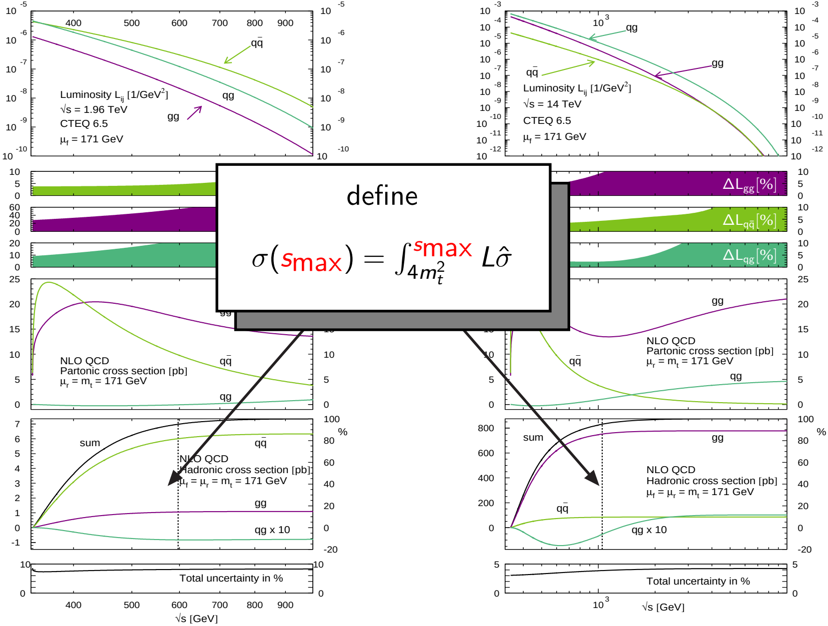
qg luminosity is dominant

but the corresponding partonic

cross section is tiny;

the *gg* channel dominates

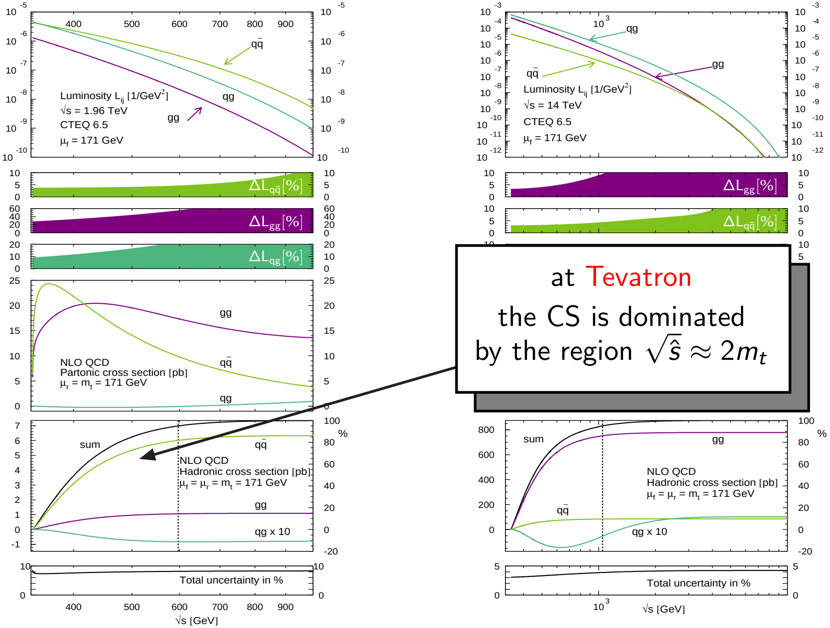
Luminosity and Partonic CS at NLO



define

$$\sigma(s_{max}) = \int_{4m_t^2}^{s_{max}} L\hat{\sigma}$$

Luminosity and Partonic CS at NLO

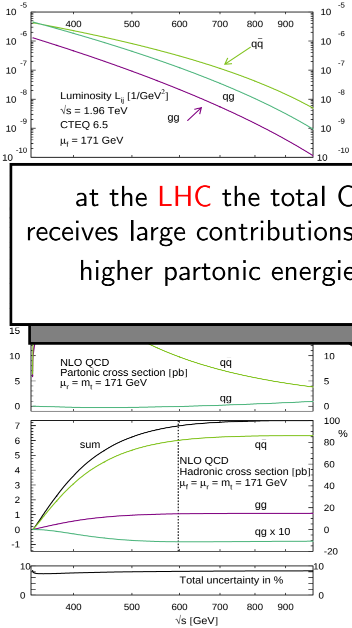


at Tevatron

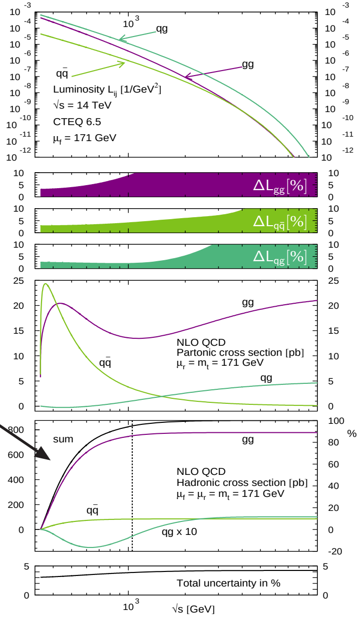
the CS is dominated

by the region $\sqrt{\hat{s}} \approx 2m_t$

Luminosity and Partonic CS at NLO



at the LHC the total CS receives large contributions from higher partonic energies



Threshold Resummation

for a threshold T_i (inclusive cross section $T = s - 4m^2$) The perturbative series for any of these (differential) cross sections can be expressed as

$$d\sigma(T) = \sum_n \sum_k^{2n} \alpha_s^n c_{n,k} \ln^k (T)$$

Resummation concerns itself with carrying out the sum above

It is often convenient to take moments

$$d\sigma(N) = \int dT\, T^N = \sum_n \sum_k^{2n} \alpha_s^n d_{n,k} \ln^k N$$

$$d\sigma(N) = C(\alpha_s) \times \exp\left(L g_0(\alpha_s L) + g_1(\alpha_s L) + \alpha_s g_2(\alpha_s L) + \cdots\right)$$

To calculate the exponent up to the term involving the function g_i corresponds to N^{i}LL resummation