NLO corrections with the OPP method[1]

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[1] In collaboration with G. Bevilacqua, M. Czakon, P. Draggiotis, M. Grazelli, I. Malamos, P. Mastrolia, G. Ossola, R. Pittau, A. van Hameren, M. Worek

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1. Introduction: Wishlists and Troubles
2. OPP Reduction
   - Rational terms
3. Automated 1-loop
   - HELAC 1-loop
4. Outlook

Introduction: LHC needs NLO

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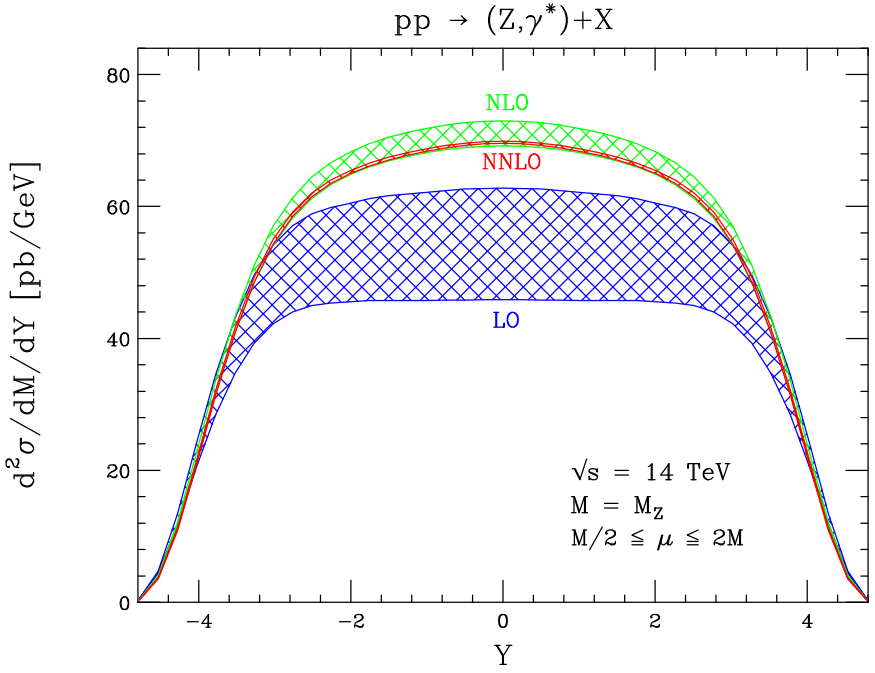

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- The experimental programs of LHC require high precision predictions for multi-particle processes (also ILC of course)
- The current need of precision goes beyond tree order. At LHC, most analyses require at least next-to-leading order calculations (NLO)
- As a result, a big effort has been devoted by several groups to the problem of an efficient computation of one-loop corrections for multi-particle processes!

NLO Wishlist Les Houches

[from G. Heinrich's Summary talk]

Wishlist Les Houches 2007
1. $pp \to V\,V +$ jet
2. $pp \to t\bar{t}\,b\bar{b}$
3. $pp \to t\bar{t} + 2$ jets
4. $pp \to W\,W\,W$
5. $pp \to V\,V\,b\bar{b}$
6. $pp \to V\,V + 2$ jets
7. $pp \to V + 3$ jets
8. $pp \to t\bar{t}\,b\bar{b}$
9. $pp \to 4$ jets

Processes for which a NLO calculation is both desired and feasible

Will we "finish" in time for LHC?

What has been done? (2005-2009)

Some recent results $\rightarrow$ Cross Sections available

- $pp \rightarrow Z\,Z\,Z$ $pp \rightarrow t\bar{t}Z$ [Lazopoulos, Melnikov, Petriello]
- $pp \rightarrow H + 2$ jets [Campbell, et al., J. R. Andersen, et al.]
- $pp \rightarrow VV + 2$ jets via VBF [Bozzi, Jäger, Oleari, Zeppenfeld]
- $pp \rightarrow VV + 1$ jet [S. Dittmaier, S. Kallweit and P. Uwer]
- $pp \rightarrow t\bar{t} + 1$ jet [S. Dittmaier, P. Uwer and S. Weinzierl]
- $pp \rightarrow V\,V\,V$ [Binoth, Ossola, Papadopoulos, Pittau and Campanario et al.]

Mostly $2 \rightarrow 3$, very few $2 \rightarrow 4$ complete calculations.

- $e^+\,e^- \rightarrow 4$ fermions [Denner, Dittmaier, Roth]
- $e^+\,e^- \rightarrow H\,H\,\nu\,\bar{\nu}$ [GRACE group (Boudjema et al.)]
- $q\,\bar{q} + gg \rightarrow t\bar{t}b\bar{b}$ [Bredenstein et al.]

This is NOT a complete list

(A lot of work has been done at NLO $\rightarrow$ calculations & new methods)

What has been done? 2009

- R. K. Ellis, K. Melnikov and G. Zanderighi, "Generalized unitarity at work: first NLO QCD results for hadronic W^+ 3jet production," arXiv:0901.4101 [hep-ph]

What has been done? 2009

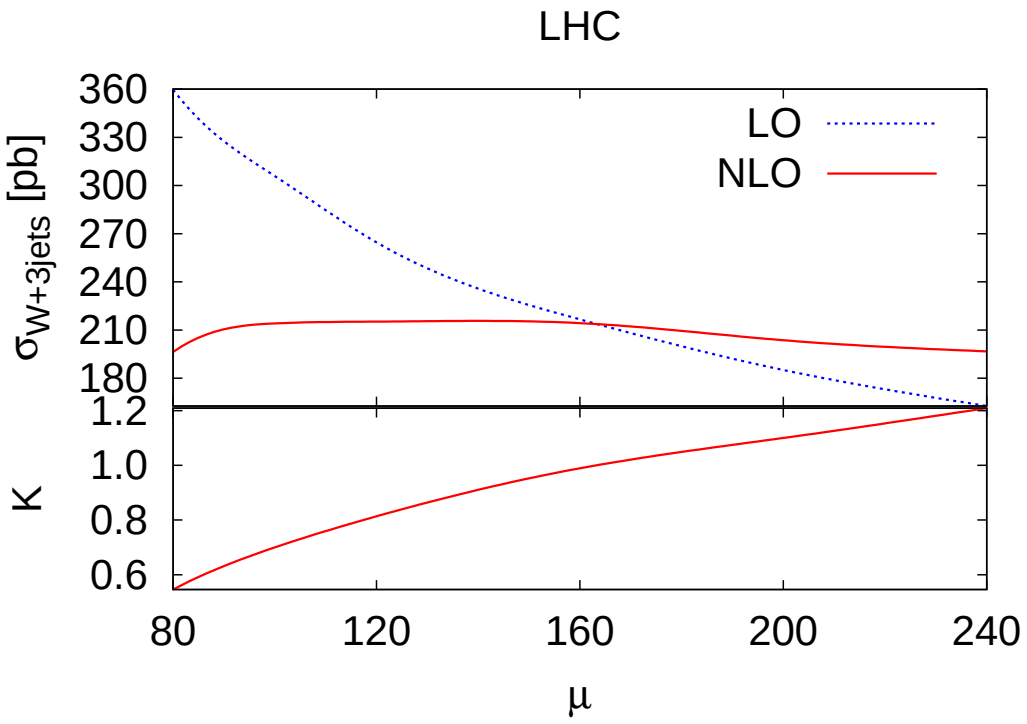

Figure 1: Inclusive W^++3 jet cross-section at the LHC and the K-factor defined as $K = \sigma_{\rm NLO}/\sigma_{\rm LO}$ as a function of the renormalization and factorization scales. Jets are defined with k_T algorithm with $R = 0.7$ and $p_T > 50$ GeV. Jet rapidities satisfy $|\eta| < 3$. The LO and NLO cross-sections are computed with CTEQ6L1 and CTEQ6M parton distributions, respectively.

What has been done? 2009

- C. F. Berger *et al.*, "Precise Predictions for $W + 3$ Jet Production at Hadron Colliders," arXiv:0902.2760 [hep-ph]

What has been done? 2009

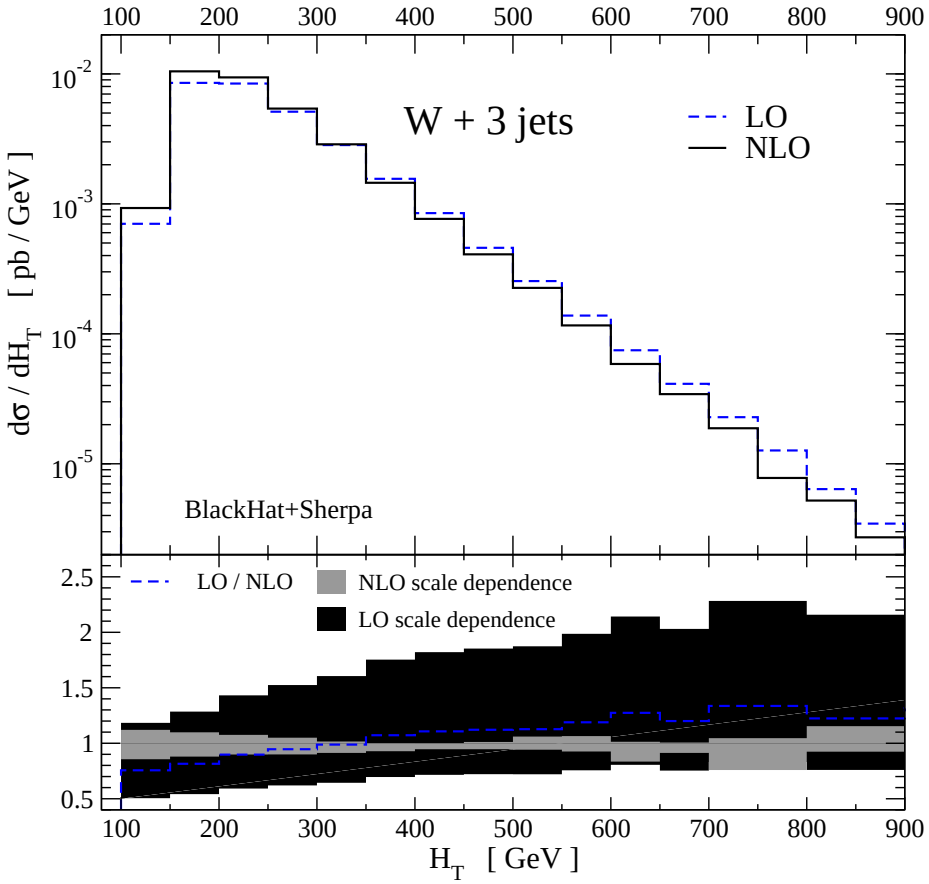

FIG. 3: The theoretical prediction for the H_T distribution in W + 3-jet production. The curves and bands are labeled as in fig. 2.

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Traditional Method: Feynman Diagrams & Passarino-Veltman Reduction:

- general applicability, major achievements
- but major problem: not designed @ amplitude level, factorially growth in complexity
- heavily based on computer-algebra simplifications

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- general applicability, major achievements
- but major problem: not designed @ amplitude level, factorially growth in complexity
- heavily based on computer-algebra simplifications

Unitarity Method: Gluing tree-amplitudes

- limited applicability
- on the positive side designed @ amplitude level
- based also heavily on analytic calculations

OPP Reduction - Intro

G. Ossola., C. G. Papadopoulos and R. Pittau, Nucl. Phys. B **763**, 147 (2007) – arXiv:hep-ph/0609007

and JHEP **0707** (2007) 085 – arXiv:0704.1271 [hep-ph]

R. K. Ellis, W. T. Giele and Z. Kunszt, JHEP **0803**, 003 (2008)

Any m-point one-loop amplitude can be written, before integration, as

$$A(\bar{q}) = \frac{\bar{N}(\bar{q})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

A bar denotes objects living in $n = 4 + \epsilon$ dimensions

$$\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$$

$$\bar{q}^2 = q^2 + \tilde{q}^2$$

$$\bar{D}_i = D_i + \tilde{q}^2$$

External momenta p_i are 4-dimensional objects

The old "master" formula

$$
\begin{aligned}
\int A &= \int \frac{\bar{N}(\bar{q})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \\
&= \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3) \\
&+ \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) C_0(i_0 i_1 i_2) \\
&+ \sum_{i_0<i_1}^{m-1} b(i_0 i_1) B_0(i_0 i_1) \\
&+ \sum_{i_0}^{m-1} a(i_0) A_0(i_0) \\
&+ \text{rational terms}
\end{aligned}
$$

OPP "master" formula - I

General expression for the 4-dim $N(q)$ at the integrand level in terms of D_i. "Cut-constructible" (CC) part of the amplitude: the one expressed in terms of scalar integrals.

OPP "MASTER" FORMULA - I

General expression for the 4-dim $N(q)$ at the integrand level in terms of D_i. "Cut-constructible" (CC) part of the amplitude: the one expressed in terms of scalar integrals.

$$
\begin{aligned}
N(q) \;=\; & \sum_{i_0<i_1<i_2<i_3}^{m-1} \Big[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \Big] \prod_{i \neq i_0,i_1,i_2,i_3}^{m-1} D_i \\
+ & \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0,i_1,i_2}^{m-1} D_i \\
+ & \sum_{i_0<i_1}^{m-1} \Big[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \Big] \prod_{i \neq i_0,i_1}^{m-1} D_i \\
+ & \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

OPP "master" formula - II

$$
\begin{aligned}
N(q) \quad = \quad & \sum_{i_0<i_1<i_2<i_3}^{m-1} \Big[d(i_0 i_1 i_2 i_3) \Big] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} [c(i_0 i_1 i_2)] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i \\
+ \quad & \sum_{i_0<i_1}^{m-1} \Big[b(i_0 i_1) \Big] \prod_{i\neq i_0,i_1}^{m-1} D_i + \sum_{i_0}^{m-1} [a(i_0)] \prod_{i\neq i_0}^{m-1} D_i
\end{aligned}
$$

- The quantities $d(i_0 i_1 i_2 i_3)$ are the coefficients of 4-point functions with denominators labeled by i_0, i_1, i_2, and i_3.
- $c(i_0 i_1 i_2)$, $b(i_0 i_1)$, $a(i_0)$ are the coefficients of all possible 3-point, 2-point and 1-point functions, respectively.

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+ \quad & \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\end{aligned}
$$

The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are the "spurious" terms

- They still depend on q (integration momentum)
- They should vanish upon integration

What is the explicit expression of the spurious term?

Spurious Terms - I

Following F. del Aguila and R. Pittau, arXiv:hep-ph/0404120

- Express any q in $N(q)$ as

$$q^\mu = -p_0^\mu + \sum_{i=1}^4 G_i \, \ell_i^\mu \ , \ \ell_i{}^2 = 0$$

$$k_1 = \ell_1 + \alpha_1 \ell_2 \, , \quad k_2 = \ell_2 + \alpha_2 \ell_1 \, , \ k_i = p_i - p_0$$
$$\ell_3{}^\mu =< \ell_1 | \gamma^\mu | \ell_2] \, , \ \ell_4{}^\mu =< \ell_2 | \gamma^\mu | \ell_1]$$

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$\rightarrow$ They give rise to d, c, b, a coefficients

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- The coefficients G_i either reconstruct denominators D_i or vanish upon integration

$\longrightarrow$ They give rise to d, c, b, a coefficients
$\longrightarrow$ They form the spurious $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ coefficients

Spurious Terms - II

- $\tilde{d}(\mathrm{q})$ term (only 1)

$$\tilde{d}(q) = \tilde{d}\, T(q)\,,$$

where $\tilde{d}$ is a constant (does not depend on q)

$$T(q) \equiv Tr[(\not{q} + \not{p}_0)\not{\ell}_1\not{\ell}_2\not{k}_3\gamma_5]$$

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$$\tilde{c}(q) = \sum_{j=1}^{j_{max}} \left\{ \tilde{c}_{1j}[(q+p_0)\cdot\ell_3]^j + \tilde{c}_{2j}[(q+p_0)\cdot\ell_4]^j \right\}$$

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- $\tilde{b}$(q) and $\tilde{a}$(q) give rise to 8 and 4 terms, respectively

General strategy

Now we know the form of the spurious terms:

$$
\begin{aligned}
N(q) \quad = \quad & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
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There is a very good set of such points: Use values of q for which a set of denominators D_i vanish $\rightarrow$ The system becomes "triangular": solve first for 4-point functions, then 3-point functions and so on

Example

$$
\begin{aligned}
N(q) &= d + \tilde{d}(q) + \sum_{i_0<i_1<i_2}^{3} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] D_{i_3} \\
&+ \sum_{i_0<i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] D_{i_2} D_{i_3} \\
&+ \sum_{i_0=0}^{3} \left[a(i_0) + \tilde{a}(q; i_0)\right] D_{i_1} D_{i_2} D_{i_3}
\end{aligned}
$$

We look for a q of the form $q^\mu = -p_0^\mu + x_i \ell_i^\mu$ such that

$$D_0 = D_1 = D_2 = D_3 = 0$$

$\rightarrow$ we get a system of equations in x_i that has two solutions $q_0^\pm$

Unitarity-like solution is derived **not** assumed !

Example

$$N(q) \quad = \quad d + \tilde{d}(q)$$

Our "master formula" for $q = q_0^{\pm}$ is:

$$N(q_0^{\pm}) = [d + \tilde{d}\, T(q_0^{\pm})]$$

$\rightarrow$ solve to extract the coefficients d and $\tilde{d}$

Unitarity-like solution is derived **not** assumed !

Example

$$
\begin{aligned}
N(q) - d - \tilde{d}(q) &= \sum_{i_0<i_1<i_2}^{3} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] D_{i_3} \\
&+ \sum_{i_0<i_1}^{3} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] D_{i_2} D_{i_3} \\
&+ \sum_{i_0=0}^{3} \left[a(i_0) + \tilde{a}(q; i_0)\right] D_{i_1} D_{i_2} D_{i_3}
\end{aligned}
$$

Then we can move to the extraction of c coefficients using

$$N'(q) = N(q) - d - \tilde{d}\, T(q)$$

and setting to zero three denominators (ex: $D_1 = 0$, $D_2 = 0$, $D_3 = 0$)

Unitarity-like solution is derived **not** assumed !

Example

$$N(q) - d - \tilde{d}(q) \quad = \quad [c(0) + \tilde{c}(q;0)]\, D_0$$

We have infinite values of q for which

$$D_1 = D_2 = D_3 = 0 \quad \text{and} \quad D_0 \neq 0$$

$\rightarrow$ Here we need 7 of them to determine $c(0)$ and $\tilde{c}(q;0)$

and so on for the b and a sectors
Unitarity-like solution is derived **not** assumed !

Rational Terms - I

- Is that possible that a 4-dimensional numerator produce rational terms?

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$$N(q) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d + \tilde{d}(q)\right] \prod_{i \neq i_0,i_1,i_2,i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c + \tilde{c}(q)\right] \prod_{i \neq i_0,i_1,i_2}^{m-1} D_i + \cdots$$

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- Finally rewrite all denominators using

$$\frac{D_i}{\bar{D}_i} = \bar{Z}_i\,, \qquad \text{with} \quad \bar{Z}_i \equiv \left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right)$$

Rational Terms - II

Expand in D-dimensions ?

$$\bar{D}_i = D_i + \tilde{q}^2$$

Rational Terms - II

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$$
\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3; \tilde{q}^2) + \tilde{d}(q; i_0 i_1 i_2 i_3; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{D}_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2; \tilde{q}^2) + \tilde{c}(q; i_0 i_1 i_2; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} \bar{D}_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1; \tilde{q}^2) + \tilde{b}(q; i_0 i_1; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1}^{m-1} \bar{D}_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0; \tilde{q}^2) + \tilde{a}(q; i_0; \tilde{q}^2) \right] \prod_{i \neq i_0}^{m-1} \bar{D}_i + \tilde{P}(q) \prod_{i}^{m-1} \bar{D}_i
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$$

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\begin{aligned}
N(q) &= \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3; \tilde{q}^2) + \tilde{d}(q; i_0 i_1 i_2 i_3; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} \bar{D}_i \\
&+ \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2; \tilde{q}^2) + \tilde{c}(q; i_0 i_1 i_2; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} \bar{D}_i \\
&+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1; \tilde{q}^2) + \tilde{b}(q; i_0 i_1; \tilde{q}^2) \right] \prod_{i \neq i_0, i_1}^{m-1} \bar{D}_i \\
&+ \sum_{i_0}^{m-1} \left[a(i_0; \tilde{q}^2) + \tilde{a}(q; i_0; \tilde{q}^2) \right] \prod_{i \neq i_0}^{m-1} \bar{D}_i + \tilde{P}(q) \prod_{i}^{m-1} \bar{D}_i
\end{aligned}
$$

$$
m_i^2 \to m_i^2 - \tilde{q}^2
$$

Rational Terms - II

Polynomial dependence on $\tilde{q}^2$

$$b(ij;\tilde{q}^2) \;=\; b(ij) + \tilde{q}^2 b^{(2)}(ij)\,, \quad c(ijk;\tilde{q}^2) \;=\; c(ijk) + \tilde{q}^2 c^{(2)}(ijk)\,.$$

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$$\int d^n\bar{q}\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j} \;=\; -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{(p_i-p_j)^2}{3}\right] + \mathcal{O}(\epsilon)\,,$$

$$\int d^n\bar{q}\frac{\tilde{q}^2}{\bar{D}_i\bar{D}_j\bar{D}_k} \;=\; -\frac{i\pi^2}{2} + \mathcal{O}(\epsilon)\,, \quad \int d^n\bar{q}\frac{\tilde{q}^4}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l} \;=\; -\frac{i\pi^2}{6} + \mathcal{O}(\epsilon)\,.$$

Rational Terms - II

Furthermore, by defining

$$\mathcal{D}^{(m)}(q,\tilde{q}^2) \equiv \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3;\tilde{q}^2) + \tilde{d}(q;i_0 i_1 i_2 i_3;\tilde{q}^2) \right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} \bar{D}_i \,,$$

the following expansion holds

$$\mathcal{D}^{(m)}(q,\tilde{q}^2) = \sum_{j=2}^{m} \tilde{q}^{(2j-4)} d^{(2j-4)}(q) \,,$$

where the last coefficient is independent on q

$$d^{(2m-4)}(q) = d^{(2m-4)} \,.$$

Rational Terms - II

In practice, once the 4-dimensional coefficients have been determined, one can redo the fits for different values of $\tilde{q}^2$, in order to determine $b^{(2)}(ij)$, $c^{(2)}(ijk)$ and $d^{(2m-4)}$.

$$\begin{aligned} \mathrm{R}_1 &= -\frac{i}{96\pi^2} d^{(2m-4)} - \frac{i}{32\pi^2} \sum_{i_0<i_1<i_2}^{m-1} c^{(2)}(i_0 i_1 i_2) \\ &- \frac{i}{32\pi^2} \sum_{i_0<i_1}^{m-1} b^{(2)}(i_0 i_1) \left(m_{i_0}^2 + m_{i_1}^2 - \frac{(p_{i_0} - p_{i_1})^2}{3} \right) . \end{aligned}$$

G. Ossola, C. G. Papadopoulos and R. Pittau,arXiv:0802.1876 [hep-ph]

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`CuTtools` publicly available code that is able to numerically evaluate both the `CC` and R_1 terms.

Rational Terms - R_2

A different source of Rational Terms, called R_2, can also be generated from the ϵ-dimensional part of $N(q)$

$$\bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}^2, \epsilon; q)$$

$$\mathrm{R}_2 \equiv \frac{1}{(2\pi)^4} \int d^n\,\bar{q} \frac{\tilde{N}(\tilde{q}^2, \epsilon; q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \equiv \frac{1}{(2\pi)^4} \int d^n\,\bar{q}\,\mathcal{R}_2$$

$$\begin{aligned} \bar{q} &= q + \tilde{q}\,, \\ \bar{\gamma}_{\bar{\mu}} &= \gamma_\mu + \tilde{\gamma}_{\tilde{\mu}}\,, \\ \bar{g}^{\bar{\mu}\bar{\nu}} &= g^{\mu\nu} + \tilde{g}^{\tilde{\mu}\tilde{\nu}}\,. \end{aligned}$$

The R_2 contribution for a given process, can be calculated in exactly the same way as the tree-order amplitude for that process, taken into account extra vertices.

Rational Terms - R_2

Only up to four-vertices are needed for the calculation of R_2 for any number of external particles, as in the case of the usual counter-terms.

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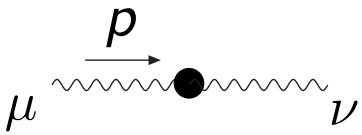
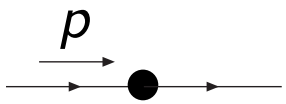
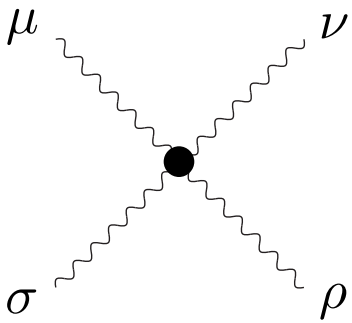
Rational counterterms for QED



$$= -\frac{ie^2}{8\pi^2}\, g_{\mu\nu}\,(2m_e^2 - p^2/3)$$



$$= \frac{ie^2}{16\pi^2}(-\not{p} + 2m_e)$$



$$= \frac{ie^4}{12\pi^2}\left(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho}\right)$$

Rational Terms - R_2

Only up to four-vertices are needed for the calculation of R_2 for any number of external particles, as in the case of the usual counter-terms.

In contrast to GKM-approach all calculations are in 4 dimensions – still producing fully d-dimensional answers.

NLO cross section

From the OPP reduction to realistic calculations:

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$$\sigma_m^{NLO} = \int_m d\sigma^B + \int_{m+1} (d\sigma^R - d\sigma^D)_{\epsilon=0} + \int_m (d\sigma^V + d\sigma^I + d\sigma^{KP})_{\epsilon=0}$$

$$d\sigma^D,\ d\sigma^I,\ d\sigma^{KP}$$

in terms of m-particle Born matrix elements, Catani-Seymour

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- Real corrections HELAC-DIPOLES $d\sigma^D,\ d\sigma^I,\ d\sigma^{KP}$
  M. Czakon, C.G. Papadopoulos, M. Worek, arXiv:0905.0883 [hep-ph]

HELAC 1-LOOP

HELAC-1L calculates virtual QCD corrections to any processes: example 6 external particles attached to the loop (decays do not count, so number of particles may be actually larger, i.e. $pp \to e^+\nu_e + 3$ jets)

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$$A(q) = \sum \frac{N_i^{(6)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_5}} + \frac{N_i^{(5)}(q)}{\bar{D}_{i_0}\bar{D}_{i_1}\cdots\bar{D}_{i_4}} + \ldots$$

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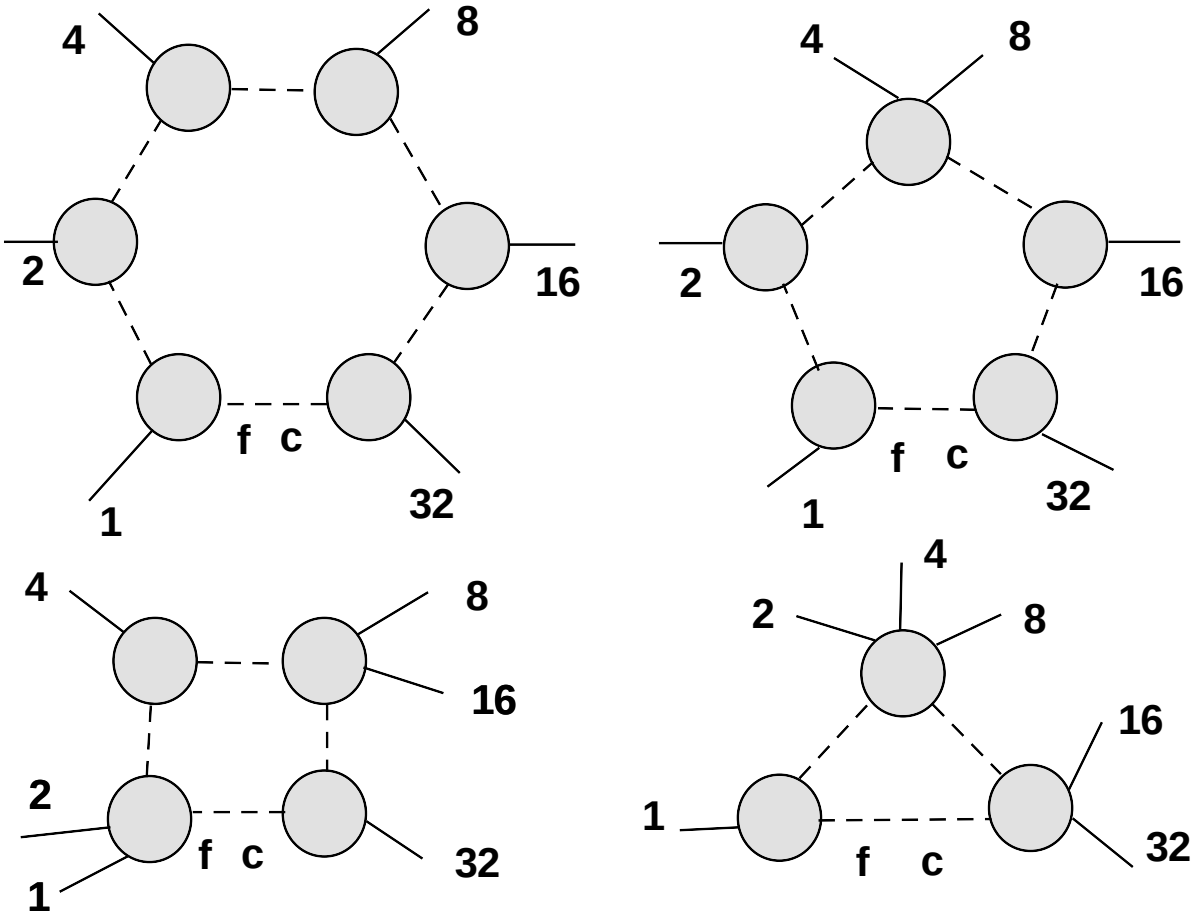
In order to apply the OPP reduction method HELAC-1L should provide numerical evaluation of all these 'numerator' functions $N_i^{(6)}(q), N_i^{(5)}(q), \ldots$, for values of the loop-momentum q provided by CuTtools.

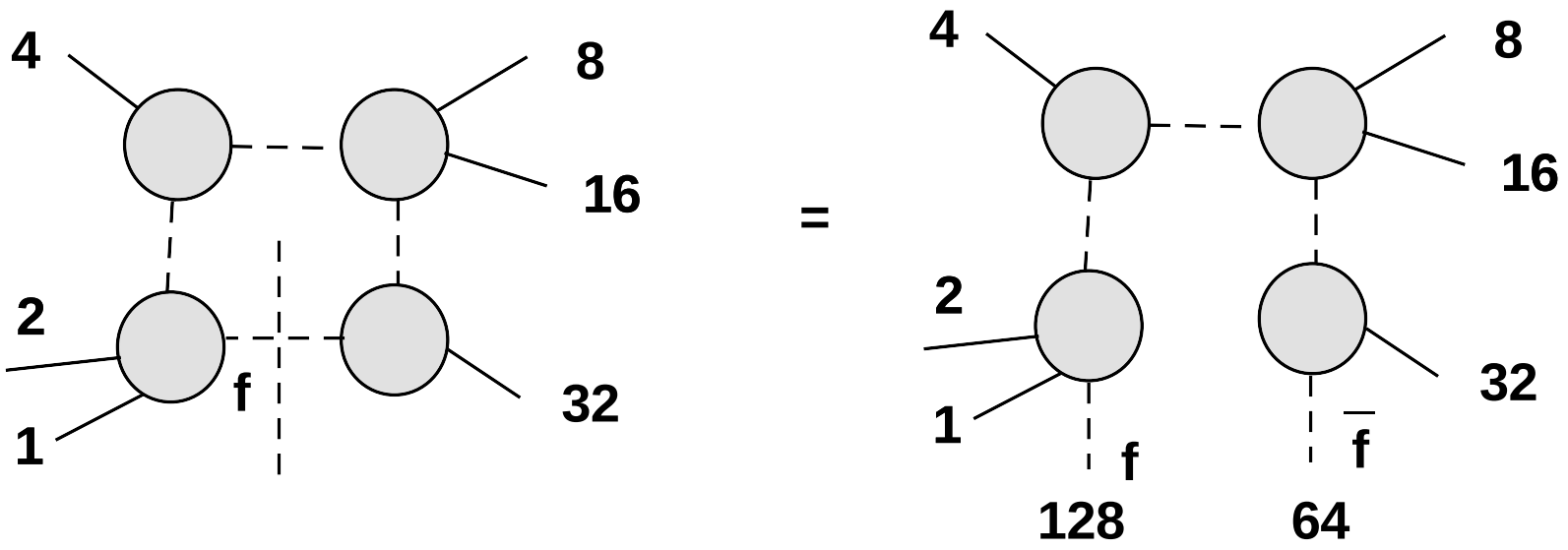
HELAC 1-LOOP

Generate all inequivalent partitions (permutations) of six, five, four, three, etc. blobs attached to the loop, and checking for all possible flavors (and colors) that can be consistently running inside



HELAC 1-LOOP

Hard cut to transform the problem to a (part) $n+2$ tree-order matrix element part (not to be confused with unitarity cuts)



Since now we have to evaluate a tree-order-like contribution, HELAC can trivially provide us with the right answer.

HELAC Color treatment

How color is treated in HELAC ?

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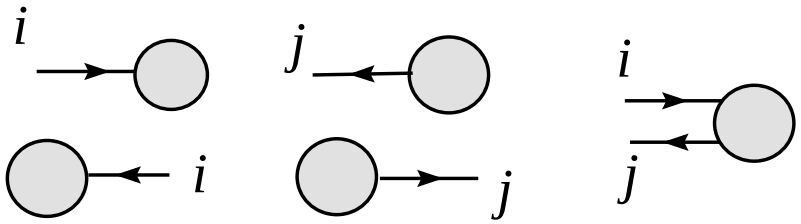
Color-connection representation (quarks and gluons treated uniformly)



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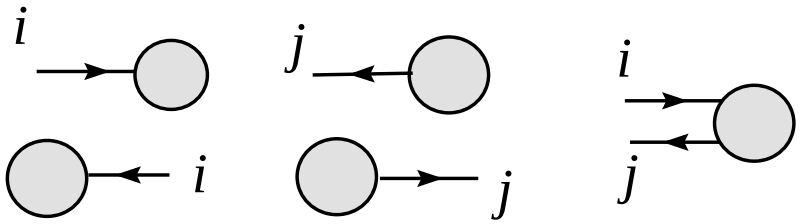

$$\mathcal{M}^{i_1,i_2,\ldots,i_k}_{j_1,j_2,\ldots,j_k} = \sum_{\sigma} \delta_{i_{\sigma_1},j_1}\delta_{i_{\sigma_2},j_2}\ldots\delta_{i_{\sigma_k},j_k}A_{\sigma}$$

The delta's are the 'color structure' or color connection and the A_σ the 'lorentz' structure or color-stripped amplitudes.

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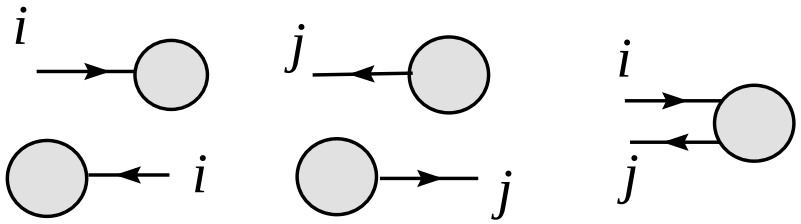

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The delta's are the 'color structure' or color connection and the A_σ the 'lorentz' structure or color-stripped amplitudes.

Important: there are Feynman rules, color-connection or color-flow FR, that allow the calculation of A_σ, once the 'color structure' is known.

HELAC Color treatment - 1 loop

Can we extend this color-connection langauge at one loop ?

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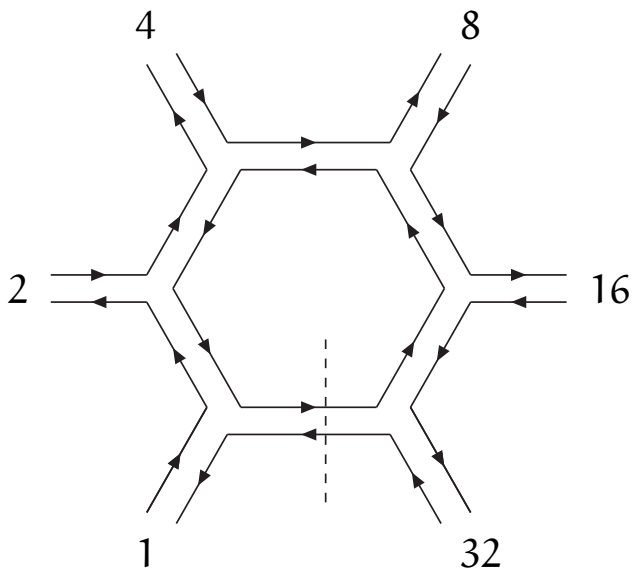
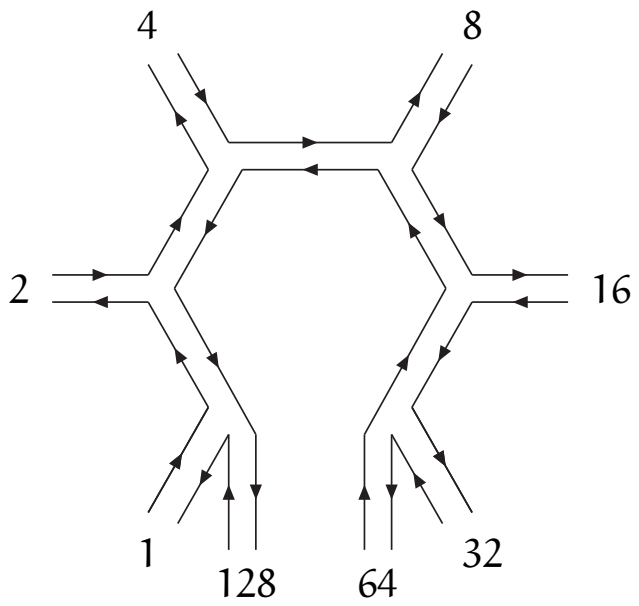
$\delta_{i_8,j_1}\delta_{i_1,j_2}\delta_{i_2,j_3}\delta_{i_3,j_4}\delta_{i_4,j_5}\delta_{i_5,j_6}\delta_{i_6,j_7}\delta_{i_7,j_8}$





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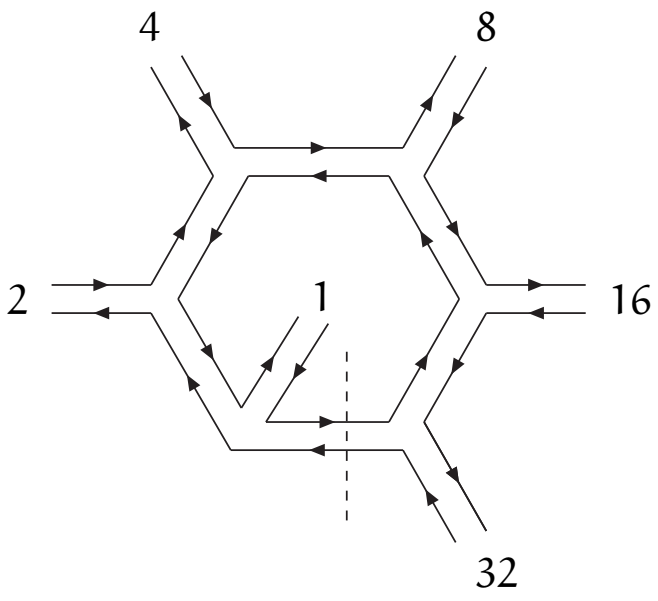

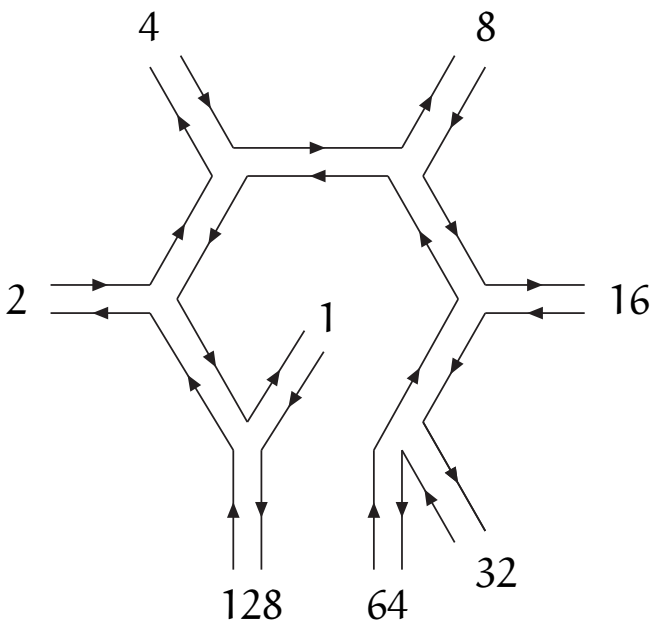

HELAC Color treatment - 1 loop

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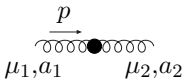
The hard cut plays an important role, since we can still use all tree-order color-connection Feynman rules:

Both 'planar' and 'non-planar' topologies have the same treatment !

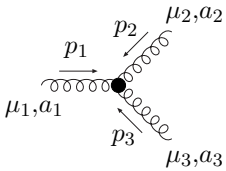
HELAC-1L calculates the full color contribution to the amplitude.

Large N_c is trivially also included, as an option, if required.

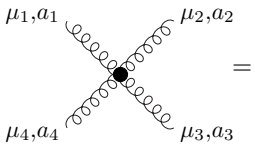
HELAC R2 COUNTER-TERMS



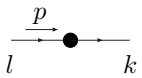
$$= \frac{ig^2 N_{col}}{48\pi^2}\,\delta_{a_1a_2}\Big[\frac{p^2}{2}g_{\mu_1\mu_2} + \lambda_{HV}\left(g_{\mu_1\mu_2}p^2 - p_{\mu_1}p_{\mu_2}\right) + \frac{N_f}{N_{col}}\left(p^2 - 6\,m_q^2\right)g_{\mu_1\mu_2}\Big]$$



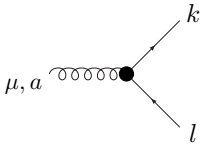
$$= -\frac{g^3 N_{col}}{48\pi^2}\left(\frac{7}{4} + \lambda_{HV} + 2\frac{N_f}{N_{col}}\right) f^{a_1a_2a_3}\,V_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3)$$



$$= -\frac{ig^4 N_{col}}{96\pi^2}\sum_{P(234)}\Big\{\Big[\frac{\delta_{a_1a_2}\delta_{a_3a_4} + \delta_{a_1a_3}\delta_{a_4a_2} + \delta_{a_1a_4}\delta_{a_2a_3}}{N_{col}} + 4\,Tr(t^{a_1}t^{a_3}t^{a_2}t^{a_4} + t^{a_1}t^{a_4}t^{a_2}t^{a_3})\,(3+\lambda_{HV}) - Tr(\{t^{a_1}t^{a_2}\}\{t^{a_3}t^{a_4}\})\,(5+2\lambda_{HV})\Big]\,g_{\mu_1\mu_2}g_{\mu_3\mu_4} + 12\frac{N_f}{N_{col}}Tr(t^{a_1}t^{a_2}t^{a_3}t^{a_4})\left(\frac{5}{3}g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_2}g_{\mu_3\mu_4} - g_{\mu_2\mu_3}g_{\mu_1\mu_4}\right)\Big\}$$

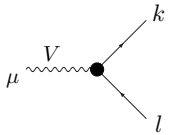


$$= \frac{ig^2}{16\pi^2}\frac{N_{col}^2 - 1}{2N_{col}}\delta_{kl}(-\not{p} + 2m_q)\,\lambda_{HV}$$

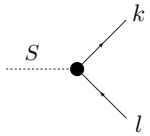


$$= \frac{ig^3}{16\pi^2}\frac{N_{col}^2 - 1}{2N_{col}}\,t^a_{kl}\gamma_\mu\,(1+\lambda_{HV})$$

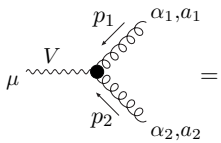
HELAC R2 terms



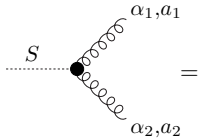
$$= -\frac{g^2}{16\pi^2}\frac{N_{col}^2-1}{2N_{col}}\,\delta_{kl}\gamma_\mu\,(v+a\gamma_5)\,(1+\lambda_{HV})$$



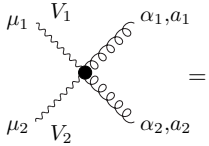
$$= -\frac{g^2}{8\pi^2}\frac{N_{col}^2-1}{2N_{col}}\,\delta_{kl}\,(c+d\gamma_5)\,(1+\lambda_{HV})$$



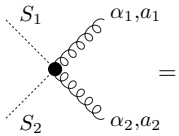
$$= a\,\frac{ig^2}{12\pi^2}\,\delta_{a_1a_2}\epsilon_{\mu\alpha_1\alpha_2\beta}\,(p_1-p_2)^\beta$$



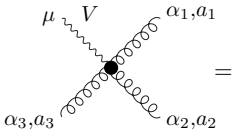
$$= c\,\frac{g^2}{8\pi^2}\,\delta_{a_1a_2}g_{\alpha_1\alpha_2}\,m_q$$



$$= -\frac{ig^2}{24\pi^2}\,\delta_{a_1a_2}(v_1v_2+a_1a_2)\,(g_{\mu_1\mu_2}g_{\alpha_1\alpha_2}+g_{\mu_1\alpha_1}g_{\mu_2\alpha_2}+g_{\mu_1\alpha_2}g_{\mu_2\alpha_1})$$



$$= \frac{ig^2}{8\pi^2}\,\delta_{a_1a_2}\,(c_1c_2-d_1d_2)\,g_{\alpha_1\alpha_2}$$



$$= -\frac{g^3}{24\pi^2}\,\{v\,Tr(t^{a_1}\{t^{a_2}t^{a_3}\})(g_{\mu\alpha_1}g_{\alpha_2\alpha_3}+g_{\mu\alpha_2}g_{\alpha_1\alpha_3}+g_{\mu\alpha_3}g_{\alpha_1\alpha_2})$$

$$-i9a\,\left[Tr(t^{a_1}t^{a_2}t^{a_3})-Tr(t^{a_1}t^{a_3}t^{a_2})\right]\,\epsilon_{\mu\alpha_1\alpha_2\alpha_3}\}$$

HELAC 1-LOOP

```
 Kmax,ncc,Nmax 0 24 0 10
 INFO ==============================================
 INFO COLOR 1 out of 6
INFO   1  12  35   7   1   1   4  -4   3   8   4   4   0   0   0   0   1   2
INFO   1  48  35   8   1   1  16  -8   5  32   8   6   0   0   0   0   1   2
INFO   2  14  -3   9   1   1  12  35   7   2  -3   2   0   0   0   1   1   2
INFO   2  14  -3   9   0   1  12  35   7   2  -3   2   0   0   0   2   1   2
INFO   2  28  -8  10   1   1  12  35   7  16  -8   5   0   0   0   1   1   2
INFO   2  28  -8  10   0   1  12  35   7  16  -8   5   0   0   0   2   1   2
INFO   3  44   8  11   1   1  12  35   7  32   8   6   0   0   0   1   1   2
INFO   3  44   8  11   0   1  12  35   7  32   8   6   0   0   0   2   1   2
INFO   2  50  -3  12   1   1  48  35   8   2  -3   2   0   0   0   1   1   2
INFO   2  50  -3  12   0   1  48  35   8   2  -3   2   0   0   0   2   1   2
INFO   2  52  -4  13   1   1  48  35   8   4  -4   3   0   0   0   1   1   2
INFO   2  52  -4  13   0   1  48  35   8   4  -4   3   0   0   0   2   1   2
INFO   3  56   4  14   1   1  48  35   8   8   4   4   0   0   0   1   1   2
INFO   3  56   4  14   0   1  48  35   8   8   4   4   0   0   0   2   1   2
INFO   1  60  35  15   1   4   4  -4   3  56   4  14   0   0   0   0   1   2
INFO   1  60  35  15   2   4  16  -8   5  44   8  11   0   0   0   0   1   2
INFO   1  60  35  15   3   4  28  -8  10  32   8   6   0   0   0   0   1   2
INFO   1  60  35  15   4   4  52  -4  13   8   4   4   0   0   0   0   1   2
INFO   2  62  -3  16   1   3  12  35   7  50  -3  12   0   0   0   1   1   2
INFO   2  62  -3  16   0   3  12  35   7  50  -3  12   0   0   0   2   1   2
INFO   2  62  -3  16   2   3  48  35   8  14  -3   9   0   0   0   1   1   2
INFO   2  62  -3  16   0   3  48  35   8  14  -3   9   0   0   0   2   1   2
INFO   2  62  -3  16   3   3  60  35  15   2  -3   2   0   0   0   1   1   2
INFO   2  62  -3  16   0   3  60  35  15   2  -3   2   0   0   0   2   1   2
 INFO ==============================================
 INFO COLOR 2 out of 6
INFO   1  12  35   7   1   1   4  -4   3   8   4   4   0   0   0   0   1   1
INFO   1  48  35   8   1   1  16  -8   5  32   8   6   0   0   0   0   1   1
```

HELAC 1-LOOP

papadopo@ aiolos:/tmp - Shell - Konsole

```
 INFO ==============================================
 INFO COLOR 4 out of 6
 INFO number of nums 143
 INFO NUM 1  of  143 10
INFO   3  96   8   9   1   1  64  35   7  32   8   6   0   0   0   1   1   2
INFO   3  96   8   9   0   1  64  35   7  32   8   6   0   0   0   2   1   2
INFO   1 112  35  10   1   1  16  -8   5  96   8   9   0   0   0   0   1   1
INFO   3 120   4  11   1   1 112  35  10   8   4   4   0   0   0   1   1   1
INFO   3 120   4  11   0   1 112  35  10   8   4   4   0   0   0   2   1   1
INFO   1 124  35  12   1   1   4  -4   3 120   4  11   0   0   0   0   1   1
INFO   2 126  -3  13   1   1 124  35  12   2  -3   2   0   0   0   1   1   1
INFO   2 126  -3  13   0   1 124  35  12   2  -3   2   0   0   0   2   1   1
INFO   2 254  -3  14   1   1 128  35   8 126  -3  13   0   0   0   1   1   2
INFO   2 254  -3  14   0   1 128  35   8 126  -3  13   0   0   0   2   1   2
INFO   6  32  16   8   4   2   1  35   8  35   4  35  -3   0   0   0   3   1
INFOYY   1
 INFO NUM 2  of  143 10
INFO   3  96   8   9   1   1  64  35   7  32   8   6   0   0   0   1   1   1
INFO   3  96   8   9   0   1  64  35   7  32   8   6   0   0   0   2   1   1
INFO   1 112  35  10   1   1  16  -8   5  96   8   9   0   0   0   0   1   1
INFO   3 120   4  11   1   1 112  35  10   8   4   4   0   0   0   1   1   1
INFO   3 120   4  11   0   1 112  35  10   8   4   4   0   0   0   2   1   1
INFO   1 124  35  12   1   1   4  -4   3 120   4  11   0   0   0   0   1   2
INFO   2 126  -3  13   1   1 124  35  12   2  -3   2   0   0   0   1   1   2
INFO   2 126  -3  13   0   1 124  35  12   2  -3   2   0   0   0   2   1   2
INFO   2 254  -3  14   1   1 128  35   8 126  -3  13   0   0   0   1   1   1
INFO   2 254  -3  14   0   1 128  35   8 126  -3  13   0   0   0   2   1   1
INFO   6  32  16   8   4   2   1  35   8  35   4  35  -3   0   0   0   3   1
INFOYY   1
 INFO NUM 3  of  143 10
INFO   2  80  -8   9   1   1  64  35   7  16  -8   5   0   0   0   1   1   2
```

HELAC 1-LOOP

papadopo@ aiolos:/tmp - Shell - Konsole

```
 INFO NUM 127  of  143 15
INFO   1  48  35   9   1   1  16  -8   5  32   8   6   0   0   0   0   1   1
INFO   3 112   3  10   1   1  48  35   9  64   3   7   0   0   0   1   1   1
INFO   3 112   3  10   0   1  48  35   9  64   3   7   0   0   0   2   1   1
INFO   1  12  35  11   1   1   4  -4   3   8   4   4   0   0   0   0   1   1
INFO   1 240  35  12   1   1 128  -3   8 112   3  10   0   0   0   0  -1   1
INFO   2 242  -3  13   1   1 240  35  12   2  -3   2   0   0   0   1   1   1
INFO   2 242  -3  13   0   1 240  35  12   2  -3   2   0   0   0   2   1   1
INFO   3 248   4  14   1   1 240  35  12   8   4   4   0   0   0   1   1   1
INFO   3 248   4  14   0   1 240  35  12   8   4   4   0   0   0   2   1   1
INFO   1 252  35  15   1   2   4  -4   3 248   4  14   0   0   0   0   1   1
INFO   4 252  35  15   2   2  12  35  11 240  35  12   0   0   0   0   1   1
INFO   2 254  -3  16   1   2  12  35  11 242  -3  13   0   0   0   1   1   1
INFO   2 254  -3  16   0   2  12  35  11 242  -3  13   0   0   0   2   1   1
INFO   2 254  -3  16   2   2 252  35  15   2  -3   2   0   0   0   1   1   1
INFO   2 254  -3  16   0   2 252  35  15   2  -3   2   0   0   0   2   1   1
INFO   2  48  15   3   3   0   0   0   0   0   0   0   0   0   0   0   2   5
INFOYY   5
 INFO NUM 128  of  143 11
INFO   1  12  35   7   1   1   4  -4   3   8   4   4   0   0   0   0   1   1
INFO   1  48  35   8   1   1  16  -8   5  32   8   6   0   0   0   0   1   1
INFO   2  28  -8   9   1   1  12  35   7  16  -8   5   0   0   0   1   1   1
INFO   2  28  -8   9   0   1  12  35   7  16  -8   5   0   0   0   2   1   1
INFO   3  56   4  10   1   1  48  35   8   8   4   4   0   0   0   1   1   1
INFO   3  56   4  10   0   1  48  35   8   8   4   4   0   0   0   2   1   1
INFO   1  60  35  11   1   3   4  -4   3  56   4  10   0   0   0   0   1   1
INFO   4  60  35  11   2   3  12  35   7  48  35   8   0   0   0   0   1   1
INFO   1  60  35  11   3   3  28  -8   9  32   8   6   0   0   0   0   1   1
INFO  25  62  -3  12   1   1  60  35  11   2  -3   2   0   0   0   1   1   1
INFO  25  62  -3  12   0   1  60  35  11   2  -3   2   0   0   0   2   1   1
INFO   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
INFOYY   1
 INFO NUM 129  of  143 12
INFO  23  12  35   7   1   1   4  -4   3   8   4   4   0   0   0   0   1   1
```

HELAC 1-loop

In summary

- For each color connection the solution of Dyson-Schwinger equation (expressing amplitude in terms of sub-amplitudes) is constructed (integer arithmetic). At the one-loop level the solution is composed by a number of structures that are treatable by CuTtools (OPP) (CC+R_1) and a number of tree-like counter-term structures (R_2)

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- When the phase-space information is available, using the solution above, the numerical evaluation of both the tree-order and one-loop amplitudes is performed

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- When the phase-space information is available, using the solution above, the numerical evaluation of both the tree-order and one-loop amplitudes is performed
- For a realistic calculation two main more ingredients are used: the helicity sampling resulting to a speed-up by a factor equal to the total number of helicity configurations and the color sampling resulting also to a huge reduction in time

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- For each color connection the solution of Dyson-Schwinger equation (expressing amplitude in terms of sub-amplitudes) is constructed (integer arithmetic). At the one-loop level the solution is composed by a number of structures that are treatable by CuTtools (OPP) (CC+R_1) and a number of tree-like counter-term structures (R_2)
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- For a realistic calculation two main more ingredients are used: the helicity sampling resulting to a speed-up by a factor equal to the total number of helicity configurations and the color sampling resulting also to a huge reduction in time
- Leading color approximation is also an option

HELAC RESULTS

$pp \to t\bar{t}b\bar{b}$			
$u\bar{u} \to t\bar{t}b\bar{b}$			
	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
HELAC-1L	-2.347908989000179E-07	-2.082520105681483E-07	3.909384299635230E-07
$I(\epsilon)$	-2.347908989000243E-07	-2.082520105665445E-07	
$gg \to t\bar{t}b\bar{b}$			
HELAC-1L	-1.435108168334016E-06	-2.085070773763073E-06	3.616343483497464E-06
$I(\epsilon)$	-1.435108168334035E-06	-2.085070773651439E-06	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
t	12.99421901255723	-9.591511769543683	75.05543670827210	190.1845561691092
$\bar{t}$	53.73271578143694	-0.2854146459513714	17.68101382654795	182.9642163285034
b	-41.57664370692741	3.895531135098977	-91.94931862397770	100.9874727883170
$\bar{b}$	-25.15029108706678	5.981395280396083	-0.7871319108423604	25.86375471407044

HELAC RESULTS

$pp \to VVb\bar{b}$ and $pp \to VV+$ 2 jets			
$u\bar{u} \to W^+W^-b\bar{b}$			
	ϵ^{-2}	ϵ^{-1}	ϵ^0
HELAC-1L	-2.493916939359002E-07	-4.885901774740355E-07	1.592538533368835E-07
$I(\epsilon)$	-2.493916939359001E-07	-4.885901774752593E-07	
$gg \to W^+W^-b\bar{b}$			
HELAC-1L	-2.686310592221201E-07	-6.078682316434646E-07	-2.431624440346638E-07
$I(\epsilon)$	-2.686310592221206E-07	-6.078682340168020E-07	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
W^+	22.40377113462118	-16.53704884550758	129.4056091248114	154.8819879118765
W^-	92.64238702192333	-0.4920930146078141	30.48443210132545	126.4095336206695
b	-71.68369328357026	6.716416578342183	-158.5329205583824	174.1159068988160
$\bar{b}$	-43.36246487297426	10.31272528177322	-1.357120667754454	44.59257156863792

HELAC RESULTS

$pp \to V+$ 3 jets			
$u\bar{d} \to W^+ggg$			
HELAC-1L	-1.995636628164684E-05	-5.935610843551600E-05	-6.235576400719452E-05
$I(\epsilon)$	-1.995636628164686E-05	-5.935610843566534E-05	
$u\bar{u} \to Zggg$			
HELAC-1L	-7.148261887172997E-06	-2.142170009323704E-05	-1.906378375774021E-05
$I(\epsilon)$	-7.148261887172976E-06	-2.142170009540120E-05	

	p_x	p_y	p_z	E
u	0	0	250	250
$\bar{d}$	0	0	-250	250
W^+	23.90724239064912	-17.64681636854432	138.0897548661186	162.5391101447744
g	98.85942812363483	-0.5251163702879512	32.53017998659339	104.0753327455388
g	-76.49423931754684	7.167141557113385	-169.1717405928078	185.8004692730082
g	-46.27243119673712	11.00479118171890	-1.448194259904179	47.58508783667868

HELAC RESULTS

$pp \to t\bar{t}+$ 2 jets			
$u\bar{u} \to t\bar{t}gg$			
	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
HELAC-1L	-6.127108113312741E-05	-1.874963444741646E-04	-3.305349683690902E-04
$I(\epsilon)$	-6.127108113312702E-05	-1.874963445081074E-04	
$gg \to t\bar{t}gg$			
HELAC-1L	-3.838786514961561E-04	-9.761168899507888E-04	-5.225385984750410E-04
$I(\epsilon)$	-3.838786514961539E-04	-9.761168898436521E-04	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
t	12.99421901255723	-9.591511769543683	75.05543670827210	190.1845561691092
$\bar{t}$	53.73271578143694	-0.2854146459513714	17.68101382654795	182.9642163285034
g	-41.57664370692741	3.895531135098977	-91.94931862397770	100.9874727883170
g	-25.15029108706678	5.981395280396083	-0.7871319108423604	25.86375471407044

HELAC RESULTS

$pp \to b\bar{b}b\bar{b}$			
$u\bar{u} \to b\bar{b}b\bar{b}$			
	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
HELAC-1L	-9.205269484951069E-08	-2.404679886692200E-07	-2.553568662778129E-07
$I(\epsilon)$	-9.205269484951025E-08	-2.404679886707971E-07	
$gg \to b\bar{b}b\bar{b}$			
HELAC-1L	-2.318436429821683E-05	-6.958360737366907E-05	-7.564212339279291E-05
$I(\epsilon)$	-2.318436429821662E-05	-6.958360737341511E-05	

	p_x	p_y	p_z	E
$u(g)$	0	0	250	250
$\bar{u}(g)$	0	0	-250	250
b	24.97040523056789	-18.43157602837212	144.2306511496888	147.5321146846735
$\bar{b}$	103.2557390255471	-0.5484684659584054	33.97680766420219	108.7035966213640
b	-79.89596300367462	7.485866671764871	-176.6948628845280	194.0630765341365
$\bar{b}$	-48.33018125244035	11.49417782256567	-1.512595929362970	49.70121215982584

Outlook

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- has changed the computational approach at one loop

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- Automatize through Dyson-Schwinger equations the full one-loop amplitudes `HELAC-1L` `CuTtools` `AVH_OLO` (Complex-mass scheme for unstable particles)

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- Automatize through Dyson-Schwinger equations the full one-loop amplitudes `HELAC-1L` `CuTtools` `AVH_OLO` (Complex-mass scheme for unstable particles)
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Future

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- has changed the computational approach at one loop
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Future

Several realistic calculations, $pp \to ttbb$, $pp \to tt{+}2$ jets, etc.

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- has changed the computational approach at one loop
- numerical but still algebraic: speed and precision not a problem

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- Automatize through Dyson-Schwinger equations the full one-loop amplitudes `HELAC-1L` `CuTtools` `AVH_OLO` (Complex-mass scheme for unstable particles)
- Automatize the real contributions `HELAC-DIPOLES`

Future

Several realistic calculations, $pp \to ttbb$, $pp \to tt{+}2$ jets, etc.

A generic NLO calculator *ante portas*

Tools 2009 ?

BlackHat

C.F. Berger, Z. Bern, L.J. Dixon, F. Febres Cordero, D. Forde, H. Ita, D.A. Kosower, D. Maitre, arXiv:0803.4180 [hep-ph]

Rocket

W. T. Giele and G. Zanderighi, arXiv:0805.2152 [hep-ph]

CutTools

G. Ossola, C. G. Papadopoulos and R. Pittau, [arXiv:0711.3596 [hep-ph]].

HELAC-1LOOP

A. van Hameren, C. G. Papadopoulos and R. Pittau [arXiv:0903.4665 [hep-ph]].

Tools 2009 ?

SHERPA-DIPOLES

Tanju Gleisberg , Frank Krauss, arXiv:0709.2881 [hep-ph]

HELAC-DIPOLES

M. Czakon, C.G. Papadopoulos, M. Worek . arXiv:0905.0883 [hep-ph]