Renormalisation of the distribution amplitudes of the B meson

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Introduction

- Distribution amplitudes well-known tools for factorisation theorems and light-cone sum rules for light mesons (ρ, π)
- Recent study of B-meson distribution amplitudes related to matrix elements $\langle 0|\bar{q}_\beta(z)\ldots(h_v)_\alpha(0)|B(p)\rangle$
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  - Factorisation for B decays

$$\phi_+ : B \to M_1 M_2, B \to V\gamma^{(*)}, B \to \gamma\ell\nu \qquad \phi_- : B \to V\ell^+\ell^-$$

Beneke et al

  - Light-cone sum rules for semileptonic form factors

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Ball et al., Khodjamirian et al.

Modelling these NP objects requires as many constraints as possible

Renormalisation and mixing when scale changes provide such (perturbative) constraints on models

2 parton B-meson DAs

Non local matrix element with a light-like separation $z_\mu = tn_{+,\mu}$

$$\langle 0|\bar{q}_\beta(z)[z,0](h_v)_\alpha(0)|B(p)\rangle$$
$$= -i\frac{\hat{f}_B(\mu)}{4}\left[(1+\not{v})\left(\tilde{\phi}_+(t) + \frac{\not{z}}{2t}[\tilde{\phi}_-(t) - \tilde{\phi}_+(t)]\right)\gamma_5\right]_{\alpha\beta}$$

with $[z,0]$ the path-ordered exponential of gluon field along n_+

$$[z,0] = P\exp\left[ig_s\int_0^z dy_\mu A^\mu(y)\right]$$
$$n_+^2 = n_-^2 = 0 \qquad n_+\cdot n_- = 2 \qquad v = p/m_B = (n_+ + n_-)/2.$$

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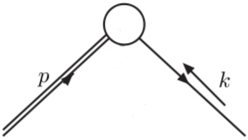
Fourier transforms of the two 2-parton DAs:



$$O_\pm^H(\omega) = \frac{1}{2\pi}\int dt e^{i\omega t}\langle 0|\bar{q}(z)[z,0]\not{n}_\pm\Gamma h_v(0)|H\rangle$$

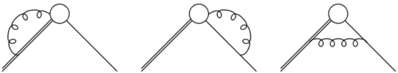
LO : prob of finding light quark with $k_+ = \omega$

Renormalisation properties of ϕ_+, ϕ_-

$$O_\pm^{bare}(\omega) = \int d\omega Z_\pm(\omega,\omega';\mu) O_\pm^{ren}(\omega';\mu) + \ldots$$

$$Z_\pm(\omega,\omega';\mu) = \delta(\omega-\omega') + \frac{\alpha_s C_F}{4\pi} z_\pm^{(1)}(\omega,\omega';\mu)$$

where ellipsis involve operators with higher number of partons



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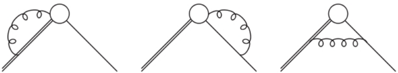

$$z_+^{(1)} = \left(\frac{4}{\varepsilon^2} - \frac{4}{\varepsilon}\log\frac{\omega}{\mu} - \frac{5}{\varepsilon}\right)\delta(\omega-\omega') - \frac{4}{\varepsilon}\left[\frac{\omega}{\omega'}\frac{\theta(\omega'-\omega)}{\omega'-\omega} + \frac{\theta(\omega-\omega')}{\omega-\omega'}\right]_+$$

$$z_-^{(1)} = z_+^{(1)} + \frac{4}{\varepsilon}\frac{\theta(\omega'-\omega)}{\omega'} \qquad d = 4-2\varepsilon$$

$1/\epsilon^2$: Sudakov logarithm terms in RGE, due to cusp between Wilson lines : time-like (heavy quark) and light-like (path-ordered exp)

Korchemsky, Lange & Neubert, Bell & Feldmann

3-parton DA

Four 3-parton DAs (quark,antiquark,gluon) Ψ_V, Ψ_A, X_A, Y_A

$$\langle 0|\bar{q}_\beta(z)[z,uz]gG_{\mu\nu}(uz)z^\nu[uz,0](h_v)_\alpha(0)|B(p)\rangle$$
$$= \frac{\hat{f}_B(\mu)M}{4}\Big[(1+\not{v})\Big[(v_\mu \not{z} - t\gamma_\mu)\left(\tilde{\Psi}_A(t,u)-\tilde{\Psi}_V(t,u)\right)$$
$$-i\sigma_{\mu\nu}z^\nu\tilde{\Psi}_V(t,u) - z_\mu\tilde{X}_A(t,u) + \frac{z_\mu\not{z}}{t}\tilde{Y}_A(t,u)\Big]\gamma_5\Big]_{\alpha\beta}.$$

with the corresponding Fourier transform

$$O_3^H(\omega,\xi) = \int \frac{dt\,du}{(2\pi)^2} e^{i(\omega+\xi u)t}\langle 0|\bar{q}(z)[z,uz]g_sG_{\mu\nu}(uz)z^\nu[uz,0]\Gamma h_v(0)|H\rangle$$

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Relations with 2 parton DAs: Beneke & Feldmann, Kawamura & al.

- Light quark e.o.m.: $\omega\phi_-^B(\omega) + \frac{d-2}{2}\int_0^\infty d\rho[\phi_+^B(\rho) - \phi_-^B(\rho)] = I(\omega)$
- Heavy and light quark e.o.m. : $\omega\phi_-^B(\omega) + (\omega - 2\bar{\Lambda})\phi_+^B(\omega) = J(\omega)$

with I (J) integro-differential functions of $\Psi_A - \Psi_V$ ($\Psi_A + X_A$ and Ψ_V)

RGE for operators

RGE (ellipsis involves operators with higher number of partons)

$$O_{\pm}^{H,bare}(\omega) = \int d\omega' Z_{\pm}(\omega,\omega';\mu) O_{\pm}^{H,ren}(\omega',\mu) + \int d\omega' d\xi' Z_{\pm,3}(\omega,\omega',\xi';\mu) O_3^{H,ren}(\omega',\xi',\mu) + \dots$$

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Any choice of H yield same RGE (short-distance ppty)

- $Z_\pm$ computed in Lange & Neubert and Bell & Feldmann with on-shell quarks $H = h(p)\bar{q}(k)$
- But then $O_3^H = \langle 0|\bar{q}(z)[z,uz]g_s G_{\mu\nu}(uz)z^\nu[uz,0]\Gamma h_v(0)|H\rangle = 0$
- No access to mixing between 2- and 3-parton DAs through RGE

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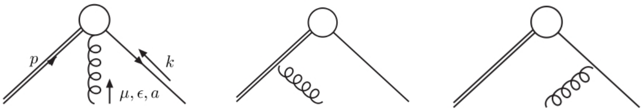
To compute $Z_{\pm,3}$

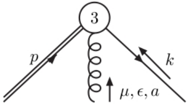
- Take $H = h(p)\bar{q}(k) g(\epsilon, q)$
- Compute $O_{\pm}^{H,bare}$ up to one loop
- Identify the UV divergences and thus determine $Z_{\pm,3}$

Principle of computation

For the matrix element of $O\pm$, we get three contributions at LO



For O_3, we have one leading-order contribution

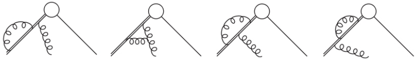


At one loop, for matrix element of $O_\pm$

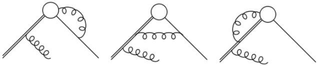
- 44 diagrams in principle
- Focus on UV divergences in dimensional regularisation
- Not all them provide divergent pieces

Diagrams for ϕ_+

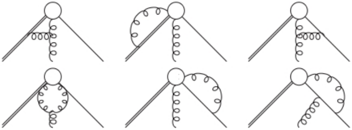
- Wave function and coupling constant renormalisation
- Diagrams corresponding to vertex renormalisation



- Diag of 2 parton DA RGE with gluon attached to external leg



- Six nontrivial diagrams



Result for ϕ_+

We collect the poles in ε and identify bare result with RGE

$$
\begin{aligned}
&\langle 0|O_\pm(\omega)|H\rangle^{bare} = Z_h^{1/2} Z_q^{1/2} Z_3^{1/2} Z_g [A+B+C]^{bare} \\
&\qquad + [2\ parton\ DA] + [Vertex\ renorm] + [Non\ trivial] \\
&= [A+B+C]^{ren}(\mu) + \frac{\alpha_s}{4\pi} \int d\omega' z_\pm^{(1)}(\omega,\omega';\mu)[A+B+C](\omega') \\
&\qquad + \frac{\alpha_s}{4\pi} \int d\omega' d\xi' z_{3\pm}^{(1)\mu}(\omega,\omega',\xi';\mu) A_{3\mu}(\omega',\xi'),
\end{aligned}
$$

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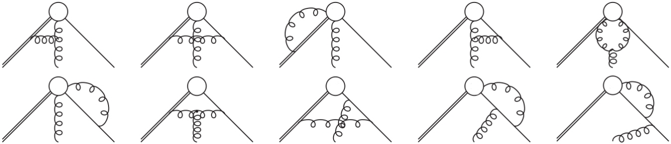
$$= [A+B+C]^{ren}(\mu) + \frac{\alpha_s}{4\pi}\int d\omega' z_\pm^{(1)}(\omega,\omega';\mu)[A+B+C](\omega')$$
$$+\frac{\alpha_s}{4\pi}\int d\omega' d\xi' z_{3\pm}^{(1)\mu}(\omega,\omega',\xi';\mu) A_{3\mu}(\omega',\xi'),$$

- we know $z_\pm^{(1)}(\omega,\omega';\mu)$ from 2-parton DA RGE
- all the poles in the computation match and we get

$$z_{3+}^{(1)}(\omega,\omega'\xi';\mu) = 0$$

ϕ_+ mixes only with itself, and not with three-parton DAs, up to one loop



More contributions for $\langle 0|O_-(\omega)|H\rangle^{bare}$ which can be recast into

$$
= [A+B+C]^{ren}(\mu) + \frac{\alpha_s}{4\pi}\int d\omega' z_-^{(1)}(\omega,\omega';\mu)[A+B+C](\omega')
$$

$$
+\frac{1}{2}\frac{\alpha_s}{4\pi} g_s \frac{1}{\varepsilon}\left[q_+ \bar{v}[\not\epsilon_\perp \not n_+ \not n_- \Gamma T^a]u - \bar{v}[\not q_\perp \not n_+ \not n_- \Gamma T^a] u \epsilon_+\right]
$$

$$
\times\Bigg\{(C_A - 2C_F)\left[\frac{1}{q_+^2}\int_{k_+}^{k_++q_+} dl_+ \left(\frac{1}{l_+} - \frac{1}{k_+}\right) + \frac{1}{(k_+ + q_+)^2}\int_0^{k_++q_+}\frac{dl_+}{k_+}\right]
$$

$$
- C_A \frac{1}{k_+}\left[\frac{1}{(k_+ + q_+)^2}\int_0^{k_++q_+} dl_+ - \frac{1}{q_+^2}\int_0^{q_+} dl_+\right]\Bigg\}
$$

$$
\times \{\delta(\omega - k_+ - q_+ + l_+) - \delta(\omega - k_+ - q_+)\},
$$

Mixing between ϕ_- and $\Psi_V - \Psi_A$

Structure corresponding to 3-parton DA contribution to RGE of ϕ_-

$$\frac{\partial \phi_-(\omega;\mu)}{\partial \log \mu} = -\frac{\alpha_s(\mu)}{4\pi} \left(\int d\omega \gamma_-^{(1)}(\omega,\omega';\mu)\phi_-(\omega';\mu) \right.$$
$$\left. + \int d\omega' d\xi' \gamma_{-,3}^{(1)}(\omega,\omega',\xi';\mu)[\Psi_A - \Psi_V](\omega',\xi';\mu) \right)$$

with the anomalous dimension

$$\gamma_{-,3}^{(1)} = 4\left[\frac{\Theta(\omega)}{\omega'}\left\{(C_A - 2C_F)\left[\frac{1}{\xi'^2}\frac{\omega-\xi'}{\omega'+\xi'-\omega}\Theta(\xi'-\omega) + \frac{\Theta(\omega'+\xi'-\omega)}{(\omega'+\xi')^2}\right]\right.\right.$$
$$\left.\left. - \, C_A\left[\frac{\Theta(\omega'+\xi'-\omega)}{(\omega'+\xi')^2} - \frac{1}{\xi'^2}\left(\Theta(\omega-\omega') - \Theta(\omega-\omega'-\xi')\right)\right]\right\}\right]_+$$

corresponding to the one-loop mixing between ϕ_- and $\Psi_A - \Psi_V$

Comments

- ϕ_+ has a special status (everywhere in factorisation analyses), confirmed by absence of mixing with other B-meson DAs: $\gamma_{+,3} = 0$

  $\Longrightarrow$different from light mesons (where conformal symmetry explains mixing between DAs)

- No contribution from 3 parton DA to regularised first positive moments

$$\langle\omega^N\rangle_\pm(\mu) = \int_0^{\Lambda_{UV}} d\omega\,\omega^N\,\phi_\pm(\omega;\mu)$$

  since $\lim_{\Lambda_{UV}\to\infty}\int_0^{\Lambda_{UV}} d\omega\,\omega^N\,z^{(1)}_{-,3}(\omega,\omega',\xi') = 0 \qquad N = 0,\ 1$

In agreement with model of Bell & Feldmann

Relations between 2- and 3-parton DAs

From equations of motion

- Light quark : $\omega\phi_-^B(\omega) + \frac{d-2}{2}\int_0^\infty d\rho[\phi_+^B(\rho) - \phi_-^B(\rho)] = I(\omega)$
- Heavy and light quarks : $\omega\phi_-^B(\omega) + (\omega - 2\bar{\Lambda})\phi_+^B(\omega) = J(\omega)$

with I (J) integro-differential functions of $\Psi_A - \Psi_V$ ($\Psi_A + X_A$ and Ψ_V)

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- Second equation difficult to make compatible with RGE, since ϕ_- mixes with $\Psi_A - \Psi_V$
  $\Longrightarrow$does not hold beyond LO from nonrelativistic models
  Bell & Feldmann, Kawamura & Tanaka
- But the first one is compatible with RGE !

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Checked in the following way

- Compute evolution kernel for $\Psi_V - \Psi_A$ (same technique as for ϕ_-)
- Take derivative with respect to $\log\mu$
- Replace the derivatives of DAs by evolution kernels

RGI of e.o.m. relation

$$\begin{aligned}
&\omega \frac{d}{d\omega} \int d\omega' d\xi' \gamma^{(1)}_{-,3}(\omega, \omega', \xi')[\Psi_A - \Psi_V](\omega', \xi'; \mu) \\
+ \quad & 2 \int d\omega' \gamma^{(1)}_{+}(\omega, \omega'; \mu) \frac{d}{d\omega'} \int_0^{\omega'} d\rho \int_{\omega'-\rho}^{\infty} \frac{d\xi}{\xi} \frac{\partial}{\partial \xi}[\Psi_A - \Psi_V](\rho, \xi; \mu) \\
= \quad & 2 \int d\omega' d\xi' \frac{d}{d\omega} \int_0^{\omega} d\rho \int_{\omega-\rho}^{\infty} \frac{d\xi}{\xi} \frac{\partial}{\partial \xi} \gamma^{(1)}_{3,3}(\rho, \xi, \omega' \xi'; \mu)[\Psi_A - \Psi_V](\omega', \xi'; \mu)
\end{aligned}$$

holds at order α_s with our RGE kernels for $\phi_+, \phi_-, \Psi_V - \Psi_A$

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holds at order α_s with our RGE kernels for $\phi_+, \phi_-, \Psi_V - \Psi_A$

The relation from light-quark e.o.m. is OK with RGE

$$
\omega \phi^B_-(\omega) + \frac{d-2}{2} \int_0^{\infty} d\rho [\phi^B_+(\rho) - \phi^B_-(\rho)] = I(\omega)
$$

$$
I(\omega; \mu) = 2 \frac{d}{d\omega} \int_0^{\omega} d\rho \int_{\omega-\rho}^{\infty} \frac{d\xi}{\xi} \frac{\partial}{\partial \xi} [\Psi_A(\rho, \xi; \mu) - \Psi_V(\rho, \xi; \mu)]
$$

Conclusions

- B-meson DAs important tools to deal with nonperturbative aspects of *B*-physics
- RGE for the two-meson parton DAs at one loop, including mixing with 3-parton DAs
- No mixing of ϕ_+ with 3-parton DAs, even though no argument from conformal symmetry
- Mixing of ϕ_- with the combination of 3-parton DAs $\Psi_V - \Psi_A$
- Relation between 2- and 3-parton DAs from light-quark e.o.m. compatible with RGE (required computing $\Psi_V - \Psi_A$ at one loop)

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JHEP 0905:091,2009 [0903.0790] and arXiv:0904.4687

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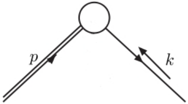
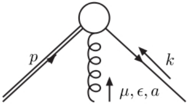
Next : use constraints to improve modelling of 2- and 3-parton DAs and study the impact on phenomenological applications ($B \to V\ell^+\ell^-$)

Back-up

Details of calculus





$\delta(\omega - k_+)\not{n}_\pm$

$-\frac{g_s}{q_+}[\delta(\omega - k_+ - q_+) - (\omega - k_+)n_+^\mu \not{n}_\pm T^a$

Compute UV div for $O_\pm^H$ at one loop as convolution of LO O_X^H and kernels

$\Longrightarrow$Treat separately the $+$ component of the loop integrals

- Integration measure split in light-cone components $d^4 l \to \frac{1}{2} dl_+ dl_- d^2\vec{l}_\perp$ where $l^\mu = \frac{1}{2}(l_+ n_-^\mu + l_- n_+^\mu) + l_\perp^\mu$
- Pick up poles in l_-
- Perform $d^2\vec{l}_\perp$ integration in $d = 2 - 2\varepsilon$
    - $1/\varepsilon$ poles from transverse integration
    - $1/\varepsilon^2$ poles (and related Sudakov double logs) from final l_+ integration

Anomalous dimension of $\Psi_V - \Psi_A$

$$
\begin{aligned}
\gamma^{(1)}_{3,3,C_A} &= 2\Bigg[\delta(\omega-\omega')\left\{\frac{\xi}{\xi'^2}\Theta(\xi'-\xi) - \left[\frac{\Theta(\xi-\xi')}{\xi-\xi'}\right]_+ - \left[\frac{\xi}{\xi'}\frac{\Theta(\xi'-\xi)}{\xi'-\xi}\right]_+\right\} \\
&+ \delta(\xi-\xi')\left\{\left[\frac{\Theta(\omega-\omega')}{\omega-\omega'}\right]_+ + \left[\frac{\omega}{\omega'}\frac{\Theta(\omega'-\omega)}{\omega'-\omega}\right]_+\right\} + \delta(\omega+\xi-\omega'-\xi') \\
&\times \left\{\frac{1}{\xi'}\Theta(\omega-\omega') - \left[\frac{\Theta(\omega-\omega')}{\omega-\omega'}\right]_+ - \left[\frac{\omega}{\omega'}\frac{\Theta(\omega'-\omega)}{\omega'-\omega}\right]_+\right\} \\
&+ \delta(\omega+\xi-\omega'-\xi')\frac{1}{\xi'(\omega'+\xi')}\Bigg\{\frac{\omega-\xi'}{\xi'}(\omega'+\xi'-\omega)\Theta(\omega-\omega') \\
&- \frac{\omega}{\omega'}(\omega'+2\xi'-\omega)\Theta(\omega'-\omega)\Theta(\omega) + \frac{\omega}{\xi'}(\omega-\xi')\Theta(\xi'-\omega)\Theta(\omega) \\
&+ \frac{\omega-\xi'}{\omega'}(\omega'+\xi'-\omega)\Theta(\omega-\xi')\Theta(\xi)\Bigg\}\Bigg] \\
\gamma^{(1)}_{3,3,C_F} &= \gamma^{(1)}_+\delta(\xi-\xi') + 4\delta(\omega+\xi-\omega'-\xi') \\
&\times \left[\frac{\xi^2}{\omega'}\frac{\Theta(\omega'-\xi)}{(\omega+\xi)^2}\Theta(\xi) + \frac{\omega}{\xi'}\frac{\Theta(\xi-\omega')}{\omega+\xi}\Theta(\omega)\left(\frac{\xi}{\omega+\xi} - \frac{\omega-\xi'}{\xi'}\right)\right]
\end{aligned}
$$

with $\gamma^{(1)}_+$ from 2-parton DA self-mixing

Diagrams at one loop