

# Minimal flavour seesaw models

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# A short tale: Chiral Pert. Theory

## Fundamental Lagrangian

$$\mathcal{L} = \sum_{i=u,d,s} q_{Li} i \not{D} q_{Li} + q_{Ri} i \not{D} q_{Ri} - M_{ij} \bar{q}_i q_j - \frac{1}{2} G_{\mu\nu} G^{\mu\nu}$$

Flavour symmetry:  $U(3)_L \times U(3)_R$

$$q_{Li} \rightarrow L q_{Li}, \quad q_{Ri} \rightarrow R q_{Ri}, \quad M \rightarrow LMR^\dagger \quad L \in U(3)_L, R \in U(3)_R$$



## Effective theory (ChPT)

$$\mathcal{L}_{\text{Ch}} = \frac{f^2}{4} \text{Tr } \partial \Sigma^\dagger \partial \Sigma - \frac{f^2 \Lambda}{2} \text{Tr } M \Sigma, \quad \Sigma \rightarrow R \Sigma L^\dagger$$

## SPURION PARADIGMA

# Flavour symmetry of the SM

Great Big Model of the Whole Universe



Effective Theory (Standard Model)

$$\mathcal{L}_{SM} = \mathcal{L}_{kin} - V(\text{Higgs}) - Y_d \bar{Q}_u H d_R - Y_u \bar{Q}_u \tilde{H} u_R - \dots$$

Flavour group :

$$G = SU(3)_{Q_L} \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e$$

$$Q_u \rightarrow L_u Q_u, \quad d_R \rightarrow R_d d_R, \dots \quad Y_d \rightarrow L_u Y_u R_d^\dagger, \dots$$

# Flavour symmetry of the SM

## Great Big Model of the Whole Universe

Probably also flavour symmetry

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# Minimal Flavour Violation

Not very useful... unless one can say something more about the fundamental theory.

Assume  $\mathcal{L}_{\text{fund.}} = \mathcal{L}(Y_u, Y_d, \dots + \text{fields})$ .

## MINIMAL FLAVOUR VIOLATION

In the low energy Lagrangian (SM) we have

$$\mathcal{L}_{\text{eff}} = \sum_i c_{d=5}^i \mathcal{O}_{d=5}^i + c_{d=6}^i \mathcal{O}_{d=6}^i + \dots$$

$$c_{d=5} \equiv c_{d=5}(Y_u, Y_d), \quad c_{d=6} \equiv c_{d=6}(Y_u, Y_d), \quad \text{etc}$$

**MFV Hypothesis**  $\equiv$  The Yukawas are the only sources (*irreducible*) of flavour violation.

R. S. Chivukula and H. Georgi, Phys. Lett. B 188, 99 (1987)



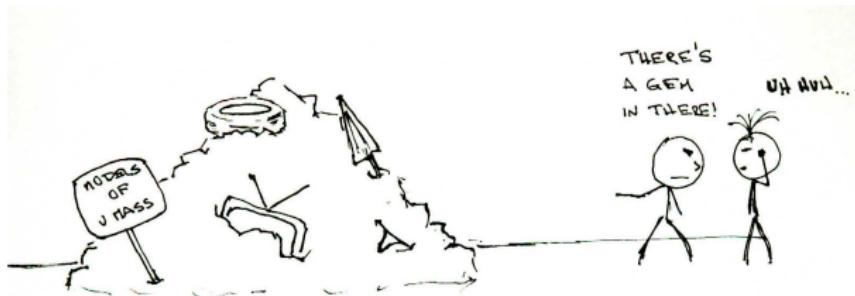
# In the case of leptons

In the lepton sector

$$\mathcal{L}_{SM} = \dots + Y_e \bar{L} \phi e_R + (?) + \sum_i c_{d=5}^i \mathcal{O}_{d=5}^i + c_{d=6}^i \mathcal{O}_{d=6}^i \dots$$

neutrino mass  $\longrightarrow c_{d=5} \equiv c_{d=5}(Y_e, ?)$ ,  $c_{d=6} \equiv c_{d=6}(Y_e, ?)$   $\longleftarrow$  flavour processes

Need mass for the neutrinos! (Model dependent, Majorana)



# Requirements for a model of MFV

- Fundamental theory that reduces to the SM.
- Tiny neutrino masses should be deducible.  $\Lambda_{LN}$  Big.
- Rare flavour processes should be measurable.  $\Lambda_{fl} \ll \Lambda_{LN}$
- Predictivity. The flavour structure of the coefficients of the  $d = 6$  operator should be fixed by that of the  $d = 5$  one.

Ex:

$$c_{d=6}^{\alpha\beta} \propto c_{d=5}^{\alpha\beta}, \quad c_{d=6}^{\alpha\beta} \propto (c_{d=5}^\dagger)^{\alpha\sigma} c_{d=5}^{\sigma\beta}, \quad \text{any other possibility?}$$

# Massive Neutrinos

Massive neutrinos  $\Rightarrow$  NEW PHYSICS!

*Mass Found in Elusive Particle;  
Universe May Never Be the Same*

*Discovery on Neutrino*

*Detecting  
Neutrinos*

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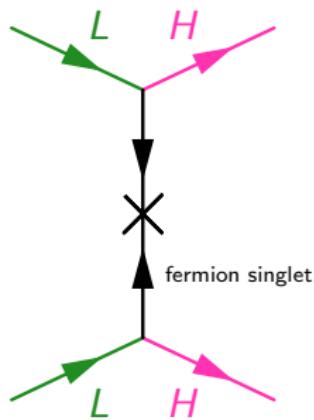
- Dirac Mass. Unnatural particular case of the following.
- Majorana Mass.  $\bar{\nu}^c_L \nu_L$ . Violates  $U(1)_Y$

Hence Weinberg operator

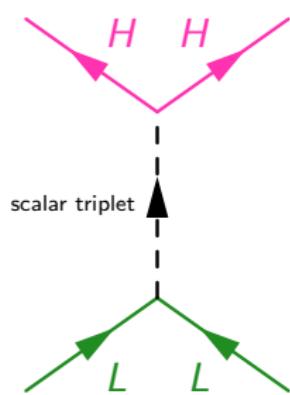
$$O_W = \frac{c_{d=5}}{\Lambda_{\text{NP}}} \bar{L}^c L H^T H$$

# Seesaw Models

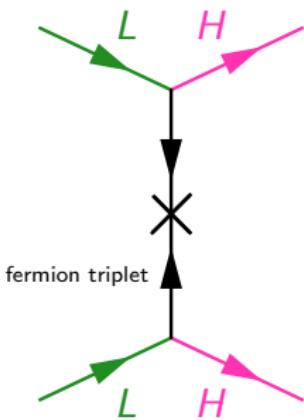
- Three types of models yield the Weinberg operator at tree level



Type I



Type II

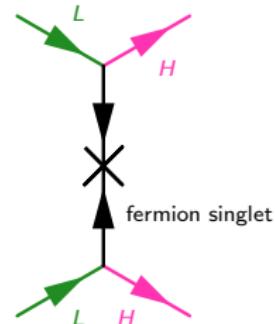


Type III

# An unsuccessful model

Standard Seesaw (Type I) doesn't work

$$\mathcal{L} = \dots - Y_N \bar{N} \phi^\dagger L_L - \Lambda_{LN} \bar{N}^c N \dots$$



- **Neutrino masses:** Ok.  $M_\nu \propto Y_N^T \frac{1}{\Lambda_{LN}} Y_N$
- **Measurable flavour:** NOT OK!.  $\Lambda_{fl} \equiv \Lambda_{LN}$
- **Predictivity:** More or less Ok.  $c_{d=5} \propto c_{d=6}$  if no CP

# Scalar mediated seesaw

$$\begin{aligned}\mathcal{L}_\Delta = & \cdots + (D_\mu \Delta)^\dagger (D^\mu \Delta) - m_\Delta^2 \Delta^\dagger \Delta + + \\ & + Y_{\Delta}^{\alpha\beta} \overline{\ell_L} (\tau \cdot \Delta) \ell_L + \mu_\Delta \tilde{\phi}^\dagger (\tau \cdot \Delta)^\dagger \phi + \dots\end{aligned}$$

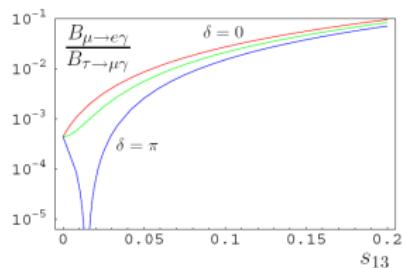
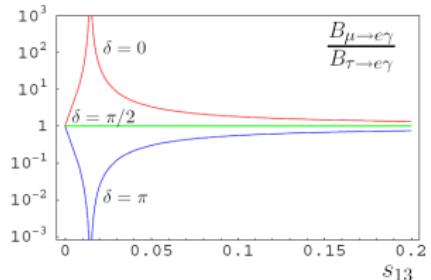
Yields the *effective coefficients*

$$c_{d=5}^{\alpha\beta} \propto Y_{\Delta\alpha\beta} \frac{\mu_\Delta}{M_\Delta^2},$$

$$c_{d=6}^{\alpha\beta\gamma\delta} \propto \frac{1}{M_\Delta^2} Y_{\Delta\alpha\beta}^\dagger Y_{\Delta\delta\gamma}.$$

Hence

$$c_{d=6}^{\alpha\beta\gamma\delta} \propto (c_{d=5}^\dagger)^{\alpha\beta} c_{d=5}^{\gamma\delta}$$



V. Cirigliano, B. Grinstein, G. Isidori, M. Wise, hep-ph/0507001.

M. B. Gavela, T. Hambye, P. Hernández, D.H., 0906.1461

# Leptonic Seesaws I

$$\mathcal{L}_{M_\nu} = \begin{pmatrix} \bar{L}_i & \bar{N}^c & \bar{N'}^c \\ 0 & Y_N^T v & \epsilon Y_N'^T v \\ Y_N v & 0 & \Lambda \\ \epsilon Y_N' v & \Lambda & 0 \end{pmatrix} \quad \begin{matrix} L_i^c \\ N \\ N' \end{matrix} \xrightarrow{\text{one generation}}$$

Lepton number violation driven by  $\epsilon$

$$c_{\alpha\beta}^{d=5} \equiv \epsilon \left( Y_N'^T \frac{1}{\Lambda} Y_N + Y_N^T \frac{1}{\Lambda} Y_N' \right)_{\alpha\beta}, \quad c_{\alpha\beta}^{d=6} \equiv \frac{1}{\Lambda^2} \left( Y_N^\dagger Y_N \right)_{\alpha\beta} + o(\epsilon)$$

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## FUNDAMENTAL

	moduli	phases
$Y_N$	3	3
$Y_N'$	3	3
$\Lambda$	1	1

vs

## LOW ENERGY

- 3 angles and 2 phases in the  $U_{PMNS}$
- 2 masses and 0 phases in  $M_\nu$
- 2 overall factors and 5 phases absorbed.

- A normalization factor apart, Yukawas are determined from the  $U_{PMNS}$  and neutrino masses!

# Leptonic Seesaw II

For both hierarchies

$$Y_N = \frac{y}{\sqrt{2}} \left[ f_1 \left( \frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} \right) U_{i3}^* + f_2 \left( \frac{\Delta m_{sol}^2}{\Delta m_{atm}^2} \right) U_{i2}^* \right]$$

- Yukawas are a linear combination of the rows of the PMNS matrix.
- Spectrum determined (modulo normal/inverted hierarchy)

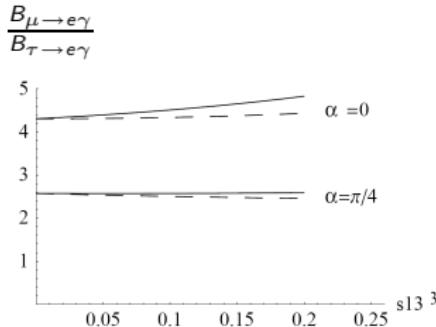
Hence, measuring  $U_{PMNS}$ , and  $\nu$  masses determines  
EVERYTHING!!

Ex:

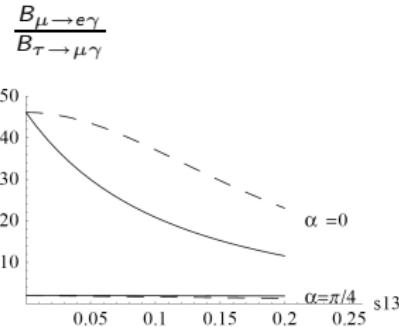
$$B_{\mu \rightarrow e\gamma} \propto |Y_{N_e} Y_{N_\mu}|^2, \quad |m_{ee}|_{IH} \simeq |s_{12}^2 e^{-2i\alpha} - c_{12}^2 e^{2i\alpha}|$$

# Leptonic Seesaw III

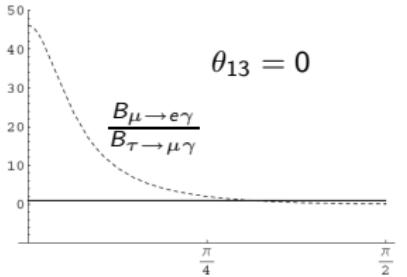
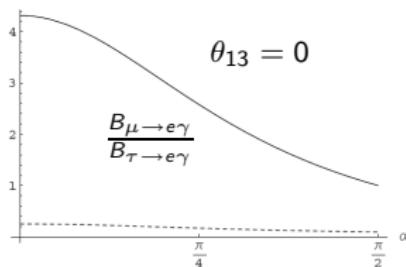
## NORMAL HIERARCHY



## INVERTED HIERARCHY



Strong dependence on the Majorana phase!



# Leptonic Seesaw IV

For the mass matrix

$$\mathcal{L}_{M_\nu} = \begin{pmatrix} \bar{L}_i & \bar{N}^c & \bar{N}'^c \\ 0 & Y_N^T v & \epsilon Y_N'^T v \\ Y_N v & \mu' & \Lambda^T \\ \epsilon Y_N' v & \Lambda & \mu \end{pmatrix} \quad \begin{matrix} L_i^c \\ N \\ N' \end{matrix} \xrightarrow{\text{one generation}}$$

Redefine

$$\tilde{Y}_N = Y'_N - \frac{\mu}{2\epsilon\Lambda} Y_N$$

to recover previous case.  $\tilde{c}_{5,6}(Y', Y) = c_{5,6}(\tilde{Y}, Y)$

- $Y_N$  can still be found using the same algorithm and  $c_{d=6}$  is determined from  $c_{d=5}$

# Conclusions

- Simple seesaw models with at least two separate scales are realizations of the MFV hypothesis in the lepton sector. Ex: Type II Seesaw, Inverse Seesaws, etc.
- Furthermore, a very simple model of realistic neutrino mass generation has been presented within the defined MFV framework. It implements as well several attracting features that include,
  - A separation of the typical scales of flavour and lepton number breaking processes.
  - A full determination of the Yukawa vectors up to overall factors.
  - Only three parameters undetermined by present data, a CP phase, a Majorana phase and  $\theta_{13}$