Higher-order QCD corrections to vector boson production at hadron colliders

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arXiv:0812.2862 & **arXiv:0903.2120**

Outline

1. Transverse momentum distribution: fixed order and resummation

2. Fully Exclusive NNLO Drell-Yan calculation

3. Conclusions and Perspectives

Motivations

The study of vector boson production is well motivated:

- Large production rates and clean experimental signatures:
  - Important for detector calibration.
  - Possible use as luminosity monitor.
- Transverse momentum distributions needed for:
  - Precise prediction for M_W.
  - Beyond the Standard Model analysis.
- Test of perturbative QCD predictions.
- Constrain for fits of PDFs.

The Drell-Yan q_T distribution

$$h_1(p_1) + h_2(p_2) \rightarrow V(M, q_T) + X \rightarrow \ell_1 + \ell_2 + X$$

$$\textit{where} \quad V = \gamma^*, Z^0, W^\pm \quad \textit{and} \quad \ell_1\ell_2 = \ell^+\ell^-, \ell\nu_\ell$$

According to the QCD factorization theorem:

$$\frac{d\sigma}{dq_T^2}(q_T,M,s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T,M,\hat{s};\alpha_S,\mu_R^2,\mu_F^2)$$

The standard fixed-order QCD perturbative expansions gives:

$$\int_{Q_T^2}^{\infty} dq_T \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \sim \alpha_S \left[c_{12} \log^2(M^2/Q_T^2) + c_{11} \log(M^2/Q_T^2) + c_{10}(Q_T) \right]$$

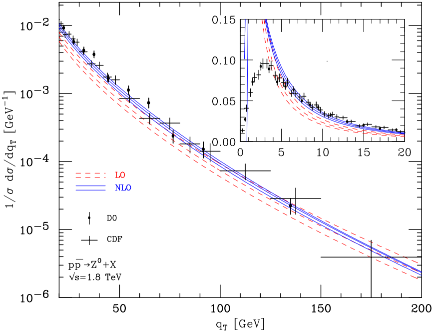
$$+ \alpha_S^2 \left[c_{24} \log^4(M^2/Q_T^2) + \cdots + c_{21} \log(M^2/Q_T^2) + c_{20}(Q_T) \right] + \mathcal{O}(\alpha_S^3)$$

Fixed order calculation theoretically justified only in the region $q_T \sim M_V$

For $q_T \rightarrow 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$: need for resummation of logarithmic corrections

Fixed order results: q_T spectrum of Drell-Yan e^+e^- pairs at $\sqrt{s} = 1.8\ TeV$

Vector boson transverse-momentum distribution known up to NLO
[Ellis et al.('83)],[Arnold,Reno('89)], [Gonsalves et al.('89)]



- CDF data: $\sigma_{tot} = 248 \pm 11\, pb$ [CDF Coll.('00)]
  D0 data: $\sigma_{tot} = 221 \pm 11\, pb$ [D0 Coll.('00)]
- Factorization and renormalization scale variations: $\mu_F = \mu_R = m_Z$, $m_Z/2 \le \mu_F, \mu_R \le 2m_Z$, $1/2 \le \mu_F/\mu_R \le 2$.
- LO and NLO scale variations bands overlap only for $q_T > 70\ GeV$.
- Good agreement between NLO results and data up to $q_T \sim 20\ GeV$.
- In the small q_T region ($q_T \lesssim 20\ GeV$) LO and NLO result diverges to $+\infty$ and $-\infty$.

In the small q_T region ($q_T \lesssim 20\ GeV$) effects of soft-gluon resummation are essential

At Tevatron 90% of the $W^\pm$ and Z^0 are produced with $q_T \lesssim 20\ GeV$

Transverse momentum resummation

[Parisi,Petronzio.('79)], [Kodaira,Trentadue('82)], [Altarelli et al.('84)], [Collins,Soper,Sterman('85)], [Catani,de Florian,Grazzini('01)]

$$\frac{d\hat{\sigma}_{ab}}{dq_T^2} = \frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2};$$

The finite component $\left(\lim_{Q_T\to 0}\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}_{ab}^{(fin)}}{dq_T^2}\right]_{f.o.} = 0\right)$ ensure to reproduce the fixed order calculation at large q_T

Resummation holds in impact parameter space:

$$\frac{d\hat{\sigma}_{ab}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}}\int_0^\infty db\,\frac{b}{2}J_0(bq_T)\,\mathcal{W}_{ab}(b,M),$$

$q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M^2/q_T^2 \gg 1 \Leftrightarrow \log Mb \gg 1$

In the Mellin moments space we have the exponentiated form:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\} \qquad \text{where} \qquad L \equiv \log\left(\frac{M^2 b^2}{b_0^2}\right)$$

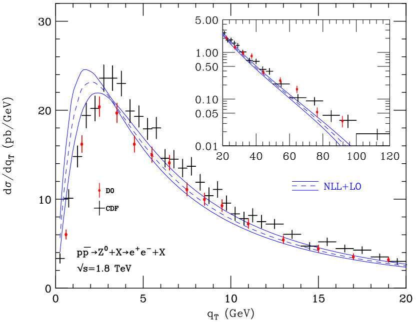
$$\mathcal{G}_N(\alpha_S, L) = L\,g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \cdots; \qquad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}(\alpha_S, M)\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_N^{(2)} + \cdots\right]$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, $(\sigma^{(0)})$; NLL ($\sim \alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

We computed the function $\mathcal{H}_N^{(2)}$ recently. In the study presented here we have performed the resummation up to NLL matched with the LO calculation.

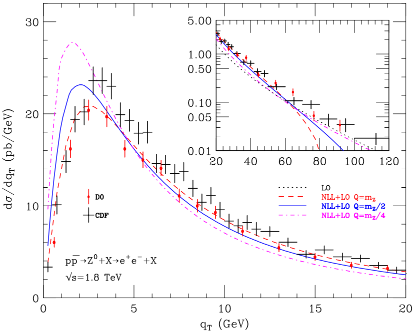
Resummed results: q_T spectrum of Drell-Yan e^+e^- pairs at $\sqrt{s} = 1.8\ TeV$

[Bozzi,Catani,G.F.,de Florian,Grazzini: arXiv:0812.2862]



- CDF data: $\sigma_{tot} = 248 \pm 11\, pb$ [CDF Coll.('00)]
  D0 data: $\sigma_{tot} = 221 \pm 11\, pb$ [D0 Coll.('00)]
- Our calculation implements γ^*/Z interference and finite-width effects. Here we use the narrow width approximation (differences within 1% level).
- Variation of factorization and renormalization scales as in customary fixed-order calculations.
- NLL+LO resummed result fits reasonably well also in the $q_T \lesssim 20\, GeV$ (without a model for non-perturbative effects).

Resummed results: q_T spectrum of Drell-Yan e^+e^- pairs at $\sqrt{s} = 1.8\ TeV$



- Resummation scale and perturbative unitarity constrain:

$$\ln\left(\frac{M^2 b}{b_0^2}\right) \rightarrow \ln\left(\frac{Q^2 b}{b_0^2} + 1\right)$$

- Integral of the NLL+LO curve reproduce the total NLO cross section to better 1%.
- NLL+LO results for different values of the resummation scale Q (estimate of higher-order logarithmic contributions).
- We vary $Q = m_Z/2, \quad m_Z/4 \leq Q \leq m_Z$: uncertainty $\pm 12 - 15\%$ in the region $q_T \gtrsim 20\ GeV$ (it dominate over the renormalization and factorization scale variations).
- We expect a sensible reduction once the complete NNLL+NLO calculation will be available.

Fully Exclusive NNLO calculation

- We have recently performed a fully exclusive NNLO calculation for vector boson production in hadron collisions [Catani,Cieri,G.F.,de Florian,Grazzini: arXiv:0903.2120], using a new version of subtraction formalism [Catani,Grazzini('07)].
- The calculation is implemented in a parton level Monte Carlo and includes the γ-Z interference, finite-width effects, the leptonic decay of the vector bosons. An analogous computation exist [Melnikov,Petriello('06)].

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; V(M, q_T) + X \;\rightarrow\; \ell_1 + \ell_2 + X$$

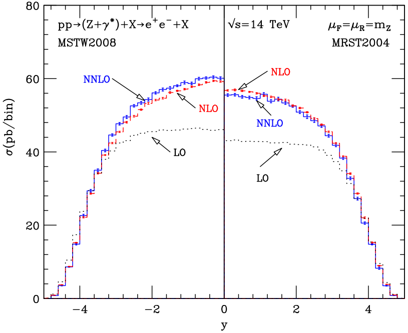
At LO the q_T of the vector boson is exactly zero. This means that for $q_T \neq 0$

$$d\sigma^{V}_{(N)NLO}|_{q_T\neq 0} = d\sigma^{V+\text{jets}}_{(N)LO} \quad ,$$

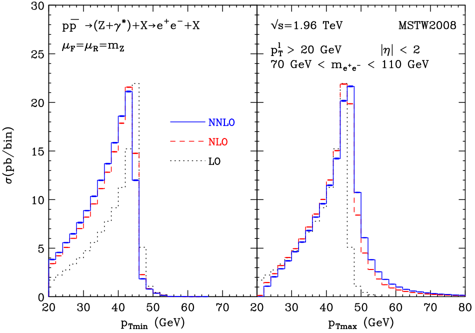
and the NNLO IR divergences can be cancelled by using the subtraction method at NLO. We treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions to q_T distributions.

$$d\sigma^{V}_{(N)NLO} = \frac{1}{\sigma^{(0)}}\mathcal{H}^{V}_{(N)NLO} \otimes d\sigma^{V}_{LO} + \left[d\sigma^{V+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO}\right] \quad ,$$

$$\textit{where} \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}(\alpha_S, M)\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}^{(1)}_N + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)}_N + \cdots\right]$$



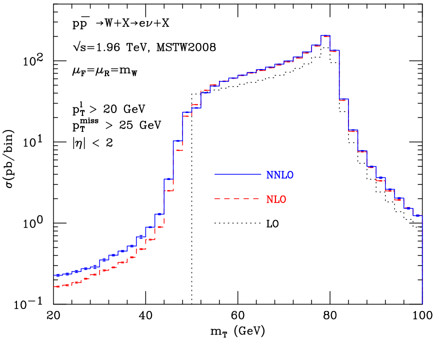
Rapidity distribution for Z production at the LHC.



Minimum (left) and maximum (right) lepton p_T distribution for Z production at the Tevatron.

- Left panel: MSTW 2008 pdf. Going from NLO to NNLO the total cross section increase by about 3%.
- Right panel: MRST 2004 pdf. Going from NLO to NNLO the total cross section decrease by about 2%.

- At LO the distributions are kinematically bounded by $p_T < Q_{max}/2$.
- The NNLO corrections make the $p_{T_{min}}$ distribution softer, and the $p_{T_{max}}$ distribution harder.



Transverse mass distribution for W production at the Tevatron:

$$m_T = \sqrt{2p_T^l p_T^{miss}(1-\cos\phi_{l\nu})}$$

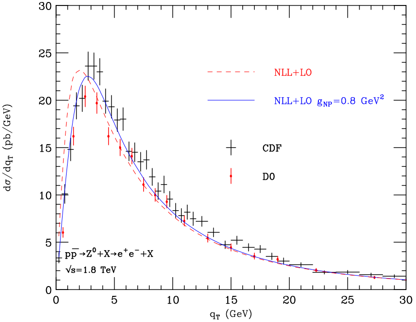
- The LO distribution is bounded at $m_T = 50$ GeV. At LO the W is produced with $q_T = 0$ therefore, the requirement $p_T^{\rm miss} > 25$ GeV sets $m_T \geq 50$ GeV.
- Around this region there are perturbative instabilities in going from LO to NLO and to NNLO.
- The origin of such instabilities is due to (integrable) logarithmic singularities in the vicinity of the boundary (Sudakov shoulder [Catani,Webber ('97)]).
- Below the boundary, the $\mathcal{O}(\alpha_S^2)$ corrections are large (for istance +40% at $m_T \sim 30$ GeV). This is not unexpected, since in this region the $\mathcal{O}(\alpha_S^2)$ result is actually only a NLO calculation.

Conclusions and Perspectives

- We have presented a study on transverse momentum distribution of Drell-Yan lepton pairs produced in hadronic collisions.
- We have applied the q_T-resummation formalism developed in [Catani,de Florian, Grazzini('01)], [Bozzi,Catani,de Florian, Grazzini('06)] performing the resummation up to NLL+LO, implementing the calculation in a numerical code.
- Study of scale dependence indicate that the NLL+LO theoretical uncertainty is relative large and is dominated by missing higher order logaritmic contributions.
- Future implementations: with the available $\mathcal{H}_N^{(2)}$ coefficient it is now possible to perform a complete NNLL+NLO calculation.
- We have presented a fully exclusive NNLO QCD calculation implemented in a parton level Monte Carlo. The program allows the user to apply arbitrary kinematical cuts on the final state and compute distribution in form of bin histograms.
- A public version of both numerical codes will be available in the near future.

Backup slides

Non perturbative effects: q_T spectrum of Drell-Yan e^+e^- pairs at $\sqrt{s}=1.8\,TeV$



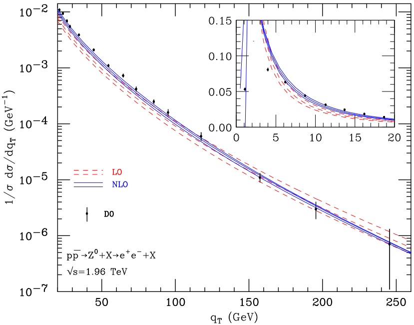
- Up to now result in a complete perturbative framework.
- Non perturbative effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \quad \to \quad \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\, S_{NP}$$

$$g_{NP} = 0.8\, GeV^2$$ [Kulesza et al.('02)]

- With NP effects there is a better agreement with the data.
- Quantitative impact of such NP effects is within perturbative uncertainties.

Fixed order results: q_T spectrum of Drell-Yan e^+e^- pairs at $\sqrt{s} = 1.96\ TeV$



- D0 data: $70\ GeV < M^2 < 110\ GeV$, normalized to unity [D0 Collaboration ('08)]
- Normalization reduces only marginally fixed order scale variations

$$\left(\frac{1}{\sigma}\frac{d\sigma}{dq_T}\right)_{(N)LO}(\mu_F,\mu_R) \equiv \frac{1}{\sigma_{(N)NLO}(\mu_F,\mu_R)}\frac{d\sigma_{(N)LO}}{dq_T}(\mu_F,\mu_R)\,.$$

- Experimental errors very small but bins are larger.
- Qualitatively same situation of Tevatron Run I data.

In the small q_T region ($q_T \lesssim 20\ GeV$) effects of soft-gluon resummation are essential

At Tevatron 90% of the $W^\pm$ and Z^0 are produced with $q_T \lesssim 20\ GeV$