Higher-order QCD corrections to vector boson production at hadron colliders

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Outline

1. Transverse momentum distribution: fixed order and resummation

2. Fully Exclusive NNLO Drell-Yan calculation

3. Conclusions and Perspectives
Motivations

The study of vector boson production is well motivated:

- Large production rates and clean experimental signatures:
  - Important for detector calibration.
  - Possible use as luminosity monitor.
- Transverse momentum distributions needed for:
  - Precise prediction for $M_W$.
  - Beyond the Standard Model analysis.
- Test of perturbative QCD predictions.
- Constrain for fits of PDFs.
The Drell-Yan $q_T$ distribution

$$h_1(p_1) + h_2(p_2) \rightarrow V(M, q_T) + X \rightarrow \ell_1 + \ell_2 + X$$

where $V = \gamma^*, Z^0, W^\pm$ and $\ell_1 \ell_2 = \ell^+ \ell^-, \ell \nu_\ell$

According to the QCD factorization theorem:

$$\frac{d\sigma}{dq_T^2}(q_T, M, s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2 f_a/h_1(x_1, \mu_F^2) f_b/h_2(x_2, \mu_F^2) \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S, \mu_R, \mu_F)$$

The standard fixed-order QCD perturbative expansions gives:

$$\int_{Q_T^2}^\infty dq_T \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \sim \alpha_S \left[ c_{12} \log^2(M^2/Q_T^2) + c_{11} \log(M^2/Q_T^2) + c_{10}(Q_T) \right]$$

$$+ \alpha_S^2 \left[ c_{24} \log^4(M^2/Q_T^2) + \cdots + c_{21} \log(M^2/Q_T^2) + c_{20}(Q_T) \right] + O(\alpha_S^3)$$

Fixed order calculation theoretically justified only in the region $q_T \sim M_V$

For $q_T \rightarrow 0$, $\alpha_S^m \log^n(M^2/q_T^2) \gg 1$: need for resummation of logarithmic corrections
Fixed order results: $q_T$ spectrum of Drell-Yan $\text{e}^+\text{e}^-$ pairs at $\sqrt{s} = 1.8 \text{ TeV}$

Vector boson transverse-momentum distribution known up to NLO

[Ellis et al. ('83)], [Arnold, Reno ('89)], [Gonsalves et al. ('89)]

- CDF data: $\sigma_{tot} = 248 \pm 11 \text{ pb}$ [CDF Coll. ('00)]
- D0 data: $\sigma_{tot} = 221 \pm 11 \text{ pb}$ [D0 Coll. ('00)]

- Factorization and renormalization scale variations:
  \[ \mu_F = \mu_R = m_Z, \quad m_Z/2 \leq \mu_F, \mu_R \leq 2m_Z, \]
  \[ 1/2 \leq \mu_F/\mu_R \leq 2. \]

- LO and NLO scale variations bands overlap only for $q_T > 70 \text{ GeV}$.

- Good agreement between NLO results and data up to $q_T \sim 20 \text{ GeV}$.

- In the small $q_T$ region ($q_T \lesssim 20 \text{ GeV}$) LO and NLO result diverges to $+\infty$ and $-\infty$.

In the small $q_T$ region ($q_T \lesssim 20 \text{ GeV}$) effects of soft-gluon resummation are essential.

At Tevatron 90% of the $W^\pm$ and $Z^0$ are produced with $q_T \lesssim 20 \text{ GeV}$.
Transverse momentum resummation

[Parisi,Petronzio.(’79)], [Kodaira,Trentadue(’82)], [Altarelli et al.(’84)], [Collins,Soper,Sterman(’85)], [Catani,de Florian,Grazzini(’01)]

\[
\frac{d\hat{\sigma}_{ab}}{dq_T^2} = \frac{d\hat{\sigma}_{ab}^{(\text{res})}}{dq_T^2} + \frac{d\hat{\sigma}_{ab}^{(\text{fin})}}{dq_T^2};
\]

The finite component \( \left( \lim_{Q_T \to 0} \int_0^{Q_T^2} dq_T^2 \left[ \frac{d\hat{\sigma}_{ab}^{(\text{fin})}}{dq_T^2} \right]_{\text{f.o.}} = 0 \) ensures to reproduce the fixed order calculation at large \( q_T \).

Resummation holds in impact parameter space:

\[
\frac{d\hat{\sigma}_{ab}^{(\text{res})}}{dq_T^2} = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(bq_T) \mathcal{W}_{ab}(b, M), \quad q_T \ll M \Leftrightarrow Mb \gg 1, \quad \log \frac{M^2}{q_T^2} \gg 1 \Leftrightarrow \log Mb \gg 1
\]

In the Mellin moments space we have the exponentiated form:

\[
\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp \left\{ \mathcal{G}_N(\alpha_S, L) \right\} \quad \text{where} \quad L \equiv \log \left( \frac{M^2b^2}{b_0^2} \right)
\]

\[
\mathcal{G}_N(\alpha_S, L) = L g^{(1)}(\alpha_S L) + g^{(2)}_N(\alpha_S L) + \frac{\alpha_S}{\pi} g^{(3)}_N(\alpha_S L) + \cdots; \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}(\alpha_S, M) \left[ 1 + \frac{\alpha_S}{\pi} \mathcal{H}^{(1)}_N + \left( \frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{(2)}_N + \cdots \right]
\]

LL \( \sim \alpha_S^n L^{n+1} \): \( g^{(1)}, (\sigma^{(0)}) \); NLL \( \sim \alpha_S^n L^n \): \( g^{(2)}_N, \mathcal{H}^{(1)}_N \); NNLL \( \sim \alpha_S^n L^{n-1} \): \( g^{(3)}_N, \mathcal{H}^{(2)}_N \).

We computed the function \( \mathcal{H}^{(2)}_N \) recently. In the study presented here we have performed the resummation up to NLL matched with the LO calculation.
Resummed results: \( q_T \) spectrum of Drell-Yan \( e^+e^- \) pairs at \( \sqrt{s} = 1.8 \text{ TeV} \)


- CDF data: \( \sigma_{tot} = 248 \pm 11 \text{ pb} \) [CDF Coll. ('00)]
  D0 data: \( \sigma_{tot} = 221 \pm 11 \text{ pb} \) [D0 Coll. ('00)]
- Our calculation implements \( \gamma^*/Z \) interference and finite-width effects. Here we use the narrow width approximation (differences within 1% level).
- Variation of factorization and renormalization scales as in customary fixed-order calculations.
- NLL+LO resummed result fits reasonably well also in the \( q_T \lesssim 20 \text{ GeV} \) (without a model for non-perturbative effects).
Resummed results: $q_T$ spectrum of Drell-Yan $e^+e^-$ pairs at $\sqrt{s} = 1.8\ TeV$

- Resummation scale and perturbative unitarity constrain:
  $$\ln\left(\frac{M^2 b}{b_0^2}\right) \rightarrow \ln\left(\frac{Q^2 b}{b_0^2} + 1\right)$$
- Integral of the NLL+LO curve reproduce the total NLO cross section to better 1%.
- NLL+LO results for different values of the resummation scale $Q$ (estimate of higher-order logarithmic contributions).
  - We vary $Q = m_Z/2$, $m_Z/4 \leq Q \leq m_Z$:
    - uncertainty $\pm 12 - 15\%$ in the region $q_T > 20\ GeV$
      (it dominate over the renormalization and factorization scale variations).
  - We expect a sensible reduction once the complete NNLL+NLO calculation will be available.
Fully Exclusive NNLO calculation

- We have recently performed a fully exclusive NNLO calculation for vector boson production in hadron collisions [Catani, Cieri, G.F., de Florian, Grazzini: arXiv:0903.2120], using a new version of subtraction formalism [Catani, Grazzini (’07)].

- The calculation is implemented in a parton level Monte Carlo and includes the $\gamma-Z$ interference, finite-width effects, the leptonic decay of the vector bosons. An analogous computation exists [Melnikov, Petriello (’06)].

\[ h_1(p_1) + h_2(p_2) \rightarrow V(M, q_T) + X \rightarrow \ell_1 + \ell_2 + X \]

At LO the $q_T$ of the vector boson is exactly zero. This means that for $q_T \neq 0$

\[ d\sigma^{V}_{(N)NLO}|_{q_T \neq 0} = d\sigma^{V+\text{jets}}_{(N)LO}, \]

and the NNLO IR divergences can be cancelled by using the subtraction method at NLO. We treat the NNLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions to $q_T$ distributions.

\[ d\sigma^{V}_{(N)NLO} = \frac{1}{\sigma(0)} \mathcal{H}^{V}_{(N)NLO} \otimes d\sigma^{V}_{LO} + \left[ d\sigma^{V+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO} \right], \]

where \[ \mathcal{H}_N(\alpha_S) = \sigma^{(0)}(\alpha_S, M) \left[ 1 + \frac{\alpha_S}{\pi} \mathcal{H}^{(1)}_N + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{(2)}_N + \cdots \right] \]
Rapidity distribution for $Z$ production at the LHC.

- Left panel: MSTW 2008 pdf. Going from NLO to NNLO the total cross section increase by about 3%.
- Right panel: MRST 2004 pdf. Going from NLO to NNLO the total cross section decrease by about 2%.

Minimum (left) and maximum (right) lepton $p_T$ distribution for $Z$ production at the Tevatron.

- At LO the distributions are kinematically bounded by $p_T < Q_{max}/2$.
- The NNLO corrections make the $p_{Tmin}$ distribution softer, and the $p_{Tmax}$ distribution harder.
The LO distribution is bounded at $m_T = 50$ GeV. At LO the $W$ is produced with $q_T = 0$ therefore, the requirement $p_T^{\text{miss}} > 25$ GeV sets $m_T \geq 50$ GeV.

Around this region there are perturbative instabilities in going from LO to NLO and to NNLO.

The origin of such instabilities is due to (integrable) logarithmic singularities in the vicinity of the boundary (Sudakov shoulder [Catani, Webber (’97)]).

Below the boundary, the $O(\alpha_S^2)$ corrections are large (for instance $+40\%$ at $m_T \sim 30$ GeV). This is not unexpected, since in this region the $O(\alpha_S^2)$ result is actually only a NLO calculation.

Transverse mass distribution for $W$ production at the Tevatron:

$$m_T = \sqrt{2p_T^l p_T^{\text{miss}}(1 - \cos \phi_{l\nu})}$$
Conclusions and Perspectives

- We have presented a study on transverse momentum distribution of Drell-Yan lepton pairs produced in hadronic collisions.
- We have applied the $q_T$-resummation formalism developed in [Catani, de Florian, Grazzini ('01)], [Bozzi, Catani, de Florian, Grazzini ('06)] performing the resummation up to NLL+LO, implementing the calculation in a numerical code.
- Study of scale dependence indicate that the NLL+LO theoretical uncertainty is relative large and is dominated by missing higher order logarithmic contributions.
- Future implementations: with the available $H_N^{(2)}$ coefficient it is now possible to perform a complete NNLL+NLO calculation.
- We have presented a fully exclusive NNLO QCD calculation implemented in a parton level Monte Carlo. The program allows the user to apply arbitrary kinematical cuts on the final state and compute distribution in form of bin histograms.
- A public version of both numerical codes will be available in the near future.
Backup slides
Non perturbative effects: $q_T$ spectrum of Drell-Yan $e^+e^-$ pairs at $\sqrt{s}=1.8$ TeV

- Up to now result in a complete perturbative framework.
- Non perturbative effects parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$:

$$\exp\{G_N(\alpha_S, \tilde{L})\} \rightarrow \exp\{G_N(\alpha_S, \tilde{L})\} S_{NP}$$

$$g_{NP} = 0.8 \text{ GeV}^2 \quad [\text{Kulesza et al. (’02)}]$$

- With NP effects there is a better agreement with the data.
- Quantitative impact of such NP effects is within perturbative uncertainties.
Fixed order results: $q_T$ spectrum of Drell-Yan $e^+e^-$ pairs at $\sqrt{s} = 1.96$ TeV

- D0 data: $70 \text{ GeV} < M^2 < 110 \text{ GeV}$, normalized to unity
  [D0 Collaboration ('08)]

- Normalization reduces only marginally fixed order scale variations

\[
\left( \frac{1}{\sigma} \frac{d\sigma}{dq_T} \right)_{\text{LO}}(\mu_F,\mu_R) \equiv \frac{1}{\sigma_{(N)NLO}(\mu_F,\mu_R)} \frac{d\sigma_{(N)LO}(\mu_F,\mu_R)}{dq_T}.
\]

- Experimental errors very small but bins are larger.

- Qualitatively same situation of Tevatron Run I data.

In the small $q_T$ region ($q_T \lesssim 20 \text{ GeV}$) effects of soft-gluon resummation are essential

At Tevatron 90% of the $W^\pm$ and $Z^0$ are produced with $q_T \lesssim 20 \text{ GeV}$