Dynamical Parton Distributions
at NNLO

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1. General framework

Standard **FOPT QCD** evolution: **FFNS** with $n_f = 3$ \( (F_{2,L}^\text{HQ} \text{ up to } \mathcal{O}(\alpha_s^2)) \)

**Chi-square + Hessian** method (experimental uncertainties):

\[
\chi^2(p) = \sum_{i=1}^{N} \left( \frac{\text{data}(i) - \text{theory}(i,p)}{\text{error}(i)} \right)^2, \\
\Delta\chi^2 = \chi^2 - \chi^2_0 \approx \frac{1}{2} \sum_{i,j=1}^{d} H_{ij}(a_i - a_i^0)(a_j - a_j^0) \leq T^2
\]

with *tolerance parameter* \( T^2 = T_{1\sigma}^2 = \frac{\sqrt{2N}}{(1.65)^2} \Rightarrow T \approx 4.5 \)

1178 **DIS** data points

\[
\left\{ \begin{array}{l}
\text{FT (SLAC,BCDMS,E665,NMC): } F_{2}^{p,n,d} \\
\text{HERA (H1,ZEUS): } \sigma^r \leftrightarrow F_{2,L,3}^p
\end{array} \right.
\]

390 **Drell-Yan** data points E866: \( \frac{d\sigma_{pp,pd}}{dM \, dx_F} \)

Inclusive jets and HQ production data not included \((GJR08)\) for consistency
2. The dynamical approach

Idea: at low-enough $Q^2$ only “valence” partons would be “resolved”

$\rightarrow$ structure at higher $Q^2$ appears *radiatively* (i.e. due to QCD *dynamics*)

$$xf(x, Q^2_0) = N x^a (1 - x)^b (1 + A\sqrt{x} + Bx)$$

**DYNAMICAL:**

$Q^2_0 < 1 \text{ GeV}^2$ optimally *determined*  
**Standard:**  
$Q^2_0 = 2 \text{ GeV}^2$ arbitrarily *fixed*

**Positive definite** input distributions  
Arbitrary fine tunning ($g < 0!!$)

$a > 0$ “valence”-like  
**Unrestricted** parameters

Connection with NP models  
Only measured region $Q^2 \geq Q^2_0$

QCD *predictions* for $x \leq 10^{-2}$  
**Extrapolations** to unmeasured region

More restrictive, less uncertainties  
Less restrictive, *smaller* $\chi^2$
3. The determination of $\alpha_s(M^2_Z)$

Only free parameter (besides masses) in QCD: acceptable agreement

However “dispersion” > uncertainties: global fits (DIS) yield smaller values

Our results:

\[ \text{dynamical: } 0.1124 \pm 0.0020 \]
\[ \text{standard: } 0.1158 \pm 0.0035 \]

Indeed dynamical results (gluon) **more constrained**!

Other NNLO determinations:

<table>
<thead>
<tr>
<th></th>
<th>AMP (06)</th>
<th>MSTW (08)</th>
<th>ABKM (09)</th>
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<tbody>
<tr>
<td></td>
<td>$0.1128 \pm 0.0015$</td>
<td>$0.1171 \pm 0.0014$</td>
<td>$0.1135 \pm 0.0014$</td>
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</tbody>
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Errors scale with $T$ + *Bayesian* treatments yield smaller estimates
4. Dynamical input distributions

Optimal determination:
\[ Q_o^2 = 0.55 \text{ GeV}^2 \]

Similar quark distributions:
- Valence distributions enhanced around \( x \simeq 0.1 \) to 0.2
- Rather flat sea input in both cases (within \( \simeq 1\sigma \))

Rather **different input gluon**!
- Even more *valencelike* (\( a_g \simeq 1 \) compared to \( a_g \simeq 0.5 \) at NLO)
- Strongly enhanced around \( x \simeq 0.1 \)
Uncertainties decrease as $Q^2$ increase: \textit{pQCD evolution}

\textbf{Valence-like} input + \textit{larger evolution} distance $\Rightarrow$ \textit{stronger constrained}

NNLO falls below NLO ($P_{gg}^{(2)}$ negative and more singular.)
6. Dynamical vs standard: sea distributions

Less noticeable differences: all distributions quite similar

Rather flat sea ($a_\Sigma \simeq 0.14$) ⇒ marginally smaller errors than “standard”

NNLO generally above NLO
7. Small-$x$ DIS data

Dynamical: QCD predictions for $x \leq 10^{-2}$; $\chi^2_{\text{DIS}} = 0.90$

Standard: Fine tuned at $Q^2_0 = 2$ GeV$^2$; $\chi^2_{\text{DIS}} = 0.87$

Higher-twist relevant at small $Q^2$
8. Typical uncertainty bands

NNLO corrections at the “few percent” \( (\chi^2_{\text{NNLO}} \simeq 0.9 \chi^2_{\text{NLO}}) \) with somewhat (not much) reduced uncertainty bands

\[ \rightarrow \text{NNLO effects small to be distinguished by the (DIS) data} \]

Other effects \( (QED, \text{factorization schemes, higher-twist . . .}) \) comparable
9. The Drell-Yan asymmetry

Closely related to the $DY$-asymmetry:

$$A_{DY} = \frac{\sigma_{pp} - \sigma_{pn}}{\sigma_{pp} + \sigma_{pn}}$$

$DY$ data instrumental in fixing $\bar{d} - \bar{u}$

Experimentally $x_1 > x_2 \leftrightarrow$ dominated by beam quark + target antiquark

NNLO effects even smaller

“standard” results very similar
10. **DIS “reduced” cross-section**

\[
\sigma_{r}^{NC} \equiv \left( \frac{2\pi \alpha^{2} Y_{+}}{xyQ^{2}} \right)^{-1} \frac{d^{2}\sigma_{NC}^{+}}{dx\,dy} = F_{2}^{NC} - \frac{y^{2}}{Y_{+}} F_{L}^{NC} + \frac{Y_{-}}{Y_{+}} x F_{3}^{NC}
\]

Usually dominated by \( F_{2}^{\gamma} \)

\[
y = \frac{Q^{2}}{s} \frac{1}{x}
\]

For fixed \( Q^{2} \) (and \( s \)) \( F_{L} \) relevant with increasing \( y \)

\( \Rightarrow \text{turnover} \) at small \( x \)

\( F_{L} \) positive
11. The perturbative stability of $F_L$

Quark and gluon contributions “compensate” in the error bars

* dynamical: perturbatively stable already at $Q^2 \gtrsim 3 \text{ GeV}^2 \rightarrow$ NNLO effects?

* standard: no clear differences due to the larger uncertainty bands
**Gluon** dominated in the small-\(x\) region

**Positive** and in complete **agreement** with measurements

Dynamical predictions more tightly constrained

Higher-twist effects may contribute for \(Q^2 \leq 2\) GeV\(^2\)
Outlook

**Dynamical** PDFs (..., GRV98, GJR08) extended to **NNLO**

Dynamical approach: more *predictive* and *smaller uncertainties*

**NNLO** predictions stable and in agreement with data ($\chi^2_{\text{NNLO}} \simeq 0.9 \chi^2_{\text{NLO}}$)

**Small effects**: NNLO/NLO usually not distinguishable (by data)

**Positive** distributions (gluon) and cross-sections ($F_L$) *possible*

Dynamical predictions for $F_L$ stable and in agreement with all data

Consistent determination (together with the distributions) of $\alpha_s(M_Z^2)$

http://doom.physik.uni-dortmund.de/pdfserver