Nonperturbative Field Theory

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1. Solving (nonperturbatively) QFT's
   - The quintessential example: sine-Gordon
   - QCD???
   - $\mathcal{N} = 4$ Super-Yang-Mills theory

2. The AdS/CFT correspondence and $\mathcal{N} = 4$ SYM

3. Why is it interesting?

4. Methods at strong coupling

5. Towards an exact solution for any coupling

6. Example: Konishi anomalous dimension

7. Outlook
The quintessential example: sine-Gordon

- sine-Gordon theory is a **two-dimensional** QFT defined by the Lagrangian

\[ S = \int d^2 x \left\{ \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{\mu^2}{\beta^2} \cos(\beta \Phi) \right\} \]

- Expanding \( \cos(\beta \Phi) \) for small \( \beta \) generates an infinite set of vertices for \( \Phi \), one can develop perturbation theory...

- There exist solitons (and anti-solitons)... classical solutions, quadratic fluctuations...

- \textit{But} the soliton-soliton S-matrix has been found analytically \textbf{for any} \( \beta \)

\[ S(\theta) = - \exp \left\{ 2 \int_0^\infty \frac{dk}{k} \sin (k \theta) \frac{\sinh(p - 1)k}{2 \cosh k \sinh pk} \right\} \]

where

\[ p = \frac{\beta^2}{8\pi - \beta^2} \]
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- How is it possible??
- The sine-Gordon theory is an example of an **integrable quantum field theory**
- These theories possess additional higher spin/nonlocal conserved charges
- As a consequence the S-matrix factorizes into $2 \rightarrow 2$ scatterings and obeys Yang-Baxter Equation which allows to *nonperturbatively* find it exactly
- All this works **only** in two dimensions! No generalization to four-dimensional theories exists...

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QCD

- Inherently difficult...
- Running coupling
- Mixture of perturbative and nonperturbative effects...

Traditional methods:
- Lattice QCD (inherently Euclidean, ‘black box’)
- Models of QCD vacuum
- Effective models (not derivable from the fundamental theory)

Look at other, ‘simpler’ gauge theories for which other methods exist...
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- $\mathcal{N} = 4$ Super Yang-Mills ($\equiv$ ordinary Yang-Mills + 4 adjoint fermions + 6 adjoint scalars + appropriate self-interactions)
- This theory is
  1. supersymmetric
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The AdS/CFT correspondence

\[ \mathcal{N} = 4 \text{ Super Yang-Mills theory} \equiv \text{Superstrings on } AdS_5 \times S^5 \]

- strong coupling
  - nonperturbative physics
  - very difficult
- weak coupling
  - ‘easy’

- (semi-)classical strings
  - or supergravity
  - ‘easy’
- highly quantum regime
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- New ways of looking at nonperturbative gauge theory physics...
- Intricate links with General Relativity...
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More theoretical perspective:

- Fascinating as it relates two completely different theoretical constructions: 4D gauge theory and string theory in 10D
- Use $\mathcal{N} = 4$ SYM as a theoretical laboratory for studying nonperturbative gauge theory physics
- In this theory one can perform quite rigorous computations at strong coupling
- The natural language of the AdS/CFT correspondence appropriate to strongly coupled $\mathcal{N} = 4$ SYM is quite new w.r.t. conventional gauge theory methods
- Try to build some new physical intuitions within this new language

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More practical perspective:

- Although its behaviour is quite different from QCD at $T = 0$ (no confinement etc.), at \textit{nonzero} temperature supersymmetry is automatically broken, the theory is deconfined.

- Use the results on strong coupling properties of $\mathcal{N} = 4$ plasma as a point of reference for analyzing/describing QCD plasma. 
  
  see talk by R. Peschanski in the Heavy-Ion session
  see plenary talk by U. Wiedemann later today

- Works directly in Minkowski signature.

- In particular many gauge-theoretical problems are translated into geometrical General Relativity like questions which are tractable (even analytically).
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- Use the results on strong coupling properties of \( \mathcal{N} = 4 \) plasma as a point of reference for analyzing/describing QCD plasma. See talk by R. Peschanski in the Heavy-Ion session, see plenary talk by U. Wiedemann later today.

- Works directly in Minkowski signature.

- In particular many gauge-theoretical problems are translated into geometrical General Relativity like questions which are tractable (even analytically).
What degrees of freedom appear on the string side???

- Vibrational modes of the string $\equiv$ particles/fields in $AdS_5 \times S^5$
- Massless modes $\equiv$ graviton$+$...
- At strong coupling massive modes are very heavy $\rightarrow$ it is enough to restrict oneself to dynamics of massless modes $\equiv$ gravity
- At weaker coupling massive string modes become important...

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How to describe strings in $AdS_5 \times S^5$?

- Consider a closed string in $AdS_5 \times S^5$:

  - The embedding coordinates of the point $(\tau, \sigma)$ are quantum fields $X^\mu(\tau, \sigma)$ on the worldsheet which has the geometry of a cylinder.
  - String theory in $AdS_5 \times S^5 \equiv$ a specific two dimensional quantum field theory defined on a cylinder (worldsheet QFT).
  - It turns out that this worldsheet QFT is integrable and one can expect to solve this theory exactly for any coupling! (recall sine-Gordon...)
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What do we mean by ‘solve’?

Local operators in gauge theory \[\langle O(x)O(y) \rangle = \frac{\text{const}}{|x - y|^{2\Delta}}\] \[\leftrightarrow\] String states in \(AdS_5 \times S^5\)

Operator dimension

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One has to find the energy levels of an integrable two-dimensional QFT...

Romuald A. Janik (Krakow) Nonperturbative Field Theory 12 / 26
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**Example: Konishi anomalous dimension**

- **Simplest nonprotected operator: the Konishi operator**

\[
\text{tr } \Phi^2_i \quad \longleftrightarrow \quad \text{tr } Z^2 X^2 + \ldots \quad \longleftrightarrow \quad \text{tr } Z D^2 Z + \ldots
\]

- When computing anomalous dimensions from two point functions there are two types of graphs:

  and

- The first class is contained in the so-called Asymptotic Bethe Ansatz of Beisert and Staudacher
- The second class are ‘wrapping interactions’ which start to appear at order \( g^{2L} \) (these are not contained in the Asymptotic Bethe Ansatz)
- Last year these wrapping graphs were computed at 4 loops by F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon
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\[ W_{B1} = \quad W_{B2} = \quad W_{B3} = \]
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\[ W_{B7} = \quad W_{B8} = \quad W_{B9} = \]

Figure C.1: Wrapping diagrams with chiral structure \( \chi(1, 2, 3) \)
Figure C.2: Wrapping diagrams with chiral structure $\chi(1, 3, 2)$

<table>
<thead>
<tr>
<th>$W_{C1} \rightarrow \ast$</th>
<th>1</th>
<th>$W_{C4} \rightarrow$ finite</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{C2} \rightarrow \ast$</td>
<td>2</td>
<td>$W_{C5} \rightarrow -W_{C3}$</td>
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Table C.2: Results of $D$-algebra for diagrams with structure $\chi(1, 3, 2)$

Figure C.3: Wrapping diagrams with chiral structure $\chi(2, 1, 3)$
Figure C.6: Wrapping diagrams with chiral structure $\chi(1)$ (continued)
\[ I_1 = J_1 = \frac{1}{(4\pi)^8} \left( -\frac{1}{24\varepsilon^4} + \frac{1}{4\varepsilon^3} - \frac{19}{24\varepsilon^2} + \frac{5}{4\varepsilon} \right) \]

\[ I_2 = \frac{1}{(4\pi)^8} \left( -\frac{1}{24\varepsilon^4} + \frac{1}{4\varepsilon^3} - \frac{19}{24\varepsilon^2} + \frac{1}{\varepsilon} \left( \frac{5}{4} - \zeta(3) \right) \right) \]

\[ I_3 = J_5 = \frac{1}{(4\pi)^8} \left( -\frac{1}{12\varepsilon^4} + \frac{1}{3\varepsilon^3} - \frac{5}{12\varepsilon^2} - \frac{1}{\varepsilon} \left( \frac{1}{2} - \zeta(3) \right) \right) \]

\[ I_4 = \frac{1}{(4\pi)^8} \left( -\frac{1}{6\varepsilon^4} + \frac{1}{3\varepsilon^3} + \frac{1}{3\varepsilon^2} - \frac{1}{\varepsilon} (1 - \zeta(3)) \right) \]

\[ I_5 = \frac{1}{(4\pi)^8} \frac{1}{\varepsilon} 5\zeta(5) \]

\[ I_6 = \frac{1}{(4\pi)^8} \left( \frac{1}{12\varepsilon^2} - \frac{7}{12\varepsilon} \right) \]

\[ I_7 = \frac{1}{(4\pi)^8} \frac{1}{\varepsilon} (-\zeta(3)) \]

\[ I_8 = \frac{1}{(4\pi)^8} \left( \frac{1}{4\varepsilon^2} - \frac{11}{12\varepsilon} \right) \]

\[ I_9 = \frac{1}{(4\pi)^8} \frac{1}{\varepsilon} \left( \frac{1}{2} \zeta(3) - \frac{5}{2} \zeta(5) \right) \]

\[ I_{10} = \frac{1}{(4\pi)^8} \frac{1}{\varepsilon} \left( -\frac{1}{2} - \frac{1}{2} \zeta(3) + \frac{5}{2} \zeta(5) \right) \]

\[ I_{11} = \frac{1}{(4\pi)^8} \frac{1}{\varepsilon} \left( -\frac{1}{4} - \frac{3}{2} \zeta(3) + \frac{5}{2} \zeta(5) \right) \]

\[ I_{12} = \frac{1}{(4\pi)^8} \frac{1}{\varepsilon} \left( -\frac{1}{8} - \frac{1}{4} \zeta(3) + \frac{5}{4} \zeta(5) \right) \]

Table C.8: Loop integrals for 4-loop wrapping diagrams. The arrows of the same type indicate contracted spacetime derivatives.
The final result for the wrapping part at 4 loops is

$\Delta_{\text{wrapping}} E = (324 + 864\zeta(3) - 1440\zeta(5))g^8$

Compute the same 4-loop anomalous dimension from string theory
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The wrapping graphs contributing at 4-loops correspond to a single ‘virtual’ graph:

$$\Delta_w^{(4\text{-loop})} = - \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} \frac{dq}{2\pi} Y_Q(q)$$

where $Y_Q(q)$ is a relatively simple rational function of $q$ and $Q$. 

[Bajnok, RJ]
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where

\[
\text{num}(Q) = 7776Q(19683Q^{18} - 78732Q^{16} + 150903Q^{14} - 134865Q^{12} + \\
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\[ \Delta = 4 + 12 g^2 - 48 g^4 + 336 g^6 + 96(-26 + 6 \zeta(3) - 15 \zeta(5)) g^8 
- 96(-158 - 72 \zeta(3) + 54 \zeta(3)^2 + 90 \zeta(5) - 315 \zeta(7)) g^{10} \]

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The agreement of the Konishi computation with the 4-loop weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT! The result came from a single diagram – in contrast to direct perturbative computations in gauge theory which are much more complex. This suggests that one can use string theory methods of AdS/CFT as an efficient calculational tool also at weak coupling. The AdS/CFT correspondence allows to use methods of exactly solvable integrable two-dimensional QFT's to study the four-dimensional supersymmetric $\mathcal{N} = 4$ gauge theory. We may be very close to the complete solution for the spectrum of the theory ($\equiv$ anomalous dimensions at any coupling).
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