Nonperturbative Field Theory

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Outline

1. **Solving (nonperturbatively) QFT's**
   - The quintessential example: sine-Gordon
   - QCD???
   - $\mathcal{N} = 4$ Super-Yang-Mills theory
2. **The AdS/CFT correspondence and $\mathcal{N} = 4$ SYM**
3. **Why is it interesting?**
4. **Methods at strong coupling**
5. **Towards an exact solution for any coupling**
6. **Example: Konishi anomalous dimension**
7. **Outlook**

The quintessential example: sine-Gordon

- sine-Gordon theory is a two-dimensional QFT defined by the Lagrangian

$$S = \int d^2x \left\{ \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi + \frac{\mu^2}{\beta^2} \cos(\beta\Phi) \right\}$$

- Expanding $\cos(\beta\Phi)$ for small β generates an infinite set of vertices for Φ, one can develop perturbation theory...
- There exist solitons (and anti-solitons).. classical solutions, quadratic fluctuations...
- *But* the soliton-soliton S-matrix has been found analytically **for any** β

$$S(\theta) = -\exp \left\{ 2 \int_0^\infty \frac{dk}{k} \sin(k\theta) \frac{\sinh(p-1)k}{2\cosh k \sinh pk} \right\}$$

where

$$p = \frac{\beta^2}{8\pi - \beta^2}$$

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- The sine-Gordon theory is an example of an **integrable quantum field theory**
- These theories posses additional higher spin/nonlocal conserved charges
- As a consequence the S-matrix factorizes into $2 \to 2$ scatterings and obeys Yang-Baxter Equation which allows to *nonperturbatively* find it exactly
- All this works **only** in two dimensions! No generalization to four-dimensional theories exists...

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QCD

- Inherently difficult...
- Running coupling
- Mixture of perturbative and nonperturbative effects...

Traditional methods:

- Lattice QCD (inherently Euclidean, 'black box')
- Models of QCD vacuum
- Effective models (not *derivable* from the fundamental theory)

Look at other, 'simpler' gauge theories for which other methods exist...

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- $\mathcal{N} = 4$ Super Yang-Mills ($\equiv$ ordinary Yang-Mills+4 adjoint fermions +6 adjoint scalars+ appropriate self-interactions)
- This theory is
  1. supersymmetric
  2. conformal (scale-invariant even at the quantum level)
- Bonus: There exists an efficient way of studying this theory at strong coupling: the AdS/CFT (Anti-deSitter/Conformal Field Theory) correspondence

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strong coupling	(semi-)classical strings
nonperturbative physics	or supergravity
very difficult	'easy'
weak coupling	highly quantum regime
'easy'	very difficult

- New ways of looking at nonperturbative gauge theory physics...
- Intricate links with General Relativity...

The AdS/CFT correspondence

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More theoretical perspective:

- Fascinating as it relates two *completely different* theoretical constructions: 4D gauge theory and string theory in 10D
- Use $\mathcal{N} = 4$ SYM as a theoretical laboratory for studying nonperturbative gauge theory physics
- In this theory one can perform quite rigorous computations at strong coupling
- The natural language of the AdS/CFT correspondence appropriate to strongly coupled $\mathcal{N} = 4$ SYM is quite new w.r.t. conventional gauge theory methods
- Try to build some new physical intuitions within this new language

$\mathcal{N} = 4$ SYM may be the 'harmonic oscillator' of four dimensional gauge theories

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More practical perspective:

- Although its behaviour is quite different from QCD at $T = 0$ (no confinement etc.), at *nonzero* temperature supersymmetry is automatically broken, the theory is deconfined
- Use the results on strong coupling properties of $\mathcal{N} = 4$ plasma as a point of reference for analyzing/describing QCD plasma

see talk by R. Peschanski in the Heavy-Ion session

see plenary talk by U. Wiedemann later today

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- In particular many gauge-theoretical problems are translated into geometrical General Relativity like questions which are tractable (even analytically)

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- Vibrational modes of the string $\equiv$ particles/fields in $AdS_5 \times S^5$
- Massless modes $\equiv$ graviton$+\ldots$
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- At weaker coupling massive string modes become important...

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Towards an exact solution for any coupling

What do we mean by 'solve'?

Local operators in gauge theory	$\longleftrightarrow$	String states in $AdS_5 \times S^5$
Operator dimension $\langle O(x)O(y)\rangle = \dfrac{const}{\lvert x-y\rvert^{2\Delta}}$	$\longleftrightarrow$	Energy of the corresponding string state

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- Simplest nonprotected operator: the Konishi operator

$$\mathrm{tr}\,\Phi_i^2 \quad \longleftrightarrow \quad \mathrm{tr}\, Z^2 X^2 + \ldots \quad \longleftrightarrow \quad \mathrm{tr}\, Z D^2 Z + \ldots$$

- When computing anomalous dimensions from two point functions there are two types of graphs:

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- The first class is contained in the so-called Asymptotic Bethe Ansatz of Beisert and Staudacher
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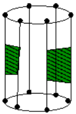

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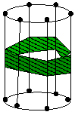

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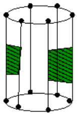

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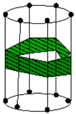

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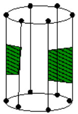

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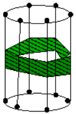

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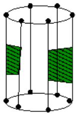

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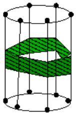

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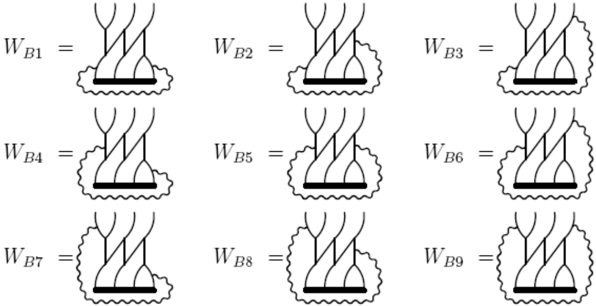


Figure C.1: Wrapping diagrams with chiral structure $\chi(1,2,3)$

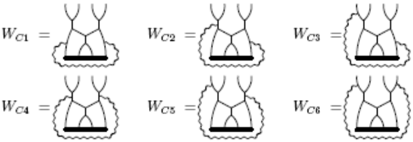


Figure C.2: Wrapping diagrams with chiral structure $\chi(1,3,2)$

$W_{C1} \rightarrow *$	1	$W_{C4} \rightarrow$ finite	
$W_{C2} \rightarrow *$	2	$W_{C5} \rightarrow -W_{C3}$	
$W_{C3} \rightarrow -W_{C5}$		$W_{C6} \rightarrow$ finite	

Table C.2: Results of D-algebra for diagrams with structure $\chi(1,3,2)$

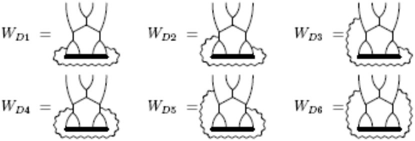
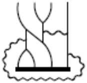
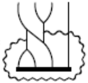
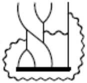


Figure C.3: Wrapping diagrams with chiral structure $\chi(2,1,3)$

$W_{E1} =$  $W_{E2} =$  $W_{E3} =$  $W_{E4} =$ 

$W_{E5} =$  $W_{E6} =$  $W_{E7} =$  $W_{E8} =$ 

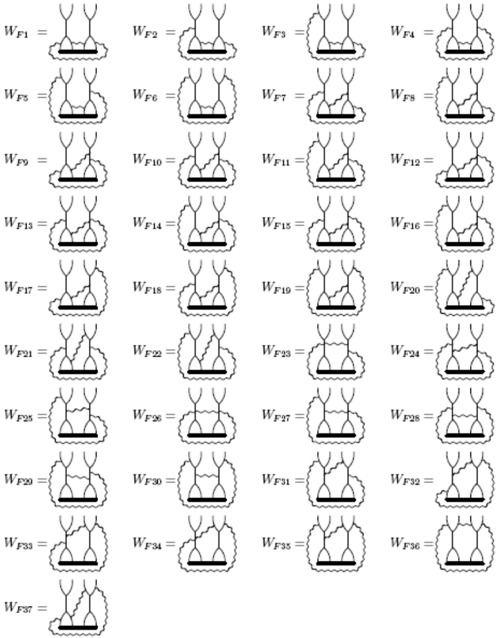
$W_{E9} =$  $W_{E10} =$  $W_{E11} =$  $W_{E12} =$ 

$W_{E13} =$  $W_{E14} =$  $W_{E15} =$  $W_{E16} =$ 

$W_{E17} =$  $W_{E18} =$  $W_{E19} =$  $W_{E20} =$ 

$W_{E21} =$  $W_{E22} =$  $W_{E23} =$  $W_{E24} =$ 

$W_{E25} =$  $W_{E26} =$  $W_{E27} =$ 


W_F1 =
W_F2 =
W_F3 =
W_F4 =
W_F5 =
W_F6 =
W_F7 =
W_F8 =
W_F9 =
W_F10 =
W_F11 =
W_F12 =
W_F13 =
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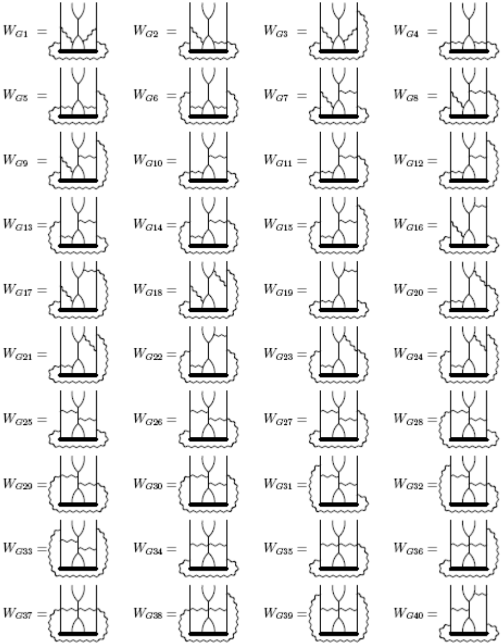
W_F36 =
W_F37 =


$W_{G1} =$
$W_{G2} =$
$W_{G3} =$
$W_{G4} =$
$W_{G5} =$
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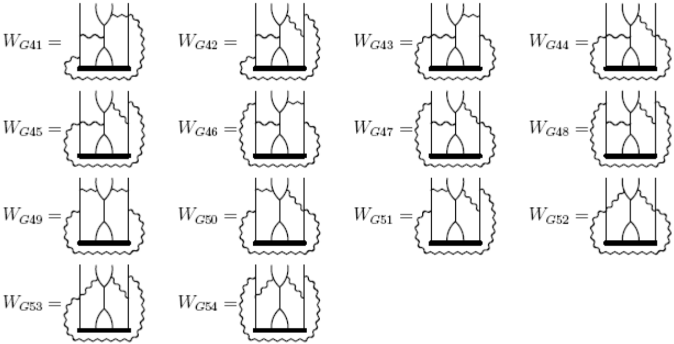
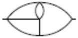


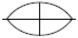
Figure C.6: Wrapping diagrams with chiral structure $\chi(1)$ (continued)

$I_1 = J_1 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{24\varepsilon^4}+\frac{1}{4\varepsilon^3}-\frac{19}{24\varepsilon^2}+\frac{5}{4\varepsilon}\Big)$

$I_2 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{24\varepsilon^4}+\frac{1}{4\varepsilon^3}-\frac{19}{24\varepsilon^2}+\frac{1}{\varepsilon}\Big(\frac{5}{4}-\zeta(3)\Big)\Big)$

$I_3 = J_5 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{12\varepsilon^4}+\frac{1}{3\varepsilon^3}-\frac{5}{12\varepsilon^2}-\frac{1}{\varepsilon}\Big(\frac{1}{2}-\zeta(3)\Big)\Big)$

$I_4 =$  $= \frac{1}{(4\pi)^8}\Big(-\frac{1}{6\varepsilon^4}+\frac{1}{3\varepsilon^3}+\frac{1}{3\varepsilon^2}-\frac{1}{\varepsilon}(1-\zeta(3))\Big)$

$I_5 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}5\zeta(5)$

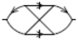
$I_6 =$  $= \frac{1}{(4\pi)^8}\Big(\frac{1}{12\varepsilon^2}-\frac{7}{12\varepsilon}\Big)$

$I_7 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}(-\zeta(3))$

$I_8 =$  $= \frac{1}{(4\pi)^8}\Big(\frac{1}{4\varepsilon^2}-\frac{11}{12\varepsilon}\Big)$

$I_9 =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(\frac{1}{2}\zeta(3)-\frac{5}{2}\zeta(5)\Big)$

$I_{10} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{2}-\frac{1}{2}\zeta(3)+\frac{5}{2}\zeta(5)\Big)$

$I_{11} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{4}-\frac{3}{2}\zeta(3)+\frac{5}{2}\zeta(5)\Big)$

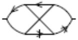
$I_{12} =$  $= \frac{1}{(4\pi)^8}\frac{1}{\varepsilon}\Big(-\frac{1}{8}-\frac{1}{4}\zeta(3)+\frac{5}{4}\zeta(5)\Big)$

Table C.8: Loop integrals for 4-loop wrapping diagrams. The arrows of the same type indicate contracted spacetime derivatives

- The final result for the wrapping part at 4 loops is

$$\Delta_{wrapping} E = (324 + 864\zeta(3) - 1440\zeta(5))g^8$$

Compute the same 4-loop anomalous dimension from string theory

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The Konishi operator from string theory

[Bajnok,RJ]

- The Konishi operator corresponds to a two particle state in the 2D worldsheet QFT of the string in $AdS_5 \times S^5$
- The wrapping graphs contributing at 4-loops correspond to a *single* 'virtual' graph:
- This corresponds to a simple expression

$$\Delta_w^{(4-loop)} = -\sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} \frac{dq}{2\pi} Y_Q(q)$$

where $Y_Q(q)$ is a relatively simple rational function of q and Q

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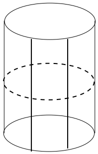

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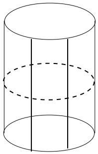

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- The integral can be easily done by residues giving a remaining sum

$$\sum_{Q=1}^{\infty}\left\{-\frac{num(Q)}{(9Q^4-3Q^2+1)^4(27Q^6-27Q^4+36Q^2+16)}+\frac{864}{Q^3}-\frac{1440}{Q^5}\right\}$$

where

$$\begin{aligned} num(Q) =& 7776Q(19683Q^{18}-78732Q^{16}+150903Q^{14}-134865Q^{12}+\\ &+1458Q^{10}+48357Q^8-13311Q^6-1053Q^4+369Q^2-10) \end{aligned}$$

- This can be summed up giving

$$\Delta_{wrapping}E=(324+864\zeta(3)-1440\zeta(5))g^8$$

which exactly agrees with the direct 4-loop perturbative computation of [F.Fiamberti, A.Santambrogio, C.Sieg and D.Zanon]

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- We extended the string computation to give the result up to 5 loops (no gauge theoretical computation so far) [Bajnok,Hegedus,RJ,Łukowski]

$$\begin{aligned}\Delta &= 4+12\,g^2-48\,g^4+336\,g^6+96(-26+6\,\zeta(3)-15\,\zeta(5))\,g^8 \\ &\quad -96(-158-72\,\zeta(3)+54\,\zeta(3)^2+90\,\zeta(5)-315\,\zeta(7))\,g^{10}\end{aligned}$$

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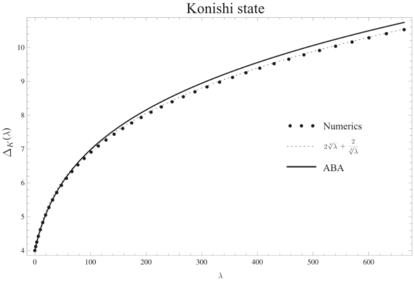

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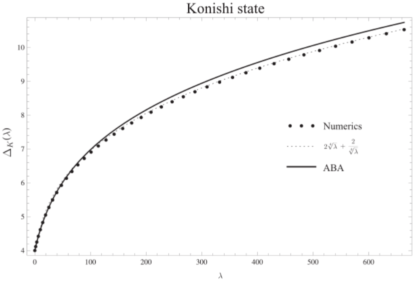

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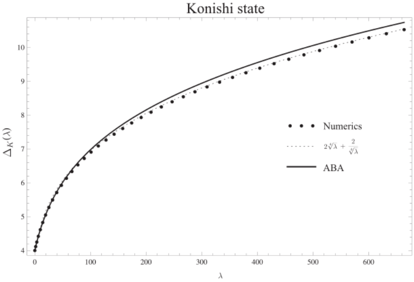

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- The agreement of the Konishi computation with the 4-loop weak coupling perturbative gauge theory result is an extremely nontrivial test of AdS/CFT!
- The result came from a single diagram – in contrast to direct perturbative computations in gauge theory which are much more complex
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