Physics from open string wavefunctions

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Outline

- Type II String Theory model building
- Open string wavefunctions
- Recent progress
- Conclusions
String Theory and Particle Physics

- String Theory has the right features for a unified framework of Gravitation and Particle Physics. Two important predictions:
  - Gravity
  - Matter fields transforming in gauge group representations

- Despite the difficulties to accommodate the Standard Model (large landscape of string vacuum solutions), a lot of progress have been carried out in last decade.

- Two major lines of attack:
  - Heterotic model building
  - Type II (D-brane) model building
D-brane model building

- Open strings lead to U(N), SO(2N), USp(N) gauge theories in the worldvolume of Dp-branes and their intersections, with fermions in (mainly) bifundamental representations.

- Several (dual) frameworks:
  - Intersecting D6-branes
  - D3/D7-branes at singularities
  - Magnetized D9-branes (type I strings)
  - ...

Intersecting D6-branes I

- E.g. consider stacks of D6-branes filling Minkowski and wrapping 3-cycles of a internal 6-torus.

\[
\prod_a U(N_a)
\]

\([\Pi_a] = (n_a^1, m_a^1) \times (n_a^2, m_a^2) \times (n_a^3, m_a^3)\]

Intersection num.: \[I_{ab} = \prod_{i=1}^3 (n_a^i m_b^i - m_a^i n_b^i)\]

- \(I_{ab}\) bifundamental chiral fermions \((N_a, \bar{N}_b)\) localized at the intersections

- Orientifold/orbifold action requires also mirror branes \(\text{symmetric/antisymmetric reps., orthogonal gauge groups...}\)

- N = 1 SUSY condition:

\[
\sum_{i=1}^3 \frac{m_a^i}{n_a^i} \tau^i = \frac{m_a^1 m_a^2 m_a^3}{n_a^1 n_a^2 n_a^3} \tau^1 \tau^2 \tau^3
\]
Intersecting D6-branes II

- E.g. 3-generation local MSSM model

\[
\begin{array}{|c|c|c|c|}
\hline
N_\alpha & (n_\alpha^1, m_\alpha^1) & (n_\alpha^2, m_\alpha^2) & (n_\alpha^3, m_\alpha^3) \\
\hline
N_a = 3 & (1, 0) & (g, 1) & (g, -1) \\
N_b = 1 & (0, 1) & (1, 0) & (0, -1) \\
N_c = 1 & (0, 1) & (0, -1) & (1, 0) \\
N_d = 1 & (1, 0) & (g, 1) & (g, -1) \\
\hline
\end{array}
\]

- One of the U(1)'s becomes massive through the Green-Schwarz mechanism

\[
U(3) \times SU(2)_L \times SU(2)_R \times U(1) \Rightarrow SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \Rightarrow SU(3) \times SU(2)_L \times U(1)_Y
\]
Magnetized D9-branes

- Intersecting D6-branes are dual to configurations of magnetized D9-branes, filling 4D Minkowski and the whole 6-dimensional internal space

\[
F_{z^k z^k} = \frac{\pi i}{\text{Im} \, \tau^k} \left( \begin{array}{cc} \frac{m_a^k}{n_a^k} I N_a \\ \frac{m_b^k}{n_b^k} I N_b \\ \cdots \end{array} \right)
\]

- Magnetization in D9-branes Higgses the gauge group: \( G_{\text{unbr}} = \prod_\alpha G_\alpha \)

\[ \Rightarrow \text{W-bosons, scalars and fermions in bifundamental reps.} \]

- SUSY condition becomes: \( J \wedge J \wedge F_2 - \frac{1}{3} F_2 \wedge F_2 \wedge F_2 = 0 \)

- Open strings are charged under \( F_2 \) \( \Rightarrow \) localization of wavefunhtions
Open string wavefunctions I

- This system admits a description in terms of field theory, where the D9-brane d.o.f. are given by 10D Super Yang-Mills

\[ S = - \int dx^{10}(\text{det } g)^{1/2}\text{Tr} \left[ \frac{1}{4} F_{MN} F^{MN} + \bar{\chi} \Gamma^M D_M \chi \right] \]

- Open strings can be understood as wavefunctions. These arise from standard KK reduction as eigenfunctions of the internal Dirac and Laplace operators

\[ D_m D^m \Phi^i = -m_\Phi^2 \Phi^i, \quad \gamma^m D_m \psi = m_\psi \psi^* \]

- Magnetized torus \( \Rightarrow \) Landau levels

- Number of massless modes given by the Dirac index \( I_{ab} \)
Open string wavefunctions II

- Solutions for a 2-tori given in terms of Jacobi theta functions

\[ \psi^{j,I_{ab}}(\tau, z) \sim \exp \left[ \frac{i\pi |I_{ab}| z \text{Im } z}{\text{Im } \tau} \right] \cdot \theta \left[ \frac{j}{|I_{ab}|} \right] (|I_{ab}| z, |I_{ab}| \tau), \quad j = 0 \ldots |I_{ab}| - 1 \]

- Under global monodromies transform with the right U(1) charge

\[ z \rightarrow z + 1, \quad \psi^{j,I_{ab}} \rightarrow e^{i\pi |I_{ab}| \text{Im } z/\text{Im } \tau} \psi^{j,I_{ab}} \]
\[ z \rightarrow z + \tau, \quad \psi^{j,I_{ab}} \rightarrow e^{i\pi |I_{ab}| \text{Im } \bar{z}/\text{Im } \tau} \psi^{j,I_{ab}} \]

- **Localization effects** (depending on W.L. moduli)

- For higher dim. factorizable tori, products of theta functions

[Cremades, Ibenez, Marchesano ‘04]
Open string wavefunctions III

- Open string wavefunctions are the target-space analogous of vertex operators in the worldsheet CFT

- O.P.E:

\[
\psi^i_{Iab} \cdot \psi^j_{Ica} = A^{-1/2} (2 \text{Im } \tau)^{1/4} \left| \frac{I_{ab}I_{ca}}{I_{bc}} \right|^{1/4}
\]

- Physical observables as overlap integrals of wavefunctions. E.g. Yukawa couplings,

\[
Y_{ijk} = \int_{T^2} \psi_i^{\dagger} \Gamma^m \psi_j \phi_k, m, f_{abc} \sim \vartheta \left[ \frac{i}{|I_{ab}|} + \frac{j}{|I_{ca}|} + \frac{k}{|I_{bc}|} \right] \left(0, \tau |I_{ab}I_{bc}I_{ca}| \right)
\]

[Cremades, Ibanez, Marchesano]
[Di Vecchia et al.]
[Antoniadis, Kumar, Panda]
Closed string fluxes I

- In the last years closed string fluxes have become a new ingredient for model building
  - Stabilization of compactification moduli
  - de Sitter vacua
  - Inflation
  - ...
- Fluxes have also interesting effects on D-branes
  - Soft-terms/moduli stabilization
  - D-terms and superpotentials
  - Instanton zero-mode lifting
  - Warp effects
Closed string fluxes II

- Perhaps better understood in the D3/D7-brane picture: 3-form fluxes

\[ G_3 = F_3 - SH_3 \]

\[
\text{backreaction (warping): } \quad ds^2 = Z^{-1/2} ds^2_{\mathbb{R}^1,3} + Z^{1/2} ds^2_{\mathcal{M}_6}
\]

- Stabilization of dilaton, c. structure moduli and D7-brane positions

- Example: global MSSM flux vacua (\(T^6/Z_2 \times Z_2\) orbifold)

\[
N_\alpha \begin{array}{c|c|c|c}
N_a = 6 & (1,0) & (3,1) & (3,-1) \\
N_b = 2 & (0,1) & (1,0) & (0,-1) \\
N_c = 2 & (0,1) & (0,-1) & (1,0) \\
N_d = 2 & (1,0) & (3,1) & (3,-1) \\
N_{h_1} = 2 & (-2,1) & (-3,1) & (-4,1) \\
N_{h_2} = 2 & (-2,1) & (-4,1) & (-3,1) \\
N_f = 8 & (1,0) & (1,0) & (1,0) \\
\end{array}
\]

[Marchesano, Shiu ‘05]

- Magnetized D7-branes

\[ G_3 = 2(d\bar{z}^1dz^2dz^3 + dz^1d\bar{z}^2dz^3 + dz^1dz^2d\bar{z}^3 + d\bar{z}^1d\bar{z}^2d\bar{z}^3) \]
Recent progress I

- We have developed techniques to compute open string wavefunctions in the presence of closed string fluxes (and magnetization)

- In terms of D9-branes (type I strings) these fluxes correspond to compactifications on elliptic fibrations ($T^2$ over a four dimensional base $B_4$) with RR 3-form fluxes (S-dual to heterotic strings with torsion)

- Simple examples given by twisted tori: $B_4 = T^4$

\[ \text{d} e^a = \frac{1}{2} f_{b c} e^b \wedge e^c \]

[Dasgupta et al.]
[Schultz]
[PGC, Grana]
[Lust et al.]
Recent progress II

- In the field theory limit there are modified internal Dirac and Laplace-Beltrami operators which account for the effect of the closed string background on the open string fluctuations.

**W-bosons:**

\[
D^m D_m W^{\alpha\beta} - 2(\partial_m \log Z) D^m W^{\alpha\beta} = -Z^{1/2} m_W^2 W^{\alpha\beta}
\]

**Charged scalars:**

\[
D^m D_m \Phi^{p,\alpha\beta} - [\nabla^M_6, \nabla^M_6 p] \Phi^{m,\alpha\beta} - 2(\partial_k \log Z) D^{[k} \Phi^{p],\alpha\beta} +
2i \Phi^{m,\alpha\beta}(F_m^{p,\alpha} - F_m^{p,\beta}) + e^{\phi/2}(\tilde{D}_m \Phi^{n,\alpha\beta}) F_n^{mp} = -Z^{1/2} m_\Phi^2 \Phi^{p,\alpha\beta}
\]

**Matter fields:**

\[
\Gamma_\text{(4)} \left( \Phi^M_6 + \frac{1}{4} e^{\phi/2} F_3 - \frac{1}{2} \partial \ln Z \right) \psi = Z^{1/4} m_\psi \mathcal{B}_6^* \psi^*
\]

- Notice that for this case there is no known worldsheet CFT, so the field theory description is the only possibility.
Recent progress III

- Solutions are found using the non-abelian Fourier transform

\[ \psi_\omega(g) = \sum_{\gamma \in \Gamma} \pi_\omega(\gamma g) \varphi(s_0) \]

- Classified by irreducible unitary reps. of the 4D gauge algebra which results from KK reduction

\[
[D_m, D_n] = -f_{mn}^p D_p + iF_{mn}^\alpha U_\alpha \\
[D_m, U_\alpha] = [U_\alpha, U_\beta] = 0
\]

- Similar observations made in other contexts
  - Fluxless CY [Douglas et al. '08]
  - WZW models
Recent progress IV

- In models with circle fibrations, wavefunctions are similar to the ones in the fluxless case but with oblique magnetization.

\[
\psi(j_1,j_2) \sim e^{i\pi(N \cdot \bar{z}) \cdot (\text{Im} \, \Omega_U)^{-1} \cdot \text{Im} \, \bar{z}} \cdot \theta \left[ \begin{array}{c} j_1 \\ 0 \end{array} \right] (N \cdot \bar{z}; N \cdot \Omega_U) e^{2\pi i(k_3x^3 + k_6x^6)}
\]

\[
\bar{z} = \left( \begin{array}{c} x^4 \\ x^2 \end{array} \right) + \Omega_U \cdot \left( \begin{array}{c} x^1 \\ x^5 \end{array} \right) \\
\Omega_U = \bar{B}^{-1} \cdot \bar{U} \cdot \bar{B} \cdot \Omega
\]

\[
N = \left( \begin{array}{cc} -I_{\alpha\beta} & -k_6M \\ k_6M & I_{\alpha\beta}^2 \end{array} \right) \\
U = \left( \begin{array}{cc} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{array} \right) \\
\tan \phi = \frac{\rho_{cl}}{\rho_{op}}
\]

- Open/closed string duality

<table>
<thead>
<tr>
<th>closed string</th>
<th>open string</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e^6 )</td>
<td>( A )</td>
</tr>
<tr>
<td>( x^6 )</td>
<td>( \Lambda )</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>( \omega_3 )</td>
</tr>
</tbody>
</table>

- O.P.E. for oblique magnetization worked out recently [Antoniadis et al. '09]
Recent progress V

- The analysis in terms of open string wavefunctions reveals that
  - D7-brane KK modes are not affected by closed string fluxes
  - D7-brane windings suffer Landau degeneracies

Mass formula:

\[ m_W^2 = \left( \frac{k_3}{R_3} \right)^2 + \left( \frac{k_6}{R_6} \right)^2 + (n + 1)\rho + (2k - n)\frac{\sigma_+ + \sigma_-}{\rho} \]

\[ k, n \in \mathbb{Z} \]
Recent progress VI

- Open string wavefunctions are affected by the warping

- For massless D7-brane (neutral) modes

\[ \psi \sim Z^p \]

whereas the kinetic terms are also rescaled

\[ \int_{\mathbb{R}^{1,3}} dx^4 \chi \partial \chi \int_{T^4} dy^4 Z^q \]

<table>
<thead>
<tr>
<th>4D Field</th>
<th>p</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>gauge boson/modulus</td>
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<td>1</td>
</tr>
<tr>
<td>gaugino/modulino</td>
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<td></td>
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<tr>
<td>Wilson line</td>
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<td>0</td>
</tr>
<tr>
<td>Wilsonino</td>
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</table>
Conclusions

- Open strings ending in Dp-branes provide a good context to embed semi-realistic chiral gauge theories in String Theory.

- In the field theory limit these are described in terms of open string wavefunctions, which play a similar role to vertex operators in CFT.

- Wavefunctions are useful for computing 4D observables in models which admit a higher dimensional sugra description.

- 4D couplings are given in terms of overlap integrals.

- Techniques for computing open string wavefunctions in flux compactifications are being developed.

- These may provide us also with hints on the 2D CFT of flux compactifications.

- Some effort along these directions is still required.
Thank you!