Bjorken Flow of the quark-gluon plasma and Gauge/Gravity Correspondence

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(Cracow University and Saclay Theory)
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- Gauge/Gravity correspondence and AdS/CFT

  Shortest Introduction

- Late Time Dynamics

  The "Holographic Bjorken Flow"

- Early Time Dynamics

  Holographic Thermalization

- Conclusion and Prospects

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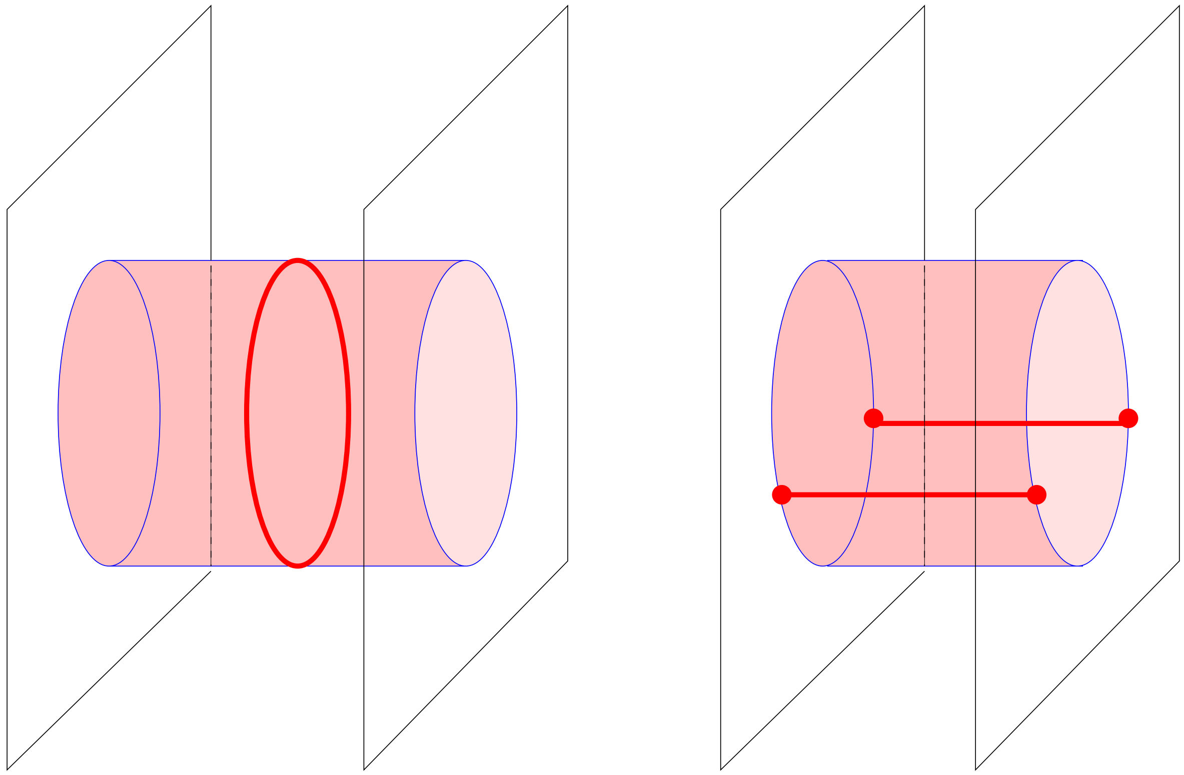
[a] M.Heller,R.J.,R.P., arXiv:0811.3113[hep-th] (Late Time,Review)
G.Beuf,M.Heller,R.J.,R.P., arXiv:0906.4423 [hep-th] (Early Time,ńew)

The Gauge-Gravity Correspondence

“Duality”: Open String $\Leftrightarrow$ Closed String



Schomerus, 2006

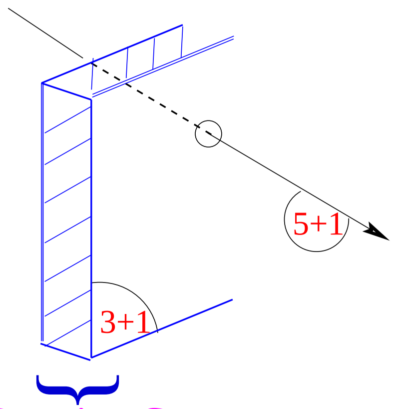
$$\begin{aligned} Closed\ String &\Leftrightarrow 1-loop\ Open\ String \\ D{-}Brane\ \text{“}Universe\text{”} &\Rightarrow Open\ String\ Ending \\ Gravity &\Leftrightarrow Gauge \\ Large/Small\ Distance &\Rightarrow Gravity/Gauge\ Correspondence \end{aligned}$$

AdS/CFT Correspondence

J.Maldacena (1998)

GRAVITY

AdS



Gravity Source

Macroscopic

$(g_s N)^{-1} \to 0$

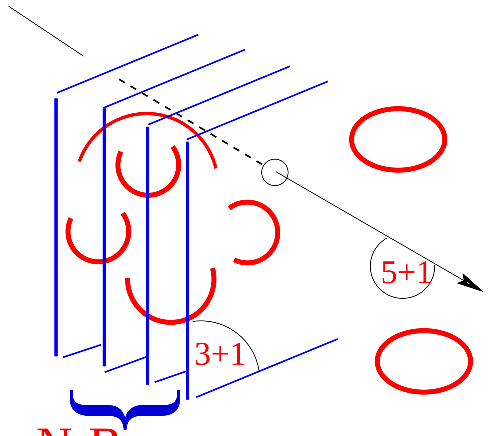
$AdS_5 \times S_5$ Superstring

Weak Gravity

Strong Gravity

GAUGE

CFT



N-Branes

Microscopic

$g_s N \to 0$

SU(N) Gauge Theory on the N-Branes

Strong Coupling

Weak Coupling

⟸⟹ Duality

WHY AdS$_5 \otimes S_5$?

- Solution of Gravity for D_3 Branes: Horowitz,Strominger, 1991

$$ds^2 = f^{-1/2}(-dt^2 + \sum_1^3 dx_i^2) + f^{1/2}(dr^2 + r^2 d\Omega_5)$$

"Physical" Branes (d=1+3) + Extra-Dimensions (d=6)

$$f = 1 + \frac{R^4}{r^4} \ ; \ R^4 = 4\pi\alpha'^2 g_{YM}^2 N_c$$

- "Maldacena breakthrough": Maldacena, 1998

$$\frac{\alpha'(\to 0)}{r(\to 0)} \to z \ , \ R \ fixed \ \Rightarrow g_{YM}^2 N_c \to \infty$$

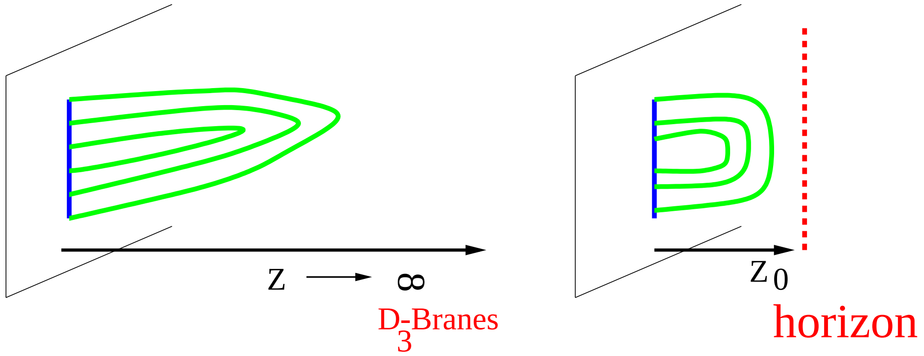
Strong coupling limit

$$ds^2 = \frac{1}{R^2 z^2}(-dt^2 + \sum_{1-3} dx_i^2 + dz^2) + R^2 d\Omega_5$$

Background Structure: AdS$_5 \otimes S_5$ (same R^2)

HOLOGRAPHY

- Holographic Principle: Brane/Bulk correspondence



$AdS_5 \otimes S_5$ $\qquad$ *Confining Geometry*

- Brane ⟶ Bulk: Holographic Renormalization

K.Skenderis (2002)

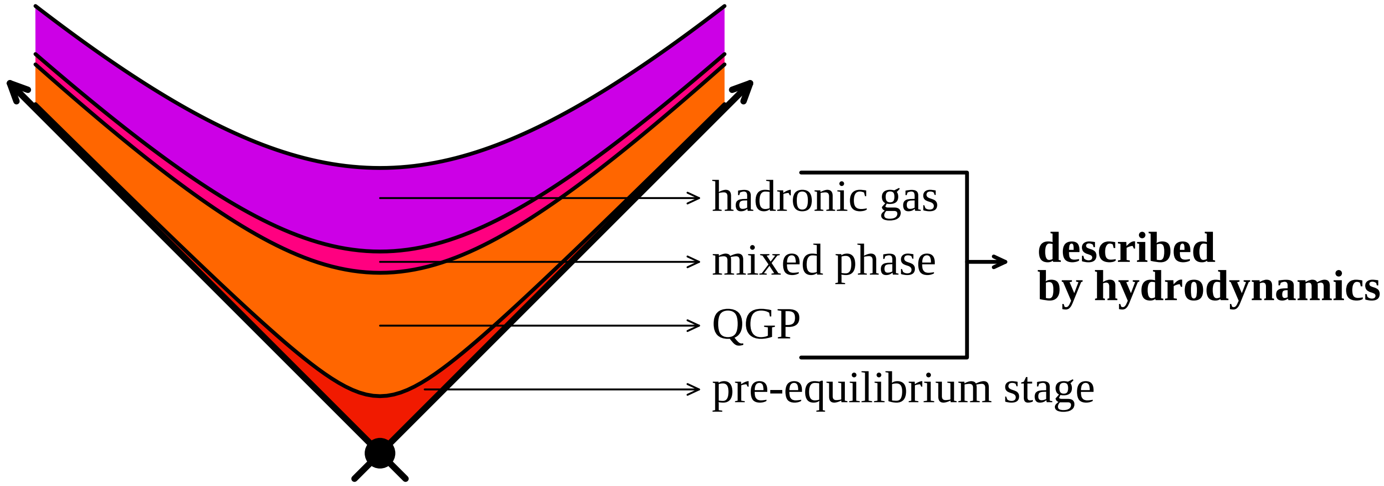
$$ds^2 = \frac{g_{\mu\nu}(z)\; dx^\mu dx^\nu + dz^2}{z^2}$$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(= \eta_{\mu\nu}) + z^2 g^{(2)}_{\mu\nu}(= 0) + z^4 \langle T_{\mu\nu}\rangle \quad + z^6 \ldots +$$

$+ z^6 \ldots +$: from Einstein Eqs.

Gauge/Gravity and QGP Dynamics

Janik, R.P. (2005)
Janik, Heller, Benincasa, Buchel...
Kovchegov, Taliotis, Albacete,...
Nakamura,Sin,Kinoshita, Mukoyama, Nakamura, Oda,Natsuume, Okamura,...
Bhattacharyya, Hubeny, Minwalla, Ranganami, Loganayagam,...
......,...



$$\tau = \sqrt{x_0^2 - x_1^2}\ ;\ y = \frac{1}{2}\log\frac{x_0 + x_1}{x_0 - x_1}\ ;\ x_T = x_2, x_3$$

Questions

- What is the Gravity Dual of a Flow?
- QGP: (almost) Perfect fluid behaviour, why?
- Universal $\frac{\eta}{S}$, Transport coefficients, Navier-Stokes,...
- Fast Pre (and Out-of)-Equilibrium stage, how and why?

AdS/CFT $\Rightarrow$ Perfect Fluid at large τ

R.Janik, RP (2005)

- Boost-invariant T^{μ}_{ν} (Bjorken, 1983)

$$T_{\mu\nu}=\begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3\frac{d}{d\tau}f(\tau)-\tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau)+\frac{1}{2}\tau\frac{d}{d\tau}f(\tau) & 0 \\ 0 & 0 & 0 & \dots \end{pmatrix}$$

- Proper-time evolution

$f(\tau) \propto \tau^{-S}$: $T_{\mu\nu}t^{\mu}t^{\nu} \geq 0 \Rightarrow 0 < s < 4$

$f(\tau) \propto \tau^{-\frac{4}{3}}$: Perfect Fluid

$f(\tau) \propto \tau^{-1}$: Free streaming

$f(\tau) \propto \tau^{-0}$: Full Anisotropy $\epsilon = p_{\perp} = -p_L$

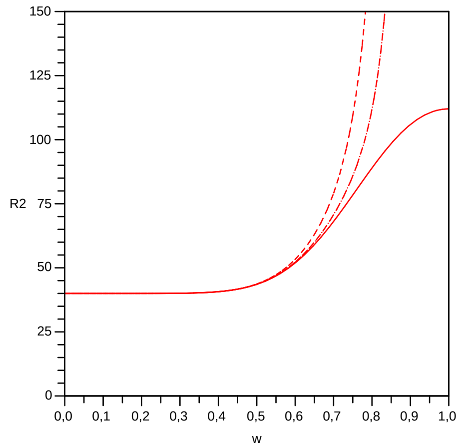
- Holographic renormalization:
  $\Rightarrow$ Existence of a Dynamical Scaling

$$\boxed{v = \frac{z}{\tau^{S/3}}}$$

The Moving Black Hole in 5-d

- Holographic Renormalization:
  ⇒ Regularity Criterium

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$$



$s = \frac{4}{3} \pm .1$

Nonsingular Dual Geometry ⇔ Perfect Fluid

- Asymptotic metric
  ⇒ Black Hole (Brane) Moving off in the 5th dimension

$$ds^2 = \frac{1}{z^2}\left[-\frac{\left(1-\frac{e_0}{3}\;\frac{z^4}{\tau^{4/3}}\right)^2}{1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}}d\tau^2+\left(1+\frac{e_0}{3}\frac{z^4}{\tau^{4/3}}\right)\left(\tau^2dy^2+dx_\perp^2\right)\right]+\frac{dz^2}{z^2}$$

$$Horizon: \; z_h(\tau) = (3/e_0)^{\frac{1}{4}} \cdot \tau^{\frac{1}{3}} \; .$$

$$Temperature: \; T(\tau) \sim 1/z_h \sim \tau^{-\frac{1}{3}}$$

$$Entropy: \; S(\tau) \sim Area \sim \tau \cdot 1/z_h^3 \sim const$$

Some recent Results

- Beyond perfect fluid

In-flow Viscosity, Relaxation time, Transport Coeff., etc...

Janik, Heller, Bak, Benincasa, Buchel, Nakamura, Sin,......
Kinoshita, Mukoyama, Nakamura, Oda,Natsuume, Okamura,...

$$\boxed{\partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{\eta}{\tau^2} + \cdots \Rightarrow \frac{\eta}{s} = \frac{1}{4\pi}}$$

- Beyond boost-invariance

General hydrodynamic equations from AdS/CFT

Bhattacharyya, Hubeny, Minwalla, Ranganami, Loganayagam,...

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ fluid} - \underbrace{2(\pi T)^{3\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2 T^{\mu\nu}_{2a} + 2T^{\mu\nu}_{2b} + (2 - \log 2)\left(\frac{1}{3}T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ order\ hydrodynamics}$$

- *Beyond hydrodynamics?*

$\Rightarrow$ *Isotropization/Thermalization problem*

$\Rightarrow$ *Look to boost-invariant early-time dynamics*

Early time Boost-Invariant Dynamics

G.Beuf, M.Heller, R.Janik, R.P. (2009)

- General Boost-Invariant Fefferman-Graham metric:

$$ds^2 = \frac{-e^{a(\tau,z)}\ d\tau^2 + \tau^2 e^{b(\tau,z)}\ dy^2 + e^{c(\tau,z)}\ dx_\perp^2}{z^2} + \frac{dz^2}{z^2}$$

- Einstein Equation(s):

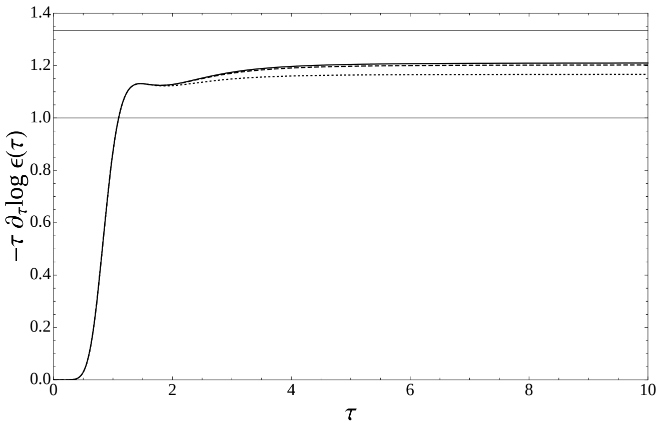
$$R_{AB} + 4G_{AB} = 0$$

- Results
  - No scaling at early time
  - The metric is singular at all times (including $\tau = 0!$):
  - The geometry should stay regular:
  - $\Rightarrow$ Initial Bulk Conditions + Constraints

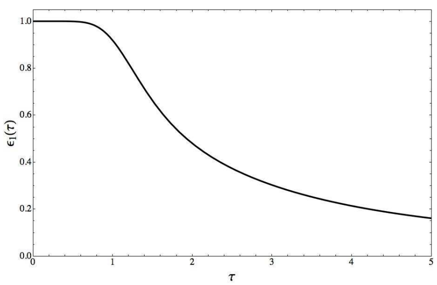
Transition to the hydrodynamic regime

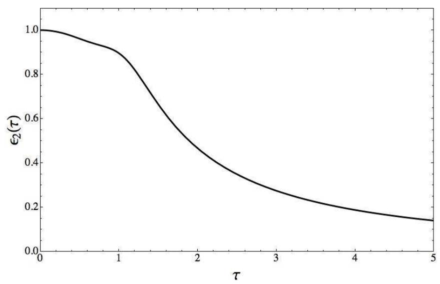
Energy Density

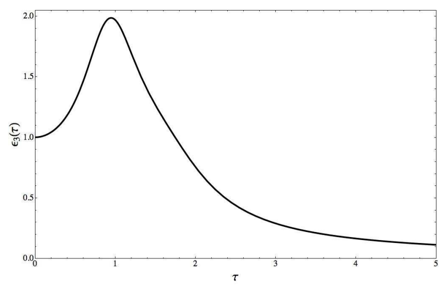
- Family Index $\mathbf{s}$: (hydro: $\mathbf{s} = \mathbf{4/3}$)



- Energy density







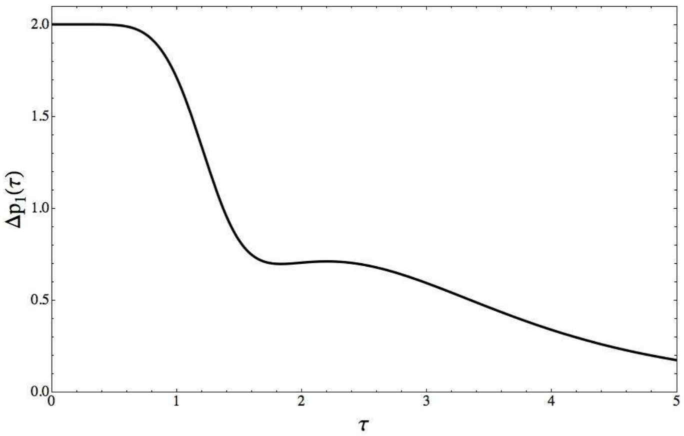
$A)$ $B)$ $C)$

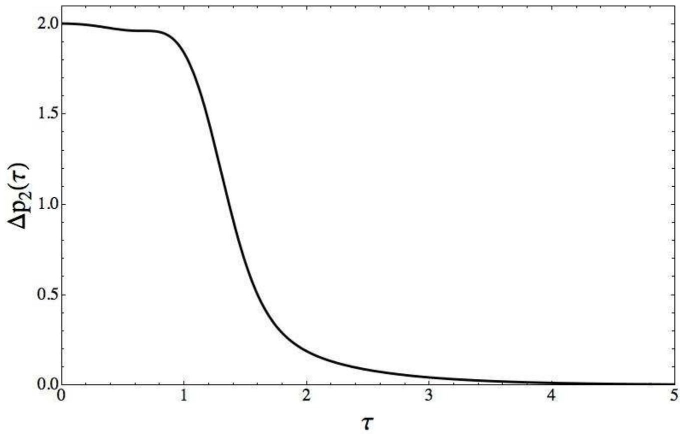
- (Classical) Positivity

$$\tau \geq \tau_0 \Rightarrow \frac{4\,\epsilon(\tau)}{\tau} \leq \epsilon'(\tau) \leq 0$$

Transition to the hydrodynamic regime

Pressure Isotropisation





Anisotropy Ratio

$$\Delta p(\tau) = 1 - \frac{p_{\parallel}(\tau)}{p_{\perp}(\tau)}$$

Prospects

In progress:

- Gauge-Gravity Correspondence
  A promising way towards QCD at strong coupling
- Results on AdS/CFT $\rightarrow S^4$QCD Hydrodynamics
  Einstein *vs*. Navier-Stokes, Thermalization,...
- Other studies
  Jet Quenching, Quark Dragging, and many others...

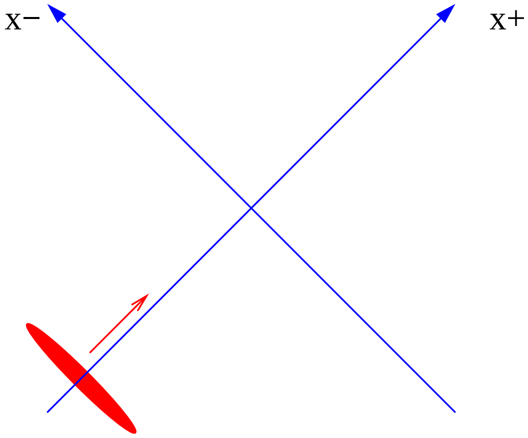
What in front of us?

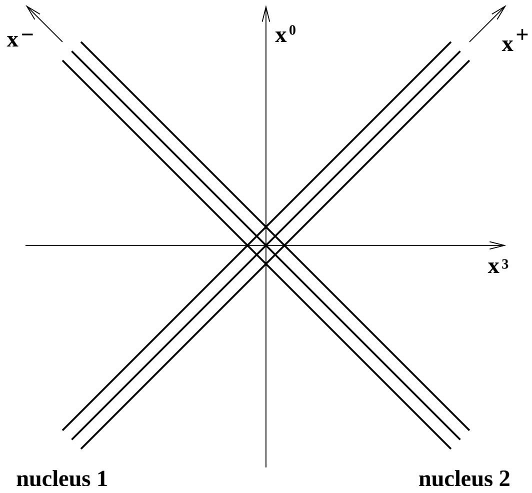
- More Theoretical Work
  Gauge/Gravity beyond AdS/CFT
- More "Translation" Work
  Formulating Observables: V_2, Diffraction, Soft/Hard Interplay
- From S^4QCD to S^0QCD ?
  Construct the "Gravity Dual" of QCD

Why all that seems to work?

EXTRA SLIDES

Initial Conditions: Shock Waves





- Dual Shock Wave

Janik,R.P.(2005)

- Dual Color Glass Condensate

G.Beuf (2009)

$$\boxed{ds^2 = \frac{-2dx^+dx^- + \mu_1 z^4 b(x^-, \mathbf{x}_\perp, z) dx^{-2} + d\mathbf{x}_\perp{}^2 + dz^2}{z^2}}$$

- Shock-wave collisions?

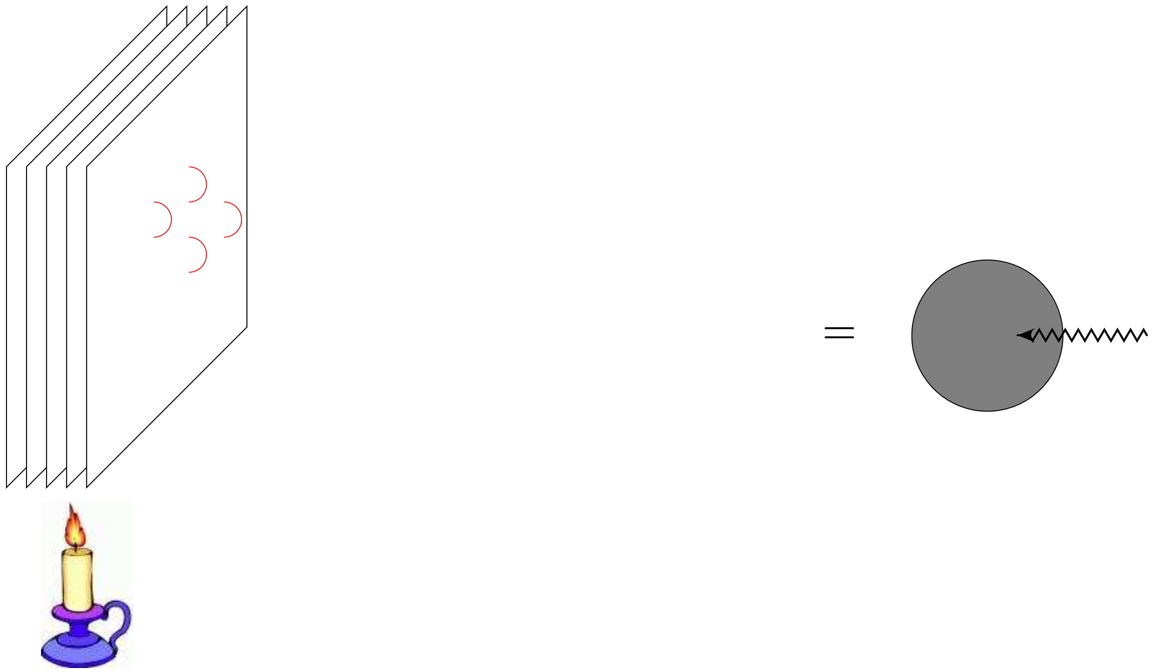
Grumiller,Romatschke ;Albacete,Kovchegov,Taliotis (2008)

But...problems...

HYDRODYNAMICS *vs.* GRAVITY

Viscosity on the light of duality

Consider a graviton that falls on this stack of N D3-branes
Will be absorbed by the D3 branes.
The process of absorption can be looked at from two different perspectives:



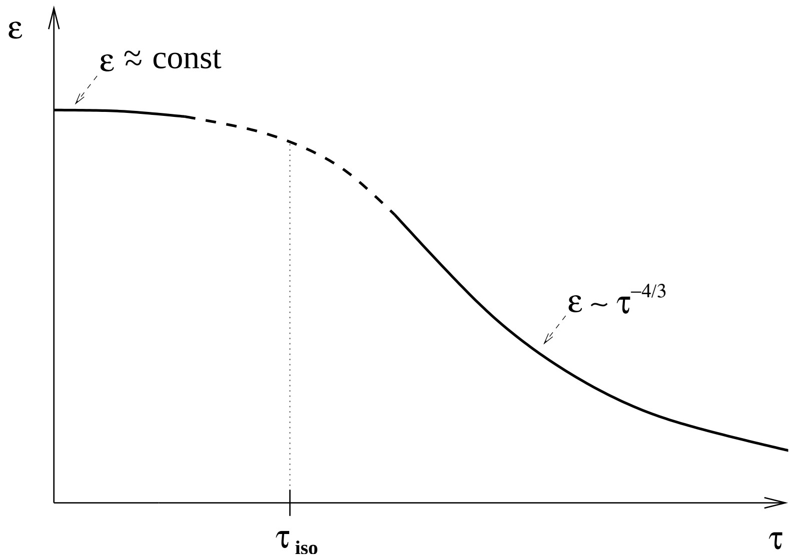
Absorption by D3 branes ($\sim$ viscosity) = absorption by black hole

$$\sigma_{abs}(\omega) \propto \int d^4x \, \frac{e^{i\omega t}}{\omega} \, \langle [T_{x_2x_3}(x), T_{x_2x_3}(0)] \rangle \Rightarrow \frac{\eta}{s} \equiv \frac{\sigma_{abs}(0)/(16\pi\ G)}{A/(4\ G)} = \frac{1}{4\pi}$$

Policastro, Son, Starinets (2001)

AdS/CFT: Anisotropy at small τ

Kovchegov, Taliotis arXiv:0705.1234



Evaluation of The Isotropization/Thermalization time

$$Matching: \ z_h^{late}(\tau) = \left(\frac{3}{e_0}\right)^{\frac{1}{4}} \equiv z_h^{early}(\tau) = \tau$$

$$Isotropization: \ \tau_{iso} = \left(\frac{3N_c^2}{2\pi^2 e_0}\right)^{3/8}$$

$$Evaluation: \epsilon(\tau) \ = \ e_0 \ \tau^{4/3}|_{\tau=.6} \sim 15 \ GeV fermi^{-3}$$

$$\Rightarrow \boxed{\tau_{iso} \sim .3 fermi}$$

EMERGENCE of the 5d BLACK HOLE

Balasubramanian,de Boer,Minic (2002)

- 4d Perfect Fluid "on the brane"

$$\langle T_{\mu\nu}\rangle \propto g^{(4)}_{\mu\nu} = \begin{pmatrix} 3/z_0^4 = \epsilon & 0 & 0 & 0 \\ 0 & 1/z_0^4 = p_1 & 0 & 0 \\ 0 & 0 & 1/z_0^4 = p_2 & 0 \\ 0 & 0 & 0 & 1/z_0^4 = p_3 \end{pmatrix}$$

- Holographic Renormalisation (Resummed) Janik,R.P. (2005)

$$ds^2 = -\frac{(1 - z^4/z_0^4)^2}{(1 + z^4/z_0^4)z^2} dt^2 + (1 + z^4/z_0^4)\frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

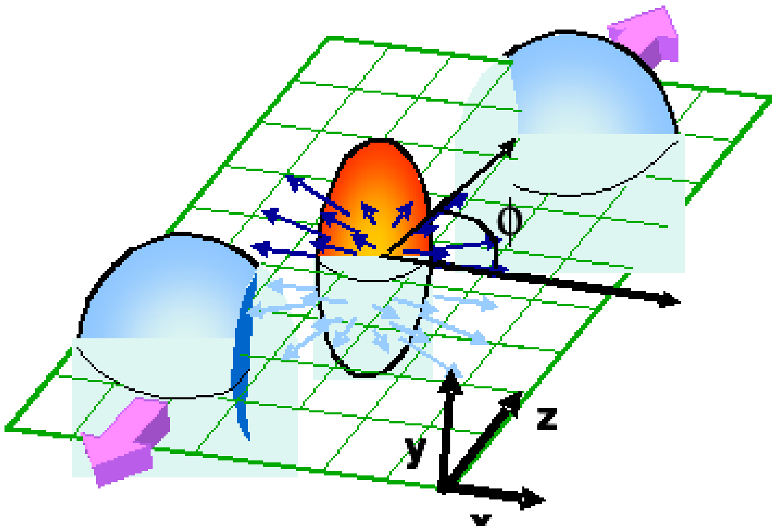
- $\Rightarrow 5d$ Black Brane with horizon at $z_0 \sim T_0^{-3}$

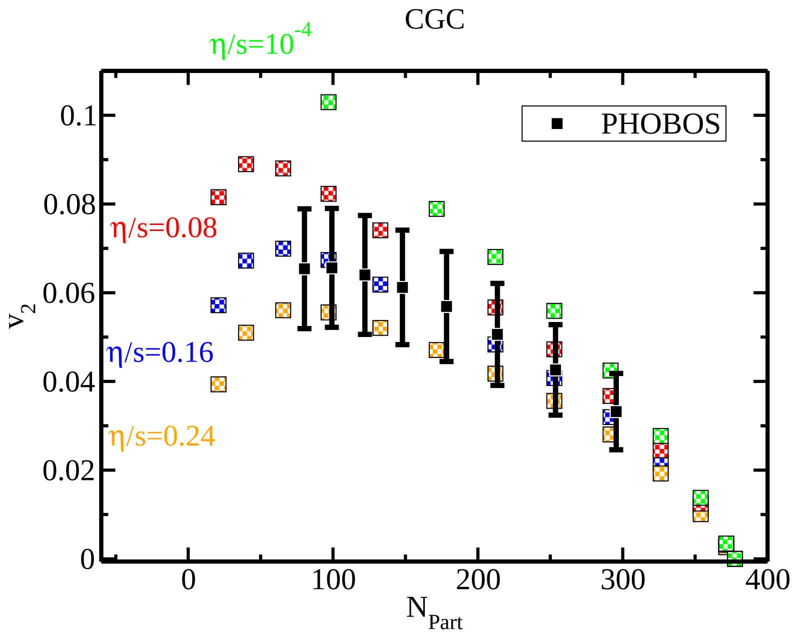
$$ds^2 = -\frac{1 - \tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2} dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1 - \tilde{z}^4/\tilde{z}_0^4}\frac{d\tilde{z}^2}{\tilde{z}^2}$$

$$z \longrightarrow \tilde{z} = z/\sqrt{1+\frac{z^4}{z_0^4}}$$

Strings $vs.$ Reality: Elliptic Flow

Ollitrault (1992)





$$\frac{\partial N}{\partial \Phi} \propto 1 + 2\, v_2 \cos 2\Phi$$

Luzum, Romatschke (2008)

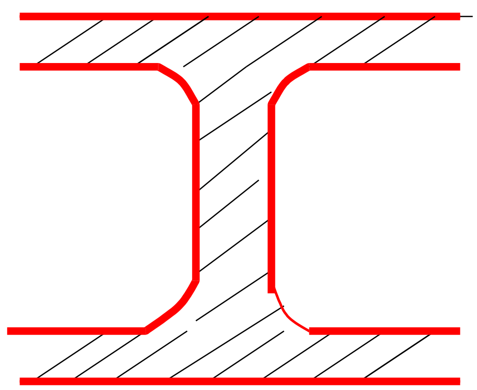
Strong Interactions and Strings

Historical Remark

Strings $\not\Leftrightarrow$ QCD $\Leftrightarrow$ Strings

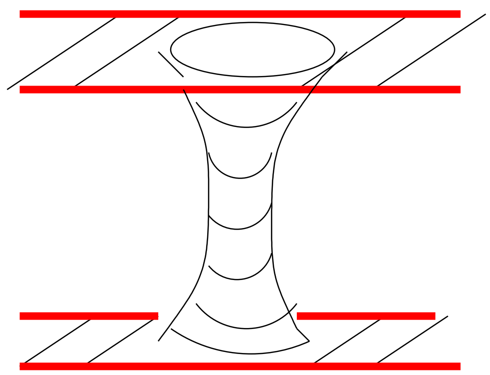
$\boxed{1968} \Rightarrow \boxed{1974} \Rightarrow \boxed{1998} \Rightarrow \boxed{2008} \ldots$

Open String



Gauge

Closed String



Gravity