Construction of the ground state of Matrix Theory: Near the Origin

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Plan

1. Matrix Theory
2. The construction of the ground state near the origin
3. Summary

Matrix Theory

- - membranes - Dirac 62'
  quantum membranes ⟶ YMQM - Goldstone, Hoppe 82'
  supermembranes ⟶ SYMQM - de Wit, Hoppe, Nicolai 87'
  -femoto-universe of YM ⟶ YMQM - Bjorken 79'
  - small volume YM ⟶ Lüscher - 82'
  -toy model - SYMQM - Claudson, Halpern 85'
  - M theory on a light cone in the IMF ⟶ SYMQM
  -Banks, Fischler, Shenker, Susskind 97'

Supermembranes

- quantum description → matrix regularization:

$$H_{reg.} = Tr\left(P_s P_s - \frac{1}{2}[X_s, X_t][X_s, X_t] - iX_s\theta\gamma^s\theta\right)$$

$$X_s = x_{sA}T_A, \quad P_s = p_{sA}T_A \quad \theta_\alpha = \theta_{\alpha A}T_A, \quad T_A \in su(N)$$

$$[x_{sA}, p_{tB}] = i\delta_{st}\delta_{AB}, \quad \{\theta_{\alpha A}, \theta_{\beta B}\} = \delta_{\alpha\beta}\delta_{AB},$$

$$\gamma^s - 16 \times 16, real$$

$$s, t = 1, \ldots 9, \quad A, B = 1, \ldots, N^2 - 1, \quad \alpha, \beta = 1, \ldots, 16$$

$$G_A|_{\mathcal{H}} = 0, \qquad G_A = f_{ABC}\left(x_{sB}p_{sC} - \frac{i}{2}\theta_{\alpha B}\theta_{\alpha C}\right)$$

-de Wit, Hoppe, Nicolai, 87'

- **Does the ground state exist?** $Q_\alpha \Psi = 0$

$$2H\delta_{\alpha,\beta} = \{Q_\alpha, Q_\beta\}, \quad Q_\beta = (-i\gamma^s_{\beta\alpha}\partial_{sA} + \frac{1}{2}\gamma^{st}_{\beta\alpha} f_{ABC} x_{sB} x_{tC})\theta_{\alpha A}$$

- Witten index computations suggest YES

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- **Can it be constructed?**

  - asymptotically (large x_{sA}) - Bach, Bordemann, Fröhlich, Graf, Halpern, Hasler, Hoppe, Konechny, Lundholm, Plefka, Schwatrz, Suter, Yau - 97'-07'

  - small x_{sA} ...

Construction of the ground state near the origin

- Taylor expansion

$$\Psi(x,\theta) = \psi^{(0)} + x_{sA}\psi^{(1)}_{sA} + \frac{1}{2}x_{sA}x_{tB}\psi^{(2)}_{sA\ tB} + \dots$$

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- $Q_\beta\Psi = 0$ gives 3 towers (for $\psi^{(3k)}$, $\psi^{(3k+1)}$ i $\psi^{(3k+2)}$)

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$$-i\gamma^s_{\beta\alpha}\theta_{\alpha A}\psi^{(3)}_{sA,tB,uC} + f_{ABC}\gamma^{tu}_{\beta\alpha}\theta_{\alpha A}\psi^{(0)} = 0 \quad \text{etc}$$

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- what about $\psi^{(0)}$? Ψ is a $SO(9)$ singlet

`-Hasler, Hoppe 02'`

- for $SU(N=2)$:

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  In the Fock space ($\theta_{\alpha A} \longrightarrow f^{\dagger}_{\hat{\alpha}A}, f_{\hat{\alpha}A}$) we look for an algebraic combination of $SU(2)$ invariants $f^{\dagger}_{\hat{\alpha}A}f^{\dagger}_{\hat{\beta}A}$ which are $SO(9)$ invariant.

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- paper-pencil derivation of $\psi^{(0)}$?

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$$2\theta_\alpha |st\rangle = \gamma^s_{\alpha\beta}|t\beta\rangle + \gamma^t_{\alpha\beta}|s\beta\rangle,$$

$$\theta_\alpha |stu\rangle = \frac{i}{\sqrt{2}}\left(\gamma^{st}_{\alpha\beta}|u\beta\rangle + \gamma^{us}_{\alpha\beta}|t\beta\rangle + \gamma^{tu}_{\alpha\beta}|s\beta\rangle\right).$$

- Define

$$\underset{44}{|||}\, 1\rangle := |su\rangle_1 |tu\rangle_2 |st\rangle_3,$$

$$\underset{844}{|||}\, 1\rangle := |suv\rangle_1 |tuv\rangle_2 |st\rangle_3 + |tuv\rangle_1 |st\rangle_2 |suv\rangle_3 + |st\rangle_1 |suv\rangle_2 |tuv\rangle_3$$

then it follows that

$$\phi := \underset{44}{|||}\, 1\rangle + \frac{13}{36} \underset{844}{|||}\, 1\rangle$$

is $SU(2)$ invariant $\rightarrow \psi^{(0)} \sim \phi$ (no **128** content)

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- $\chi \sim \phi$!

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- Taylor expansion $\longrightarrow$ $\Psi(0) = \phi$ (two ways)