

Construction of the ground state of Matrix Theory: Near the Origin

Maciej Trzetrzelewski,
Jagiellonian University

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Plan

1. Matrix Theory
2. The construction of the ground state near the origin
3. Summary

Matrix Theory

- ▶ - membranes - Dirac 62'
 - quantum membranes \rightarrow YMQM - Goldstone, Hoppe 82'
 - supermembranes \rightarrow SYMQM - de Wit, Hoppe, Nicolai 87'
 - femoto-universe of YM \rightarrow YMQM - Bjorken 79'
 - small volume YM \rightarrow Lüscher - 82'
 - toy model - SYMQM - Claudson, Halpern 85'
 - M theory on a light cone in the IMF \rightarrow SYMQM
 - Banks, Fischler, Shenker, Susskind 97'

Supermembranes

- ▶ quantum description \rightarrow matrix regularization:

$$H_{reg.} = Tr \left(P_s P_s - \frac{1}{2} [X_s, X_t] [X_s, X_t] - i X_s \theta \gamma^s \theta \right)$$

$$X_s = x_{sA} T_A, \quad P_s = p_{sA} T_A, \quad \theta_\alpha = \theta_{\alpha A} T_A, \quad T_A \in su(N)$$

$$[X_{sA}, p_{tB}] = i \delta_{st} \delta_{AB}, \quad \{\theta_{\alpha A}, \theta_{\beta B}\} = \delta_{\alpha\beta} \delta_{AB},$$

$$\gamma^s - 16 \times 16, \text{ real}$$

$$s, t = 1, \dots, 9, \quad A, B = 1, \dots, N^2 - 1, \quad \alpha, \beta = 1, \dots, 16$$

$$G_A|_{\mathcal{H}} = 0, \quad G_A = f_{ABC} \left(x_{sB} p_{sC} - \frac{i}{2} \theta_{\alpha B} \theta_{\alpha C} \right)$$

-de Wit, Hoppe, Nicolai, 87'

- ▶ Does the ground state exist? $Q_\alpha \Psi = 0$

$$2H\delta_{\alpha,\beta} = \{Q_\alpha, Q_\beta\}, \quad Q_\beta = (-i\gamma_{\beta\alpha}^s \partial_{sA} + \frac{1}{2}\gamma_{\beta\alpha}^{st} f_{ABC} X_s B X_t C)\theta_{\alpha A}$$

- Witten index computations suggest YES

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- ▶ Can it be constructed?

- asymptotically (large x_{sA}) - Bach, Bordemann, Fröhlich, Graf, Halpern, Hasler, Hoppe, Konechny, Lundholm, Plefka, Schwartz, Suter, Yau - 97'-07'

- small x_{sA} ...

Construction of the ground state near the origin

- ▶ Taylor expansion

$$\Psi(x, \theta) = \psi^{(0)} + x_{sA} \psi_{sA}^{(1)} + \frac{1}{2} x_{sA} x_{tB} \psi_{sA tB}^{(2)} + \dots$$

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$$\gamma_{\beta\alpha}^s \theta_{\alpha A} \psi_{sA, tB}^{(2)} = 0,$$

$$-i \gamma_{\beta\alpha}^s \theta_{\alpha A} \psi_{sA, tB, uC}^{(3)} + f_{ABC} \gamma_{\beta\alpha}^{tu} \theta_{\alpha A} \psi^{(0)} = 0 \quad \text{etc}$$

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- ▶ what about $\psi^{(0)}$? Ψ is a $SO(9)$ singlet

-Hasler, Hoppe 02'

- ▶ for $SU(N = 2)$:

$$\underbrace{f_{ABC}\theta_{\alpha A}\theta_{\alpha B}}_{SU(2)}\psi^{(0)} = 0, \quad \underbrace{\gamma_{\alpha\beta}^{st}\theta_{\alpha A}\theta_{\alpha A}}_{SO(9)}\psi^{(0)} = 0$$

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- ▶ existence and uniqueness of $\psi^{(0)}$

- J. Wosiek 05'

In the Fock space $(\theta_{\alpha A} \rightarrow f_{\hat{\alpha}A}^\dagger, f_{\hat{\alpha}A})$ we look for an algebraic combination of $SU(2)$ invariants $f_{\hat{\alpha}A}^\dagger f_{\hat{\beta}A}^\dagger$ which are $SO(9)$ invariant.

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- ▶ paper-pencil derivation of $\psi^{(0)}$?

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$$2\theta_{\alpha} |st\rangle = \gamma_{\alpha\beta}^s |t\beta\rangle + \gamma_{\alpha\beta}^t |s\beta\rangle,$$

$$\theta_{\alpha} |stu\rangle = \frac{i}{\sqrt{2}} \left(\gamma_{\alpha\beta}^{st} |u\beta\rangle + \gamma_{\alpha\beta}^{us} |t\beta\rangle + \gamma_{\alpha\beta}^{tu} |s\beta\rangle \right).$$

- ▶ Define

$$\begin{array}{c} ||| \\ 44 \end{array} 1 \rangle := |su\rangle_1 |tu\rangle_2 |st\rangle_3,$$

$$\begin{array}{c} ||| \\ 844 \end{array} 1 \rangle := |suv\rangle_1 |tuv\rangle_2 |st\rangle_3 + |tuv\rangle_1 |st\rangle_2 |suv\rangle_3 + |st\rangle_1 |suv\rangle_2 |tuv\rangle_3$$

then it follows that

$$\phi := \begin{array}{c} ||| \\ 44 \end{array} 1 \rangle + \frac{13}{36} \begin{array}{c} ||| \\ 844 \end{array} 1 \rangle$$

is $SU(2)$ invariant $\rightarrow \psi^{(0)} \sim \phi$ (no **128** content)

- ▶ an independent approach - explicit construction in the Fock space given by $\theta_\alpha \rightarrow \lambda_{\hat{\alpha}}, \lambda_{\hat{\alpha}}^\dagger$

$$\theta_{\hat{\alpha}} = \frac{1}{\sqrt{2}}(\lambda_{\hat{\alpha}} + \lambda_{\hat{\alpha}}^\dagger), \quad \theta_{\hat{\alpha}+8} = \frac{1}{i\sqrt{2}}(\lambda_{\hat{\alpha}} - \lambda_{\hat{\alpha}}^\dagger).$$

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$$b_{AB} := \lambda_{\alpha A} \lambda_{\alpha B}, \quad b_{AB}^i := \Gamma_{\alpha\beta}^i \lambda_{\alpha A} \lambda_{\beta B}.$$

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$$r_1 = v_1 v_1 v_1 |0\rangle, \quad r_2 = v_1 v_1 v_2 |0\rangle, \quad r_3 = v_1 v_2 v_2 |0\rangle, \quad r_4 = v_2 v_2 v_2 |0\rangle, \\ r_5 = v_1 w_1 |0\rangle, \quad r_6 = v_2 w_1 |0\rangle, \quad r_7 = v_1 w_2 |0\rangle.$$

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$$\chi = 326304r_1 + 488136r_2 + 72612r_3 + 1377r_4 + 114576r_5 - 176528r_6 + 10296r_7,$$

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- ▶ $\chi \sim \phi!$

Summary

- ▶ SYMQM:

$$2\delta_{\alpha\beta}H = \{Q_\alpha, Q_\beta\}, \quad Q_\beta = \left(-i\gamma_{\beta\alpha}^s \partial_{sA} + \frac{1}{2}\gamma_{\beta\alpha}^{st} f_{ABC} X_{sB} X_{tC}\right)\theta_{\alpha A}$$

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- does the ground state exist?
- what is it?
- other bound states?
- large N limit?

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- ▶ Taylor expansion $\rightarrow \Psi(0) = \phi$ (two ways)