## Nonleptonic charmless B decays (and their search at LHCb)

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$$

## Introduction

$B_{c}^{+}(\bar{b} c)$ very interesting meson

- Only a few experimental features investigated up to now: mass, width, bounds on a few channel ( $\mathrm{J} / \psi$ with 1 or $3 \pi, D^{*+} \bar{D}^{0} \ldots$ )
- Will be produced and studied at LHCb
- Shares dynamical features with the better known quarkonia, but different decays (only weak interaction and not strong interaction)
- Theoretical investigations on lifetime, decay constants, semileptonic form factors (OPE, sum rules, lattice...)


## Introduction

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Less investigated: non-leptonic charmless decays into light mesons

- Pure annihilation processes, similar to $B_{d} \rightarrow K^{+} K^{-}$, $B_{d} \rightarrow D_{s}^{-} K^{+}, B_{u} \rightarrow D_{s}^{-} K^{0}$
- Sheds light on theoretical methods used to assess annihilation for other (non annihilation-dominated) non-leptonic modes
- Decays within the reach of LHCb


## Non-leptonic charmless $B_{c}$ decays

SDG, J. He, E. Kou, P. Robbe, arXiv:0907.2256 [hep-ph]


32 decay channels if lightest pseudoscalar and vector octets

$$
\begin{array}{ccc} 
& S=1 & S=0 \\
P P & K^{+} \pi^{0}, K^{+} \eta, K^{+} \eta^{\prime}, K^{0} \pi^{+} & \pi^{+} \pi^{0}, \pi^{+} \eta, \pi^{+} \eta^{\prime}, K^{+} \bar{K}^{0} \\
V V & K^{*+} \rho^{0}, K^{*+} \phi, K^{*+} \omega, K^{* 0} \rho^{+} & \rho^{+} \rho^{0}, \rho^{+} \phi, \rho^{+} \omega, K^{*+} \bar{K}^{* 0} \\
V P & K^{*+} \pi^{0}, K^{*+} \eta, K^{*+} \eta^{\prime}, K^{* 0} \pi^{+} & \rho^{+} \pi \pi^{0}, \rho^{+} \eta, \rho^{+} \eta^{\prime}, K^{*+} \bar{K}^{0} \\
& \rho^{0} K^{+}, \phi K^{+}, \omega K^{+}, \rho^{+} K^{0} & \rho^{0} \pi^{+}, \phi \pi^{+}, \omega \pi^{+}, \bar{K}^{* 0} K^{+}
\end{array}
$$

## Matrix element

$\left\langle h_{1} h_{2}\right| \mathcal{H}_{\text {eff }}\left|B_{c}\right\rangle$ with $\mathcal{H}_{\text {eff }}=-\frac{G_{F}}{\sqrt{2}}\left[V_{u d} V_{c b}^{*} \mathcal{O}^{\Delta S=0}+V_{u S} V_{c b}^{*} \mathcal{O}^{\Delta S=1}\right]$ with current-current operators with following $S U(3)$ tensor structure

$$
\begin{array}{ll}
\mathcal{O}^{\Delta S=0}=\bar{u} \gamma_{\mu}\left(1-\gamma_{5}\right) d \bar{c} \gamma^{\mu}\left(1-\gamma_{5}\right) b & \left(Y, I, I_{3}\right)=(0,1,1) \\
\mathcal{O}^{\Delta S=1}=\bar{u} \gamma_{\mu}\left(1-\gamma_{5}\right) S \bar{c} \gamma^{\mu}\left(1-\gamma_{5}\right) b & \left(Y, I, I_{3}\right)=(0,1 / 2,1 / 2)
\end{array}
$$

with $\left(Y, I, I_{3}\right)=$ hypercharge, isospin and isospin projection

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Wigner-Eckart th: all decay amplitudes in terms of 3 reduced matrix elts

- $S=\left\langle 8_{S}\left\|\mathcal{O}^{8}\right\| 1\right\rangle$ from symmetric product of the 2 octet mesons.
- $A=\left\langle 8_{A}\left\|\mathcal{O}^{8}\right\| 1\right\rangle$ from antisymmetric product
- $I=\left\langle 8_{\|}\left\|\mathcal{O}^{8}\right\| 1\right\rangle$ from product of an octet and a singlet mesons The values of the reduced matrix elements depend on $\mathcal{O}^{\Delta S=0}$ vs $\mathcal{O}^{\Delta S=1}$ and nature of final state ( $P P, V P$ or $V V$ )


## Symmetry properties of outgoing states

Wigner-Eckart theorem : compute Clebsch-Gordan coefficients for projection of given $8 \times 8$ and $8 \times 1$ final state onto octet operators

CG coeff $=$ usual $S U(2)$ coeff $\times$ so-called isoscalar coeff
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Symmetry properties of wave function for out-going states

- $P P$ ( 1 amplitude): wave function of final state symmetric, only $S$
- $V P$ ( 1 amplitude): contributions from $S$ and $A$
- VV (3 amplitudes):
- wave function symmetric for $S$ and $D$ waves: $S$ contributes
- wave function antisymmetric for $P$ wave: $A$ contributes


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Symmetry properties of wave function for out-going states

- $P P$ ( 1 amplitude): wave function of final state symmetric, only $S$
- VP (1 amplitude): contributions from $S$ and $A$
- VV (3 amplitudes):
- wave function symmetric for $S$ and $D$ waves: $S$ contributes
- wave function antisymmetric for $P$ wave: $A$ contributes

If states with $\eta, \eta^{\prime}, \phi, \omega$, I involved each time $S$ is
NB : Single-angle scheme for $\eta, \eta^{\prime}, \omega, \phi$ to get ideal mixing for vector and fair phenomenological description of pseudoscalars

## PP modes

| Mode | Amplitude | Mode | Amplitude |
| :---: | :---: | :---: | :---: |
| $K^{+} \pi^{0}$ | $\sqrt{\frac{3}{10}} S^{P P}$ | $\pi^{+} \pi^{0}$ | 0 |
| $K^{0} \pi^{+}$ | $\sqrt{\frac{3}{5}} S^{P P}$ | $K^{+} \bar{K}^{0}$ | $\sqrt{\frac{3}{5}} S^{P P}$ |
| $K^{+} \eta$ | $-\frac{2}{3 \sqrt{5}} S^{P P}+\frac{\sqrt{2}}{3} I^{P P}$ | $\pi^{+} \eta$ | $\frac{4}{3 \sqrt{5}} S^{P P}+\frac{\sqrt{2}}{3} I^{P P}$ |
| $K^{+} \eta^{\prime}$ | $\frac{1}{3 \sqrt{10}} S^{P P}+\frac{4}{3} I^{P P}$ | $\pi^{+} \eta^{\prime}$ | $-\frac{1}{3} \sqrt{\frac{2}{5}} S^{P P}+\frac{4}{3} I^{P P}$ |

- Amplitudes must be multiplied by $G_{F} / \sqrt{2}$ and CKM factor $V_{u D} V_{c b}^{*}$
- Relation $A\left(B_{c}^{+} \rightarrow K^{0} \pi^{+}\right)=\sqrt{2} A\left(B_{c}^{+} \rightarrow K^{+} \pi^{0}\right)=\hat{\lambda} A\left(B_{c}^{+} \rightarrow K^{+} \bar{K}^{0}\right)$ with Cabibbo suppression $\hat{\lambda}=V_{u s} / V_{u d}$.
- Similar results for $V P$ and $P P$ results


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- Similar results for $V P$ and $P P$ results

Knowing $S, A, I$ for one type of mesons (PP, VP, VV) is enough to compute all amplitudes in $S U(3)$ limits How to estimate these quantities ? 2 ways...

## First way: Estimate from $B^{0} \rightarrow K^{+} K^{-}$(1)

Two pure annihilation known processes (annihil not negligible)

$$
\operatorname{Br}\left(B^{0} \rightarrow K^{+} K^{-}\right)=\left(0.15_{-0.10}^{+0.11}\right) \times 10^{-6}, \operatorname{Br}\left(B^{0} \rightarrow D_{s}^{-} K^{+}\right)=(3.9 \pm 2.2) \times 10^{-}
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Assuming naive factorization between initial and final states, final-state contribution cancels out in ratio of amplitudes


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\frac{B r\left(B_{c} \rightarrow K^{+} \bar{K}^{0}\right)}{\operatorname{Br}\left(B^{0} \rightarrow K^{+} K^{-}\right)} \simeq \underbrace{\left(\frac{V_{c b}}{V_{u b}}\right)^{2}}_{\sim 100} \underbrace{\left(\frac{f_{B_{c}}}{f_{B}}\right)^{2}}_{\sim 4} \underbrace{\frac{\tau_{B_{c}}}{\tau_{B_{d}}}}_{\sim 0.3} \frac{1}{\xi^{2}}
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$\xi$ : fudge factor to take into account that $B^{0} \rightarrow K^{+} K^{-}$with $t$-channel $W$ vs $B_{c} \rightarrow K^{+} \bar{K}^{0}$ with $s$-channel

- One gluon exchange: $\xi=C_{1} / C_{2} \simeq 4$
- Non perturbative effects: since $C_{2}<C_{1}, C_{2}$ more sensitive than $C_{1}$ to nonperturbative effects, so that $\xi$ closer to 1


## First way: Estimate from $B^{0} \rightarrow K^{+} K^{-}$(2)

$$
\operatorname{Br}\left(B_{c} \rightarrow K^{+} \bar{K}^{0}\right) \simeq \operatorname{Br}\left(B^{0} \rightarrow K^{+} K^{-}\right) \frac{1.2 \times 10^{2}}{\xi^{2}} \gtrsim \operatorname{Br}\left(B^{0} \rightarrow K^{+} K^{-}\right) \times 7.5
$$

- from $\operatorname{Br}\left(B^{0} \rightarrow K^{+} K^{-}\right)$, lower limit of $\operatorname{Br}\left(B_{c} \rightarrow K^{+} \bar{K}^{0}\right)=O\left(10^{-6}\right)$
- leading to $S^{P P} \gtrsim 0.085 \mathrm{GeV}^{3}\left[(P P)=\left(\bar{K}^{0} K^{+}\right)\right]$
- Assume a (naive) dimensional estimate $\left|S^{P P}\right| \simeq \sqrt{2}\left|S^{P V}\right| \simeq\left|S_{0}^{V V}\right|$
- Use Zweig rule to determine the singlet contributions I
$\left[\operatorname{Br}\left(B_{c}^{+} \rightarrow \phi \pi^{+}\left(\rho^{+}\right)\right)\right.$very suppressed]


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$\left[\operatorname{Br}\left(B_{c}^{+} \rightarrow \phi \pi^{+}\left(\rho^{+}\right)\right)\right.$very suppressed]
$\left[B_{d}\right.$ annihil] $\quad B R\left(\phi K^{+}\right) \simeq \mathcal{O}\left(10^{-7}-10^{-8}\right), \quad B R\left(\bar{K}^{* 0} K^{+}\right) \simeq \mathcal{O}\left(10^{-6}\right)$

$$
B R\left(\bar{K}^{0} K^{+}\right) \simeq \mathcal{O}\left(10^{-6}\right), \quad B R\left(\bar{K}^{* 0} K^{*+}\right) \simeq \mathcal{O}\left(10^{-6}\right)
$$

## Second way: QCD factorisation (1)

QCD factorisation estimate for weak annihilation for $B$ into light mesons (neglect all transverse momenta and take only one gluon exchange)


$$
\left\langle h_{1} h_{2}\right| \mathcal{H}_{\text {eff }}\left|B_{C}\right\rangle=i \frac{G_{F}}{\sqrt{2}} V_{c b}^{*} V_{u d(s)} f_{B_{c}} f_{n_{1}} f_{h_{2}} \frac{C_{F}}{N_{C}^{2}} C_{2} A_{i}^{1}\left(h_{1} h_{2}\right)
$$

$A_{i}^{1}\left(h_{1} h_{2}\right)$ convolution of kernel with twist-3 and -4 distrib ampl of mesons

$$
\begin{aligned}
A_{i}^{1}= & \pi \alpha_{s} \int d x d y\left\{\phi_{M_{1}}(y) \phi M_{2}(x)\left[\frac{1}{y\left[(\bar{x}+y) z_{b}-\bar{x} y\right]}-\frac{1}{\bar{x}\left[(\bar{x}+y) z_{c}-\bar{x} y\right]}\right]\right. \\
& \left.+r^{M_{1}} r^{M_{2}} \phi_{m_{1}}(y) \phi_{m_{2}}(x)\left[\frac{2\left(1-z_{b}\right)}{(\bar{x}+y) z_{b}-\bar{x} y}-\frac{2\left(1-z_{c}\right)}{(\bar{x}+y) z_{c}-\bar{x} y}\right]\right\}
\end{aligned}
$$

for $M_{1}$ and $M_{2}$ pseudoscalars (sign changes if $M_{1}$ and/or $M_{2}$ vectors)

## Second way: QCD factorisation (2)

$$
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\end{aligned}
$$

- Asymptotic twist-3 and twist-4 distribution amplitudes, where $y(x)$ part of long. momentum of $M_{1}\left(M_{2}\right)$ carried by valence quark
- Chiral enhancement factors for twist-4 pseudoscalar

$$
r^{\pi}=\frac{2 m_{\pi}^{2}}{m_{b} \times 2 m_{q}} \quad r^{K}=\frac{2 m_{K}^{2}}{m_{b}\left(m_{q}+m_{s}\right)} \quad\left(r^{v}=\frac{2 m_{V}}{m_{b}} \frac{f_{V}^{\perp}}{f_{V}}\right)
$$

- $b$ and $c$-quark masses relative masses

$$
z_{b}=\frac{m_{b}}{m_{b}+m_{c}} \quad z_{c}=1-z_{b}=\frac{m_{c}}{m_{b}+m_{c}}
$$

## Second way: QCD factorisation (3)

A few differences in $B_{c}$ decays compared to the $B_{u, d}$ decays

- $B_{c}$ much simpler: only $O_{2}$ contributes, only CKM factor $V_{c b}^{*} V_{u D}$
- Recover QCD factorisation estimates for $\mathrm{O}_{2}$ weak-annihilation in $B_{u}$ decays when taking $z_{c} \rightarrow 0, z_{b} \rightarrow 1$ )


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- no endpoint singularities/long-distance divergences for $B_{c}$ (div. preventing from estimating annihil in $B_{u, d}$ decays accurately)


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Can compute contribution and recast it into $\operatorname{SU}(3)$ analysis

$$
S^{V P}=0.036 \mathrm{GeV}^{3} \quad S^{P P}=0.021 \mathrm{GeV}^{3} \quad S_{0}^{V V}=0.025 \mathrm{GeV}^{3}
$$

$$
\begin{array}{ll}
\text { [One-gluon] } & B R\left(\phi K^{+}\right)=5 \times 10^{-9}, \quad B R\left(\bar{K}^{* 0} K^{+}\right)=9.0 \times 10^{-8} \\
& B R\left(\bar{K}^{0} K^{+}\right)=6.3 \times 10^{-8}, B R\left(\bar{K}^{* 0} K^{*+}\right)=9.1 \times 10^{-8}
\end{array}
$$

## Comparison of the two methods

$\operatorname{Br}\left(B_{C} \rightarrow \bar{K}^{* 0} K^{+}\right)_{Q C D F} \sim 10^{-7} \quad \operatorname{Br}\left(B_{c} \rightarrow \bar{K}^{* 0} K^{+}\right)_{B_{d}} \simeq 10^{-6}$

- $B_{d}$ annihilation
- charm quark treated as light
- takes into account some long-distance effects
- very naive relation between matrix elements of $O_{1}$ and $O_{2}$, and between $P P, V P$ and $V V$ modes.
- QCD factorisation
- charm quark treated as heavy
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- assumes dominance from specific (one-gluon exchange) diagrams


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Rather different estimates: QCDF one order below $B_{d}$ annihilation

- In agreement with what is seen for pure annihilation $B_{d} \rightarrow K^{+} K^{-}$ $\operatorname{Br}\left(B_{d} \rightarrow K^{+} K^{-}\right)_{Q C D F} \sim 10^{-8} \quad \operatorname{Br}\left(B_{d} \rightarrow K^{+} K^{-}\right)_{B_{d}} \simeq 10^{-7}$
- Final-state interaction may increase significantly factorisation based estimates $\left(B \rightarrow K \chi_{c}\right.$ and $\left.D_{s}^{+} \rightarrow \rho^{0} \pi^{+}\right)$


## Charmless nonleptonic $B_{c}$ decays at LHCb

Which channel for LHCb ?

- all two-body PP final states contain one neutral particle
- LHCb has good efficiencies for charged $K$ - and/or $\pi$-tags (when avoiding low- $p_{T}$ neutral particles) at LHCb
- small width of $\phi$ and $\bar{K}^{* 0}$ simplifies reconstruction

$$
\begin{aligned}
& B_{c}^{+} \rightarrow \phi K^{+}, \bar{K}^{* 0} K^{+}, \bar{K}^{0} \pi^{+}, \rho^{0} K^{+}, \rho^{0} \pi^{+}, \phi \pi^{+} \\
& \text {(with } \phi \rightarrow K^{+} K^{-}, \bar{K}^{* 0} \rightarrow K^{-} \pi^{+}, \rho^{0} \rightarrow \pi^{+} \pi^{-} \text {) }
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- Zweig rule forbids $B_{c}^{+} \rightarrow \phi \pi^{+}$
- $B_{c}^{+} \rightarrow \rho^{0} \pi^{+}$channel comes only from the $A$ (asymmetric) amplitude, subdominant in QCDF.
- $\Delta S=1$ channels Cabibbo suppressed


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- $\Delta S=1$ channels Cabibbo suppressed
$B_{c}^{+} \rightarrow \bar{K}^{* 0} K^{+}$channel might be the best candidate


## LHCb "analysis"

Theoretical estimate of the $B_{c}$ cross section, dominated by $g g$-fusion

- $g g \rightarrow b \bar{b}$ followed by fragmentation (dominant at low- $p_{T}$ )
- $\bar{b} \rightarrow B_{C} \bar{c}$ (dominant at large $p_{T}$ )
- Significant uncertainties on LHCb reference $\sigma\left(B_{c}\right)=0.4 \mu \mathrm{~b}$


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- Detailed simulations necessary in order to estimate sensitivity


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- Selection criteria and trigger efficiencies different for each channel
- Detailed simulations necessary in order to estimate sensitivity

Such a study has been done for $B_{c} \rightarrow J / \psi \pi^{+}$

- From expected $\operatorname{Br}\left(B_{c} \rightarrow J / \psi \pi^{+}\right) \simeq 1 \%$, over 1000 events after one year run of LHCb
- scaling this observation to our process, $\operatorname{Br}\left(B_{c}^{+} \rightarrow \bar{K}^{* 0} K^{+}\right)=10^{-6}$ yields a few events per year at LHCb.

$$
\operatorname{Br}\left(B_{C} \rightarrow \bar{K}^{* 0} K^{+}\right)_{Q C D F} \sim 10^{-7} \quad \operatorname{Br}\left(B_{C} \rightarrow \bar{K}^{* 0} K^{+}\right)_{B_{d}} \simeq 10^{-6}
$$

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$B_{c}^{+}$interesting probe of QCD dynamics: heavy-quark system with different masses, unstable due to weak interactions

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## Conclusions

$B_{c}^{+}$interesting probe of QCD dynamics: heavy-quark system with different masses, unstable due to weak interactions

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- Simple $S U(3)$ relations between 32 processes
- Two different estimates
- Naive relation with (pure annihilation) $B^{0} \rightarrow K^{+} K^{-}$(treating $c$ as light, including long-distance effects)
- Estimate à la QCD factorisation (treating $c$ as heavy, essentially short distances)
- One order of magnitude of difference between the two approaches


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Analysis of LHCb data may set first experimental limits on some of these decays ( $B_{c} \rightarrow \bar{K}^{* 0} K^{+}$)
providing information on annihilation mechanisms at work

## Back-up

## Octet-singlet mixing

$\eta, \eta^{\prime}, \omega, \phi=$ mixtures of $S U(3)$ octet ( $\eta^{8}$ or $\omega^{8}$ ) and singlet ( $\eta^{0}$ or $\omega^{0}$ )

$$
\left|\eta^{8}\right\rangle,\left|\omega^{8}\right\rangle \propto|u \bar{u}+d \bar{d}-2 s \bar{s}\rangle / \sqrt{6} \quad\left|\eta^{0}\right\rangle,\left|\omega^{0}\right\rangle \propto|u \bar{u}+d \bar{d}+s \bar{s}\rangle / \sqrt{3}
$$

For our purpose (order of magnitudes), one-angle scheme

$$
\begin{aligned}
|\eta(\omega)\rangle & =\cos \theta_{p(v)}\left|\eta^{8}\left(\omega^{8}\right)\right\rangle+\sin \theta_{p(v)}\left|\eta^{0}\left(\omega^{0}\right)\right\rangle \\
\left|\eta^{\prime}(\phi)\right\rangle & =-\sin \theta_{p(v)}\left|\eta^{8}\left(\omega^{8}\right)\right\rangle+\cos \theta_{p(v)}\left|\eta^{0}\left(\omega^{0}\right)\right\rangle
\end{aligned}
$$

(Simple) values for the angles: $\tan \theta_{p}=1 /(2 \sqrt{2}), \quad \tan \theta_{v}=\sqrt{2}$.

- Ideal mixing of vector meson

$$
\omega=(u \bar{u}+d \bar{d}) / \sqrt{2} \quad \phi=s \bar{s}
$$

- Non-ideal mixing of $\left(\eta, \eta^{\prime}\right)$, linked to the $U(1)_{A}$ anomaly, in broad agreement with phenomenology (e.g., $J / \psi$ radiative decays)

$$
\eta=(u \bar{u}+d \bar{d}-s \bar{s}) / \sqrt{3} \quad \eta^{\prime}=(u \bar{u}+d \bar{d}+2 s \bar{s}) / \sqrt{6}
$$

## VP modes: strange modes

$$
\begin{array}{cc}
\text { Mode } & \text { Amplitude } \\
K^{+*} \pi^{0} & \frac{1}{2} \sqrt{\frac{3}{5}} S^{V P}+\frac{1}{2 \sqrt{3}} A^{V P} \\
\omega K^{+} & -\frac{1}{2 \sqrt{15}} S^{V P}-\frac{1}{2 \sqrt{3}} A^{V P}+\sqrt{\frac{2}{3}} I^{V P} \\
\rho^{0} K^{+} & \frac{1}{2} \sqrt{\frac{3}{5}} S^{V P}-\frac{1}{2 \sqrt{3}} A^{V P} \\
\phi K^{+} & \frac{1}{\sqrt{30}} S^{V P}+\frac{1}{\sqrt{6}} A^{V P}+\frac{1}{\sqrt{3}} I^{V P} \\
K^{*+} \eta & -\frac{1}{3} \sqrt{\frac{2}{5}} S^{V P}+\frac{\sqrt{2}}{3} A^{V P}+\frac{1}{3} I^{V P} \\
\rho^{+} K^{0} & \sqrt{\frac{3}{10}} S^{V P}-\frac{1}{\sqrt{6}} A^{V P} \\
K^{*+} \eta^{\prime} & \frac{1}{6 \sqrt{5}} S^{V P}-\frac{1}{6} A^{V P}+\frac{2 \sqrt{2}}{3} I^{V P} \\
K^{* 0} \pi^{+} & \sqrt{\frac{3}{10}} S^{V P}+\frac{1}{\sqrt{6}} A^{V P}
\end{array}
$$

## VP modes: non-strange modes

Mode
$\rho^{+} \pi^{0}$ Amplitud
$\frac{1}{\sqrt{3}} A^{V P}$
$\rho^{0} \pi^{+}$
$-\frac{1}{\sqrt{3}} A^{V P}$
$\rho^{+} \eta \quad \frac{2}{3} \sqrt{\frac{2}{5}} S^{V P}+\frac{1}{3} I^{V P}$
$\rho^{+} \eta^{\prime}$
$-\frac{1}{3 \sqrt{5}} S^{V P}+\frac{2 \sqrt{2}}{3} I^{V P}$

Mode
Amplitude
$\omega \pi^{+}$ $\frac{1}{\sqrt{15}} S^{V P}+\sqrt{\frac{2}{3}} I^{V P}$ $\phi \pi^{+} \quad-\sqrt{\frac{2}{15}} S^{V P}+\frac{1}{\sqrt{3}} I^{V P}$
$K^{*+} \bar{K}^{0} \quad \sqrt{\frac{3}{10}} S^{V P}-\frac{1}{\sqrt{6}} A^{V P}$
$\bar{K}^{* 0} K^{+} \quad \sqrt{\frac{3}{10}} S^{V P}+\frac{1}{\sqrt{6}} A^{V P}$

The previous tables yield the simple relations

$$
\begin{aligned}
A\left(B_{c}^{+} \rightarrow K^{* 0} \pi^{+}\right) & =\sqrt{2} A\left(B_{c}^{+} \rightarrow K^{*+} \pi^{0}\right)=\hat{\lambda} A\left(B_{c}^{+} \rightarrow \bar{K}^{* 0} K^{+}\right) \\
A\left(B_{c}^{+} \rightarrow \rho^{+} K^{0}\right) & =\sqrt{2} A\left(B_{c}^{+} \rightarrow \rho^{0} K^{+}\right)=\hat{\lambda} A\left(B_{c}^{+} \rightarrow K^{*+} \bar{K}^{0}\right)
\end{aligned}
$$

## VV modes

- 3 configurations, labeled by (common) helicity of final mesons
- Weak inter left-handed and $h$ conserved in high-energy QCD $\Longrightarrow$ Longitudinal ampl $(h=0)$ dominates transverse (linked to $h \pm 1$ )

$$
\begin{array}{cccl}
\text { Mode } & S, D \text { Amplitudes } & P \text { Amplitude } & \\
K^{*+} \rho^{0} & \sqrt{\frac{3}{10}} S_{0,2}^{V V} & \frac{1}{\sqrt{6}} A_{1}^{V V} & \\
K^{*+} \omega & -\frac{1}{\sqrt{30}} S_{0,2}^{V V}+\frac{2}{\sqrt{3}} I_{0,2}^{V V} & -\frac{1}{\sqrt{6}} A_{1}^{V V} & \\
K^{*+} \phi & \sqrt{\frac{1}{15}} S_{0,2}^{V V}+\sqrt{\frac{2}{3}} I_{0,2}^{V V} & \sqrt{\frac{2}{3}} A_{1}^{V V} & \text { distinguishing } \\
K^{* 0} \rho^{+} & \sqrt{\frac{3}{5}} S_{0,2}^{V V} & \frac{1}{\sqrt{3}} A_{1}^{V V} & \text { partial waves by } \\
\rho^{+} \rho^{0} & 0 & \sqrt{\frac{2}{3}} A_{1}^{V V} & \ell=0,1,2 \\
\rho^{+} \omega & \sqrt{\frac{2}{15}} S_{0,2}^{V V}+\frac{2}{\sqrt{3}} V_{0,2}^{V V} & 0 & \\
\rho^{+} \phi & -\frac{2}{\sqrt{15}} S_{0,2}^{V V}+\sqrt{\frac{2}{3}} I_{0,2}^{V V} & 0 & \\
K^{*+} \bar{K}^{* 0} & \sqrt{\frac{3}{5}} S_{0,2}^{V V} & -\frac{1}{\sqrt{3}} A_{1}^{V V} & \\
A\left(K^{* 0} \rho^{+}\right)=\sqrt{2} A\left(K^{*+} \rho^{0}\right) & \hat{\lambda} A\left(K^{*+} \bar{K}^{0}\right)=\sqrt{2}(-1)^{\ell} A\left(K^{*+} \rho^{0}\right)
\end{array}
$$

## Zweig rule (1)

$S, A$ related to the $8 \times 8, I$ to $1 \times 8$ in Wigner-Eckhart
Zweig rule for the $\Delta S=0$ processes involving $\phi$ :

- $B_{c}^{+} \rightarrow \phi \pi^{+}\left(\rho^{+}\right)$from non-planar diagrams since $\phi=s \bar{s}$
- suppressed: at least three gluons perturbatively, $1 / N_{c}$-suppressed

$A\left(B_{c}^{+} \rightarrow \phi \pi^{+}\right)=A\left(B_{c}^{+} \rightarrow \phi \rho^{+}\right)=0 \Longrightarrow I^{V P}=\sqrt{\frac{2}{5}} S^{V P}, I_{0,2}^{V V}=\sqrt{\frac{2}{5}} S_{0,2}^{V V}$
In principle, possible also for $\eta, \eta^{\prime}$, but $1 / N_{c}$-suppressed contrib related to anomalv can be larme here
S. Descotes-Genon (LPT-Orsay)


## Zweig rule (2)

| Mode | Amplitude | Mode | Amplitude |
| :---: | :---: | :---: | :---: |
| $K^{*+} \eta$ | $\frac{\sqrt{2}}{3} A^{V P}$ | $\rho^{+} \eta$ | $\sqrt{\frac{2}{5}} S^{V P}$ |
| $K^{*+} \eta^{\prime}$ | $\frac{3}{2 \sqrt{5}} S^{V P}-\frac{1}{6} A^{V P}$ | $\rho^{+} \eta^{\prime}$ | $\frac{1}{\sqrt{5}} S^{V P}$ |
| $\omega K^{+}$ | $\frac{1}{2} \sqrt{\frac{3}{5}} S^{V P}-\frac{1}{2 \sqrt{3}} A^{V P}$ | $\omega \pi^{+}$ | $\sqrt{\frac{3}{5}} S^{V P}$ |
| $\phi K^{+}$ | $\sqrt{\frac{3}{10}} S^{V P}+\frac{1}{\sqrt{6}} A^{V P}$ | $\phi \pi^{+}$ | 0 |
|  | $A\left(B_{C}^{+} \rightarrow \rho^{+} \eta\right)=\sqrt{2} A\left(B_{c}^{+} \rightarrow \rho^{+} \eta^{\prime}\right)$ |  |  |

Mode $S, D$ Amplitudes $P$ Amplitude

| $K^{*+} \omega$ | $\sqrt{\frac{3}{10}} S_{0,2}^{V V}$ | $-\frac{1}{\sqrt{6}} A_{1}^{V V}$ |
| :---: | :---: | :---: |
| $K^{*+} \phi$ | $\sqrt{\frac{3}{5}} S_{0,2}^{V V}$ | $\sqrt{\frac{2}{3}} A_{1}^{V V}$ |
| $\rho^{+} \omega$ | $\sqrt{\frac{6}{5}} S_{0,2}^{V V}$ | 0 |
| $\rho^{+} \phi$ | 0 | 0 |

## Identification with results from QCD factorisation

Expressions for all the decay channels if we identify $S, I, A$ and the $O_{2}$ reduced coefficients $b_{2}\left(h_{1}, h_{2}\right)=\frac{C_{F}}{N_{C}^{2}} C_{2} A_{i}^{1}\left(h_{1} h_{2}\right)$.

$$
\left\langle h_{1} h_{2}\right| \mathcal{H}_{\mathrm{eff}}\left|B_{c}\right\rangle=i \frac{G_{F}}{\sqrt{2}} V_{c b}^{*} V_{u d(s)} f_{B_{c}} f_{h_{1}} f_{h_{2}} b_{2}\left(h_{1}, h_{2}\right)
$$

Beneke, Buchalla, Neubert and Sachrajda

- Take QCDf expressions for the decay amplitudes of $B_{u}$ decay into relevant final state
- Pick up the $\mathrm{O}_{2}$ contribution (only one remaining for $B_{C}$

$$
\begin{array}{cl}
S^{P P}=\sqrt{\frac{5}{3}} N_{P P} b_{2}(P P) & I^{P P}=\sqrt{\frac{2}{3}} N_{P P} b_{2}(P P) \\
S^{V P}=\sqrt{\frac{5}{6}} N_{V P}\left(b_{2}(P V)+b_{2}(V P)\right) & A^{V P}=\sqrt{\frac{3}{2}} N_{V P}\left(b_{2}(P V)-b_{2}(V P)\right) \\
I^{V P}=\sqrt{\frac{1}{3}} N_{V P}\left(b_{2}(P V)+b_{2}(V P)\right) & S_{S, D}^{V V}=\sqrt{\frac{5}{3}} N_{V V} b_{2}^{S, D}(V V) \\
\text { S. Descotes-Genon (LPT-OIsay) } & \ldots . .
\end{array}
$$

## $B_{c}$ production at LHCb

- LHC pp collider large cross section for the $b \bar{b}$ hadro-production
- Some collisions (not many) produce $B_{C}$ (less than 1\%)
- LHCb experiment ideal to study it

Theoretical estimate of the $B_{c}$ cross section, dominated by $g g$-fusion

- $g g \rightarrow b \bar{b}$ followed by fragmentation (dominant at low- $p_{T}$ )
- $\bar{b} \rightarrow B_{C} \bar{c}$ (dominant at large $p_{T}$ )


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$\mathcal{O}\left(\alpha_{s}^{4}\right)$ computation of the $\bar{b} \rightarrow B_{c} b \bar{c}$
- $\sigma\left(p p \rightarrow B_{c}^{+} X\right) \simeq 0.3-0.8 \mu b$ for LHCb
- error from theoretical inputs such as choice of the $\alpha_{s}$ scale and $B_{C}$ distribution function
- additional systematics from higher-twist and radiative corrections ?

Take value for the cross section at $\mathrm{LHCb} \sigma\left(B_{c}\right)=0.4 \mu \mathrm{~b}$

