Heavy hadron spectroscopy in a Salpeter model with AdS/QCD inspired potential

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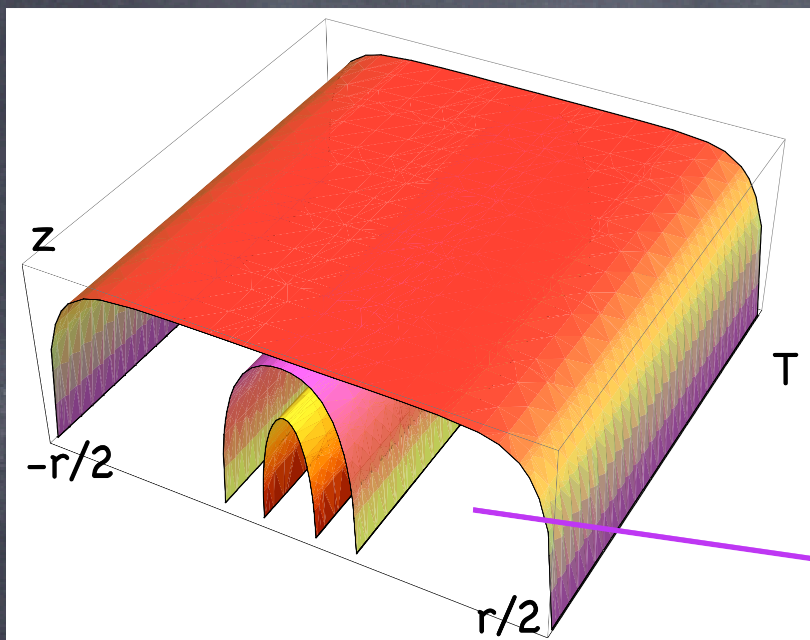
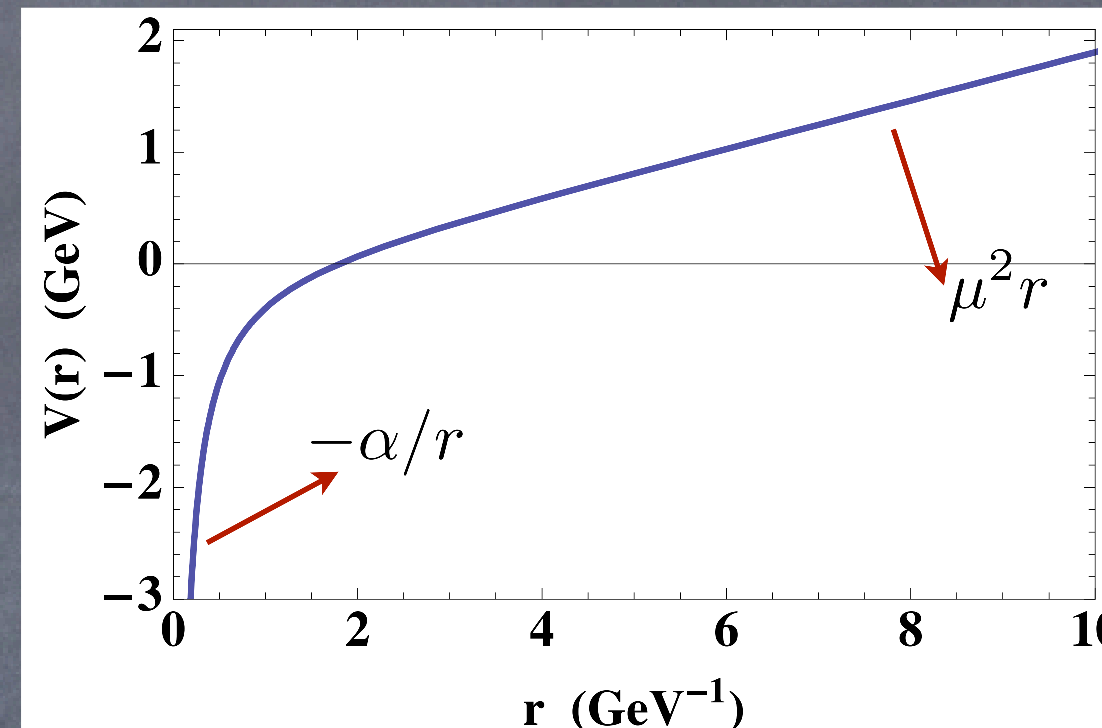
EPS - HEP 2009
16-22 July 2009, Krakow

AdS/QCD correspondence: new approach to non-perturbative QCD

compute $Q\bar{Q}$ potential:



rectangular
Wilson loop



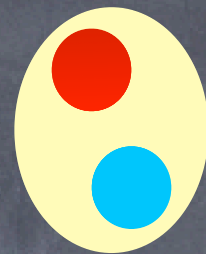
O.Andreev and V.I.Zakharov,
PRD 74, 025023 (2006)

Previous determinations: interpolation (Cornell potential)
lattice techniques
NRQCD
...

We introduce it in a potential model with relativistic kinematics (Salpeter equation) to compute hadron spectra

... not only mesons:

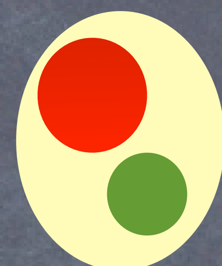
meson:



$$3 \times \bar{3} = 1 + 8$$

$$\hookrightarrow V_{Q\bar{Q}} = V_{\mathrm{AdS/QCD}}$$

diquark:

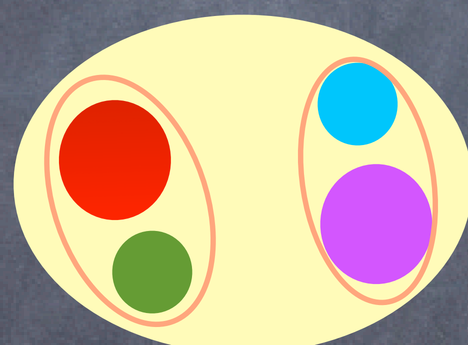


$$3 \times 3 = \bar{3} + 6$$

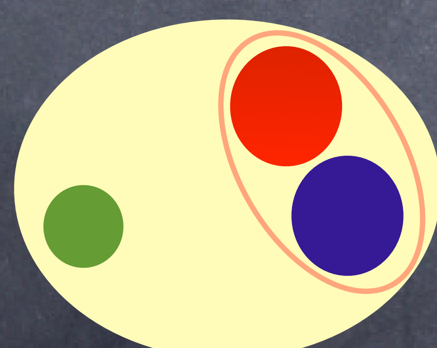
$$\hookrightarrow V_{QQ} = \frac{1}{2} V_{Q\bar{Q}}$$

one gluon exchange approximation

tetraquark:



baryon:



$$3 \times \bar{3} = 1 + 8$$

$$\hookrightarrow V_{Q\bar{Q}}$$

account for finite size of constituents!

use AdS/QCD inspired potential

Plan:

- ✓ description of the model
- ✓ fit of parameters from meson spectrum and predictions
- ✓ meson decay widths
- ✓ doubly heavy baryon masses

Ingredients:

1. Salpeter equation $\left(\sqrt{m_1^2 - \nabla^2} + \sqrt{m_2^2 - \nabla^2} + V(r)\right)\psi(\mathbf{r}) = M\,\psi(\mathbf{r})$

constituents' masses

hadron wave function

hadron mass

2. $Q\bar{Q}$ Potential $V(r) = V^{color}(r) \; + \; V^{spin}(r) \; + \; V_0$

$$A_Q \frac{\delta(r)}{m_1 m_2}(\mathbf{S}_1 \cdot \mathbf{S}_2), \;\; \delta(r) = \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 \exp^{-\sigma^2 r^2}$$

different values at the c and b scale

AdS/QCD potential:

$$\begin{cases} r(\lambda) = 2\sqrt{\frac{\lambda}{c}} \int_0^1 dv\, v^2\; \mathrm{e}^{\lambda(1-v^2)/2} \left(1 - v^4\; \mathrm{e}^{\lambda(1-v^2)}\right)^{-1/2} \\ V(\lambda) = \frac{g}{\pi}\sqrt{\frac{c}{\lambda}} \left(-1 + \int_0^1 dv\, v^{-2} \left[\mathrm{e}^{\lambda v^2/2} \left(1 - v^4\; \mathrm{e}^{\lambda(1-v^2)}\right)^{-1/2} - 1 \right.\right. \end{cases}$$

cutoff: $V(r) = V(r_M) \quad \text{if} \;\; r < r_M = \frac{4\pi\Lambda}{3M} \qquad (\text{if } m_1 = m_2, \; \Lambda = 1)$

regulate the singularity of the wave function

3. Multhopp method: integral equation $\longrightarrow$ set of linear equations

$$\sum_{m=1}^{N} B_{km}\,\phi(\theta_k) = M\,\phi(\theta_m)$$

$$\phi(\theta_k) = u_0(-\cot\theta_k)\,, \qquad \theta_k = \frac{k\pi}{N+1} \qquad (k = 1, \cdots N)$$

Meson spectrum

M.V.Carlucci,F.G.,G.Nardulli,M.Pellicoro, S.Stramaglia Eur.Phys.J.C 57, 569 (2008)

fit of the parameters and predictions:

Flavor	Level	$J=0$			$J=1$		
		Particle	Th. mass	Exp. mass	Particle	Th. mass	Exp. mass
$c\bar{q}$	$1S$	D	1.862	1.867	D^*	2.027	2.008
	$2S$		3.393			2.598	2.622
	$3S$		2.837			2.987	
$c\bar{s}$	$1S$	D_s	1.973	1.968	D_s^*	2.111	2.112
	$2S$		2.524			2.670	
	$3S$		2.958			3.064	
$c\bar{c}$	$1S$	η_c	2.990	2.980	J/ψ	3.125	3.097
	$2S$		3.591	3.637		3.655	3.686
	$3S$		3.994			4.047	4.039
$b\bar{q}$	$1S$	B	5.198	5.279	B^*	5.288	5.325
	$2S$		5.757			5.819	
	$3S$		6.176			6.220	
$s\bar{b}$	$1S$	B_s	5.301	5.366	B_s^*	5.364	5.412
	$2S$		5.856			5.896	
	$3S$		6.266			6.296	
$b\bar{c}$	$1S$	B_c	6.310	6.286	B_c^*	6.338	
	$2S$		6.869			6.879	
	$3S$		7.221			7.228	
$b\bar{b}$	$1S$	η_b	9.387		Υ	9.405	9.460
	$2S$		10.036			10.040	10.023
	$3S$		10.369			10.371	10.355
	$4S$		10.619			10.620	10.579

in good agreement with the subsequent measurement by BaBar!

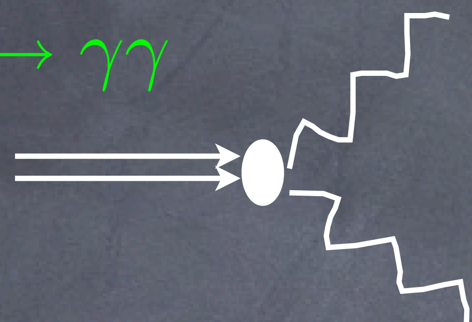
$$M_{\eta_b} = 9388.9^{+3.1}_{-2.3} \pm 2.7 \text{ MeV}$$

PRL 101, 071801 (2008)

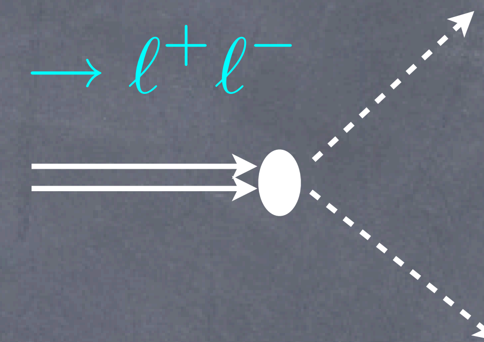
Decay widths

F.G., PRD 78, 117501 (2008)

$\eta_{b,c}(nS) \to \gamma\gamma$



$\psi, \Upsilon\,(nS) \to \ell^+\ell^-$



Effective lagrangians:

$$\mathcal{L}_{eff}^{\gamma\gamma} = -i\,c_1(\bar{q}\,\gamma^\sigma\gamma^5 q)\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}A^\rho \qquad \mathcal{L}_{eff}^{\ell\bar{\ell}} = -c_2(\bar{q}\,\gamma^\mu q)(\ell\gamma_\mu\bar{\ell})$$

$$c_1 = \frac{Q^2\,4\pi\,\alpha_{em}}{M^2} \qquad c_2 = \frac{Q\,4\pi\,\alpha_{em}}{M^2}$$

Widths:

$$\Gamma_{\gamma\gamma} = \frac{4\pi\,Q^4\alpha_{em}^2 f_P^2}{M} \qquad \Gamma_{\ell^+\ell^-} = \frac{4\pi\,Q^2\alpha_{em}^2 f_V^2}{3\,M}$$

Decay constants:

$$\langle 0|A_{ij}^\mu|P(k)\rangle = ik^\mu Q_{ij} f_P$$

meson flavor matrix

$$\langle 0|V_{ij}^\mu|V(k,\lambda)\rangle = \epsilon(\lambda)^\mu Q_{ij} m_V f_V$$

helicity

polarization vector

$$f_P = \sqrt{3}\frac{1}{2\pi M}\int_0^{+\infty} dk\;k\;\tilde{u}_0(k) N^{\frac{1}{2}}\left[1 - \frac{k^2}{(E_i+m_i)(E_j+m_j)}\right]$$

$$f_V = \sqrt{3}\frac{1}{2\pi M}\int_0^{+\infty} dk\;k\;\tilde{u}_0(k) N^{\frac{1}{2}}\left[1 + \frac{k^2}{3(E_i+m_i)(E_j+m_j)}\right]$$

$$N = \frac{(E_i+m_i)(E_j+m_j)}{E_j E_i}$$

Particle	$\Gamma_{\gamma\gamma}$(KeV)	[1]	[2]	[3]	[4]
η_c	4.252	7.46	7.18	7.14±0.95	5.5
η'_c	3.306	4.1	1.71	4.44±0.48	1.8
η''_c	1.992		1.21		
η_b	0.313	0.560	0.230	0.384±0.047	0.350
η'_b	0.151	0.269	0.070	0.191±0.025	0.150
η''_b	0.092	0.208	0.040		0.100

[1] J.P.Lansberg and T.N.Pham, PRD 75, 017501 (2007)
[2] O.Lakhina and E.S.Swanson, PRD 74, 014012 (2006)
[3] C.S.Kim et al., Phys. Lett. B 606, 323 (2005)
[4] D.Ebert et al., Mod. Phys. Lett. A 18, 601 (2003)

Some experimental data:

$$\eta_c : \begin{array}{l} \Gamma = 27.4 \pm 2.9 \text{ MeV} \\ \Gamma_{\gamma\gamma}/\Gamma = (1.8 + 0.6 - 0.5)\, 10^{-4} \end{array} \Rightarrow \Gamma_{\gamma\gamma} \sim 4.9 \text{ KeV}$$

Partial width (evaluation by from various determinations) = $7.2 \pm 0.7 \pm 2.0$ KeV
(not included in summary tables of pdg)

Particle	$\Gamma_{\ell^+\ell^-}$ (KeV)	Exp.
J/ψ	4.080	5.55± 0.14±0.02
ψ'	2.375	2.38 ± 0.04
ψ''	0.836	0.86±0.07
Υ	1.237	1.340 ± 0.018
$\Upsilon(2S)$	0.581	0.612 ± 0.011
$\Upsilon(3S)$	0.270	0.443± 0.008
$\Upsilon(4S)$	0.212	0.272± 0.029

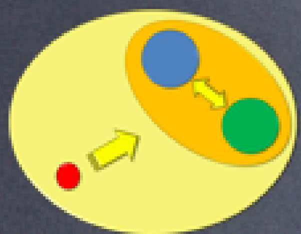
$\longrightarrow$ D-wave component?

Doubly Heavy Baryons: 2-body problem: quark + diquark (3 x $\bar{3}$)

F.G., PRD 79, 094002 (2009)

interaction: $$\tilde{V}(r) = \frac{1}{N}\int d\mathbf{R}\ |\psi_d(\mathbf{R})|^2 V(|\mathbf{R}-\mathbf{r}|)$$

$$N = \int d\mathbf{R}\ |\psi_d(\mathbf{R})|^2$$



					n.r. quark model q-q-q			quark model d-q		sum rules	quenched lattice QCD	
Particle	State	J^P	q-d content	Mass (GeV)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Ξ_{cc}	$1S$	$\frac{1}{2}^+$	$q\{cc\}_{1S}$	3.547	3.579	3.676	3.612	3.620	3.48	4.26	3.562	3.549
	$2S$			4.183	3.876							
	$3S$			4.640								
Ξ^*_{cc}	$1S$	$\frac{3}{2}^+$	$q\{cc\}_{1S}$	3.719	3.656	3.753	3.706	3.727	3.61	3.90	3.625	3.641
	$2S$			4.282	4.025							
	$3S$			4.719								
Ω_{cc}	$1S$	$\frac{1}{2}^+$	$s\{cc\}_{1S}$	3.648	3.697	3.815	3.702	3.778	3.59	4.25	3.681	3.663
	$2S$			4.268	4.112							
	$3S$			4.714								
Ω^*_{cc}	$1S$	$\frac{3}{2}^+$	$s\{cc\}_{1S}$	3.770	3.769	3.876	3.783	3.872	3.69	3.81	3.737	3.734
	$2S$			4.334	4.160							
	$3S$			4.766								

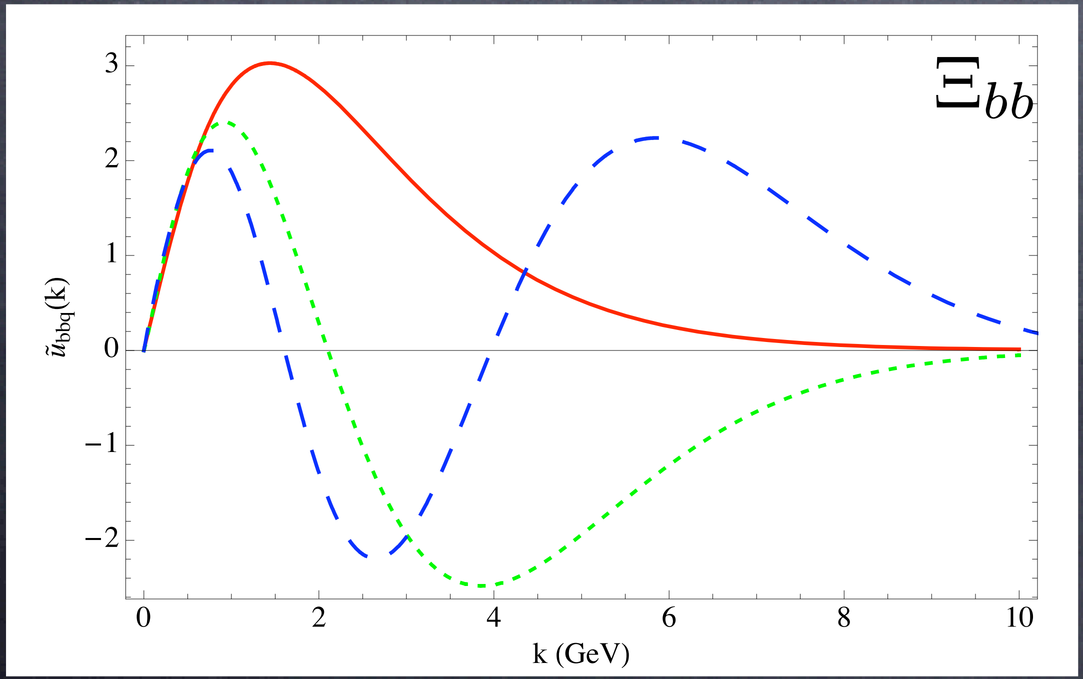
experimental observation: $\Xi^+_{cc} \to \Lambda_c K^- \pi^+$ PRL 89, 112001 (2002)

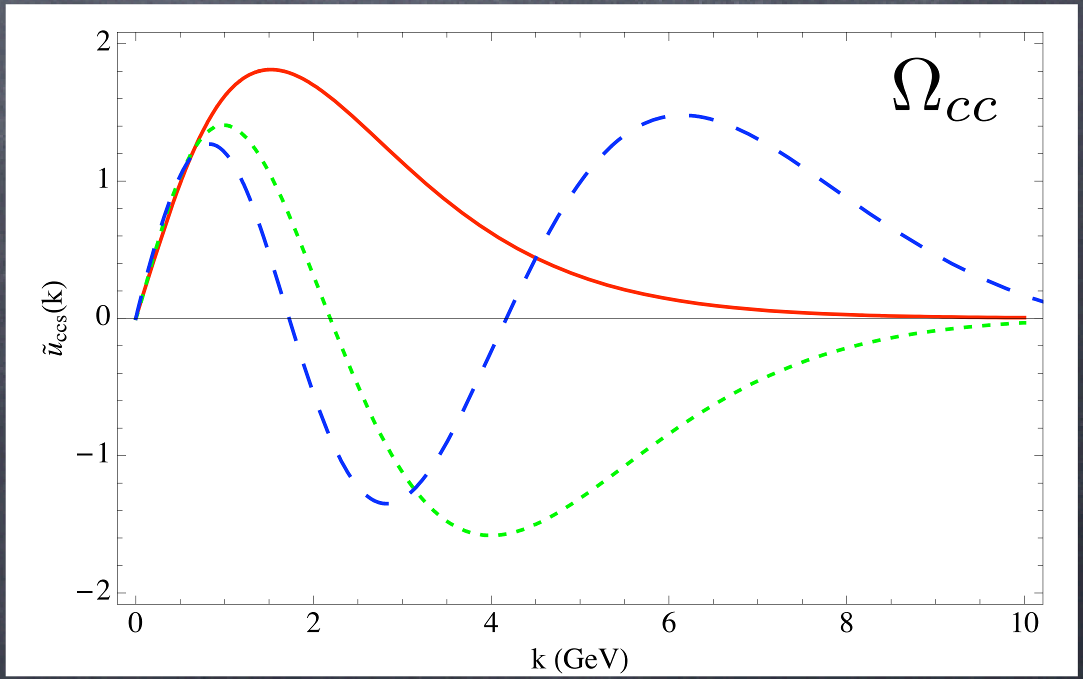
(Selex Collaboration) $\Xi^+_{cc} \to p D^+ K^-$ PL B 628, 18 (2005)

$$M_{\Xi_{cc}} = 3518.9 \pm 0.9\ \text{MeV}$$

Particle	State	J^P	q-d content	Mass (GeV)	[1]	[2]	[3]	[4]	[5]	[6]	[9]
Ξ_{bb}	$1S$	$\frac{1}{2}^+$	$q\{bb\}_{1S}$	10.185	10.189	10.340	10.197	10.202	10.09	9.78	10.127
	$2S$			10.751	10.586						
	$3S$			11.170							
Ξ^*_{bb}	$1S$	$\frac{3}{2}^+$	$q\{bb\}_{1S}$	10.216	10.218	10.367	10.236	10.237	10.13	10.35	10.151
	$2S$			10.770	10.501						
	$3S$			11.184							
Ω_{bb}	$1S$	$\frac{1}{2}^+$	$s\{bb\}_{1S}$	10.271	10.293	10.454	10.260	10.359	10.18	9.85	10.225
	$2S$			10.830	10.604						
	$3S$			11.240							
Ω^*_{bb}	$1S$	$\frac{3}{2}^+$	$s\{bb\}_{1S}$	10.289	10.321	10.486	10.297	10.389	10.20	10.28	10.246
	$2S$			10.839	10.622						
	$3S$			11.247							

[1] A.Valcarce et al., Eur. Phys. J. A 37, 217 (2008)
[2] W.Roberts et al., Int. J. Mod. Phys. A 23, 2817 (2008)
[3] C.Albertus et al., Eur. Phys. J. A 32, 183 (2007)
[4] D.Ebert et al., Phys. Rev. D 66, 014008 (2002)
[5] V.V.Kiselev et al., Phys. Usp. 45, 455 (2002)
[6] J.R.Zhang et al., Phys. Rev. D 78, 094007 (2008)
[7] N.Mathur et al., Phys. Rev. D 66, 014502 (2002)
[8] J.M.Flynn et al., JHEP 0307, 066 (2003)
[9] R.Lewis et al., Phys. Rev. D 79, 014502 (2009)





Additional energetic level: baryon = quark + 2S diquark

For doubly heavy baryons it is lower than 2S-baryons

Baryon	J^P	Quark-diquark content	Mass (GeV)	[2]	[4]	[5]
Ξ_{cc}	$\frac{1}{2}^+$	$q\{cc\}_{2S}$	3.893	4.029	3.910	3.812
Ξ^*_{cc}	$\frac{3}{2}^+$	$q\{cc\}_{2S}$	4.021	4.042	4.027	3.944
Ω_{cc}	$\frac{1}{2}^+$	$s\{cc\}_{2S}$	3.992	4.180	4.075	
Ω^*_{cc}	$\frac{3}{2}^+$	$\mathrm{s}\{cc\}_{2S}$	4.105	4.188	4.174	
Ξ_{bb}	$\frac{1}{2}^+$	$q\{bb\}_{2S}$	10.453	10.576	10.441	10.373
Ξ^*_{bb}	$\frac{3}{2}^+$	$q\{bb\}_{2S}$	10.478	10.578	10.482	10.413
Ω_{bb}	$\frac{1}{2}^+$	$s\{bb\}_{2S}$	10.538	10.693	10.610	
Ω^*_{bb}	$\frac{3}{2}^+$	$s\{bb\}_{2S}$	10.556	10.721	10.645	

Comparison with HQET

Doubly heavy baryons $\longrightarrow$ 1/m$_{\{QQ\}}$ expansion:

(analogous to the 1/m$_Q$ expansion for heavy baryons)

$$M_{\{QQ\}q} = m_{\{QQ\}} + \Lambda + \frac{\lambda_1}{2m_{\{QQ\}}} + A_Q d_H \frac{\lambda_2}{2m_{\{QQ\}}} \qquad d_H = \mathbf{S}_{\{QQ\}} \cdot \mathbf{S}_q$$

Inferred relations:

$$\underbrace{\Xi^*_{QQ}}_{J^P = 3/2^+} - \underbrace{\Xi_{QQ}}_{J^P = 1/2^+} = A_Q \frac{3\lambda_2}{4m_{\{QQ\}}}$$

$$\frac{\Xi^*_{bb} - \Xi_{bb}}{\Xi^*_{cc} - \Xi_{cc}} = \frac{A_b m_{\{cc\}}}{A_c m_{\{bb\}}} \qquad \frac{\Xi^{*2}_{bb} - \Xi^2_{bb}}{\Xi^{*2}_{cc} - \Xi^2_{cc}} = \frac{A_b}{A_c}$$

ok!

Mass splitting hierarchy:

$$(\Xi^*_{cc} - \Xi_{cc}) > (\Omega^*_{cc} - \Omega_{cc}) > (\Xi^*_{bb} - \Xi_{bb}) > (\Omega^*_{bb} - \Omega_{bb})$$

Conclusions

AdS/QCD correspondence: new tool for exploring the non-perturbative regime of QCD

Example: compute the quark-antiquark static potential
Application in a potential model with results:

* reproduce meson spectra
* predictions for some meson masses and decay constants
* predictions for doubly heavy baryon masses

Prospects:

➡ strong decay widths of baryons
➡ $1/\alpha$ corrections to the AdS/QCD potential

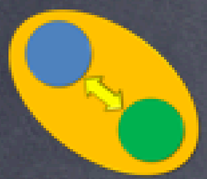
THANK YOU!

Tetraquark

Consider tetraquark=diquark+antidiquark

State	Mass	Mass	Mass
{qs}	0.980	[qs]	0.979
{ss}	1.096		
{cq}	2.168	[cq]	2.120
{cs}	2.276	[cs]	2.235
{cc}	3.414		
{bq}	5.526	[bq]	5.513
{bs}	5.630	[bs]	5.619
{bc}	6.741	[bc]	6.735
{bb}	10.018		

Diquark



$$3 \times 3 = \bar{3} + 6$$

attractive

{ } spin 1

[ ] spin 0

Tetraquark

$$3 \times \bar{3} = 1 + 8$$

the same as for mesons!

$\Rightarrow$ use the same potential

diquark not point-like $\longrightarrow$ convolution with diquark wave function

$$\tilde{V}(R) = \frac{1}{N} \int d\mathbf{r_1} \int d\mathbf{r_2}\ |\psi(\mathbf{r_1})|^2 |\psi(\mathbf{r_2})|^2 V(|\mathbf{R} + \mathbf{r_1} - \mathbf{r_2}|)$$

$$N = \int d\mathbf{r_1} \int d\mathbf{r_2}\ |\psi_{12}(\mathbf{r_1})|^2 |\psi_{34}(\mathbf{r_2})|^2 \qquad \psi_{ij}(r) = \frac{u_0(r)}{r\sqrt{4\pi}}$$

$(\mathbf{cq})(\bar{\mathbf{c}}\bar{\mathbf{q}})$ spectrum (GeV)

J^{PC}	Flavor content	Mass	Mass$^{(1)}$	Mass$^{(2)}$	Exp. (?) State	Exp. (?) Mass$^{(3)}$
0^{++}	$[cq][\bar{c}\bar{q}]$	3.857	3.812	3.723		
1^{++}	$([cq]\{\bar{c}\bar{q}\} + [\bar{c}\bar{q}]\{cq\})/\sqrt{2}$	3.899	3.871	$3.872^{\dagger}$	$X(3872)$	3.8712 ± 0.0004
1^{+-}	$([cq]\{\bar{c}\bar{q}\} - [\bar{c}\bar{q}]\{cq\})/\sqrt{2}$	3.899	3.871	3.754		
0^{++}	$\{cq\}\{\bar{c}\bar{q}\}$	3.729	3.852	3.832		
1^{+-}	$\{cq\}\{\bar{c}\bar{q}\}$	3.833	3.890	3.882		
2^{++}	$\{cq\}\{\bar{c}\bar{q}\}$	3.988	3.968	3.952	$Y(3940)$	$3.943 \pm 0.011 \pm 0.013$

First radial excitation of 1^{+-}: 4.421 GeV and 4.418 GeV
(indentified with $Z^{+}(4430)$ in arXiv:0708.3997, Maiani, Polosa and Riquer)

(1) D. Ebert, R.N. Faustov and V.O.Galkin, Phys. Lett. B 634, 214 (2006) [arXiv:hep-ph/0512230].
(2) L.Maiani, F.Piccinini, A.D.Polosa and V.Riquer, Phys. Rev. D 71, 014028 (2005) [arXiv:hep-ph/0412098].
(3) S.K.Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003) [arXiv:hep-ex/0309032].
V.M.Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 93, 072001 (2004) [arXiv:hep-ex/0312021].
D.Acosta et al. [CDF II Collaboration], Phys. Rev. Lett. 93, 072001 (2004) [arXiv:hep-ex/0312021].
B.Aubert et al. [BABAR Collaboration], Phys. Rev. D 71, 071103 (2005) [arXiv:hep-ex/0406022].
K.Abe et al., arXiv:hep-ex/0505037. K.Abe et al., arXiv:hep-ex/0505038.

† means that the experimental value is used as an input in this case

$(\mathbf{bq})(\bar{\mathbf{b}}\mathbf{q})$ spectrum (GeV)

J^{PC}	Flavor content	Mass	Mass(1)
0^{++}	$[bq][\bar{b}\bar{q}]$	10.260	10.471
$1^{+\pm}$	$([bq]\{\bar{b}\bar{q}\} \pm [\bar{b}\bar{q}]\{bq\})/\sqrt{2}$	10.284	10.492
0^{++}	$\{bq\}\{\bar{b}\bar{q}\}$	10.264	10.473
1^{+-}	$\{bq\}\{\bar{b}\bar{q}\}$	10.275	10.484
2^{++}	$\{bq\}\{\bar{b}\bar{q}\}$	10.296	10.534

$(\mathbf{cq})(\bar{\mathbf{q}}\mathbf{s})$ spectrum (GeV)

J^{P}	Flavor content	Mass	Mass(2)	
0^{+}	$[cq][\bar{q}\bar{s}]$	2.840	2.371	$D_s(2371)$?
0^{+}	$\{cq\}\{\bar{q}\bar{s}\}$	2.503	2.424	
1^{+}	$\{cq\}[\bar{q}\bar{s}]$	2.880	2.410	
1^{+}	$\{cq\}\{\bar{q}\bar{s}\}$	2.748	2.462	$D_s(2457)$?
1^{+}	$[cq]\{\bar{q}\bar{s}\}$	2.841	2.571	
2^{+}	$\{cq\}\{\bar{q}\bar{s}\}$	2.983	2.648	$X(2632)$?

ko!