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PSI

HEPTOOLS

Tools and Precision Calculations for Physics Discoveries at Colliders

Electroweak corrections to W+jet hadroproduction including leptonic W-boson decays

Alexander Mück

in collaboration with

A. Denner (PSI), S. Dittmaier (Freiburg),

T. Kasprzik (MPI Munich)

EPS HEP 2009

Krakow, 16th July 2009

based on Denner, Dittmaier, Kasprzik, AM [arXiv:0906.1656]

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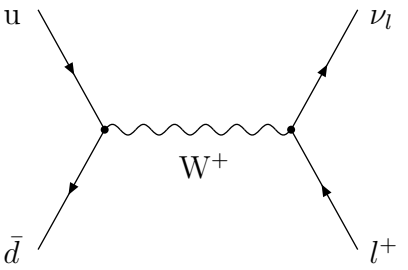
Outline

- W production at the LHC
- W+jet production: $pp \to W + \text{jet} \to l\nu_l + \text{jet}$
  - motivation and theoretical status
  - Electroweak and QCD corrections
  - Numerical results for the LHC
- Conclusions

W Production at the LHC

Charged-current Drell-Yan:

$$pp \longrightarrow W^{\pm} \longrightarrow l^{\pm}\nu_l$$



- clean signal: lepton + missing p_{T}
- huge cross section: $\sigma_W = 30$ nb (5 nb after basic cuts)
- very useful: M_W, Γ_W, luminosity, PDFs, calibration

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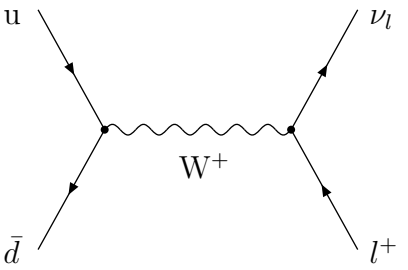

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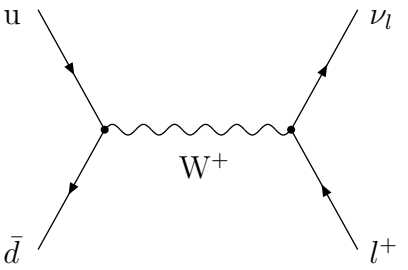

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- EW: NLO, leading higher order contributions
- combined EW and QCD corrections?

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{MC@NLO}} + (\mathrm{d}\sigma_{\mathrm{EW}}^{\mathrm{HORACE}} - \mathrm{d}\sigma_{\mathrm{Born}})_{\mathrm{HERWIG-PS}} \quad \text{etc.}$$

Balossini et al. [arXiv:0907.0276]

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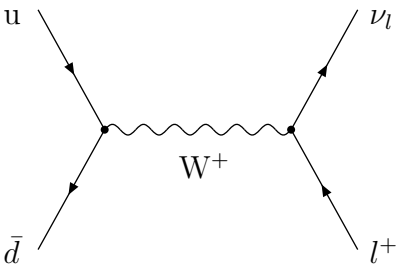

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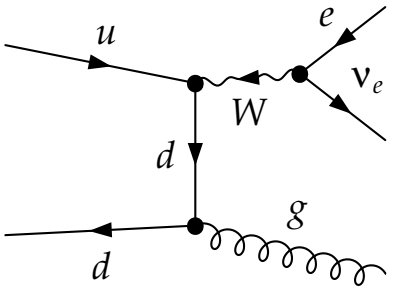
  hard QCD radiation + EW corrections?

  $\Rightarrow$ look at EW corrections for W+jet production

W+jet production

$pp \to l\nu_l + \text{jet}$:



- large cross section
  (~ 1nb after basic cuts)
- dominant SM channel for high p_T leptons
- precision tests for jet dynamics
- W+jet(s) important background for many searches
  (high p_T lepton, missing energy, jet(s))

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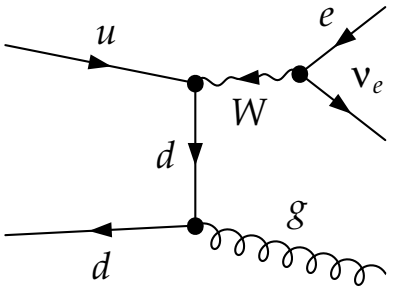

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Theoretical status:

- NLO QCD corrections known and available

DYRAD: Giele et al. [hep-ph/9302225]
MCFM: Campbell,Ellis [hep-ph/0202176]
and as part of NNLO single W: Melnikov, Petriello [hep-ph/0609070]
Catani et al. [arXiv:0903.2120]

W+jet production

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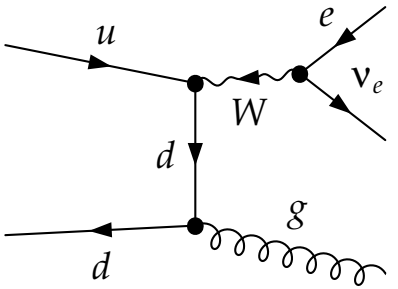

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Theoretical status:

- NLO QCD corrections known and available
- EW corrections for stable (on-shell) W bosons

Kühn, Kulesza,Pozzorini,Schulze [hep-ph/0703283], [arXiv:0708.0476]
Hollik,Kasprzik,Kniehl [arXiv:0707.2553]

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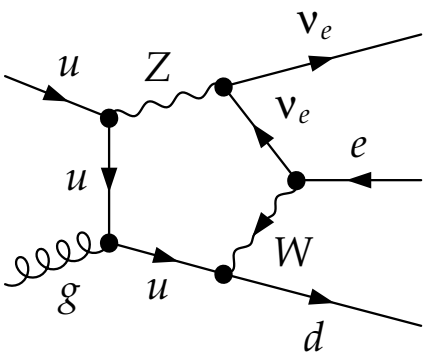
EW corrections

Complete EW corrections calculated

Denner, Dittmaier, Kasprzik, AM [arXiv:0906.1656]



+

$\mathcal{O}(100)$ diagrams

per partonic channel

- physical final state
- all off-shell effects included
- part of the $\mathcal{O}(\alpha\alpha_s)$ corrections for incl. W production

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Complete EW corrections calculated

Denner, Dittmaier, Kasprzik, AM [arXiv:0906.1656]

- stable reduction scheme for tensor integrals

  Denner, Dittmaier [hep-ph/0509141]

  - avoid inverse Gram determinants for pentagon reduction
  - expand around vanishing determinants in critical phase-space regions

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- complex mass scheme for resonances

  Denner, Dittmaier, Roth, Wieders [hep-ph/0505042]

  - use complex W and Z masses everywhere by means of complex renormalization:

    $$M^2_{\mathrm{W},0} = \mu^2_{\mathrm{W}} + \delta\mu^2_W$$

    with: $M^2_{\mathrm{W},0}$ = bare mass
    μ^2_{W} = ren. complex mass
    $\delta\mu^2_W$ = complex counterterm

  - $\Rightarrow$ complex $s^2_{\mathrm{W}} = 1 - \mu^2_{\mathrm{W}}/\mu^2_{\mathrm{Z}}$
  - loop-integrals for complex masses needed

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- dipole subtraction for infrared divergencies

  Catani, Seymour [hep-ph/9605323]

  Dittmaier [hep-ph/9904440]

  Dittmaier,Kabelschacht,Kasprzik [arXiv:0802.1405]

  - subtraction formalism also for non-collinear safe observables
    e.g. for bare muons (without lepton–photon recombination)
    $\Rightarrow$ muon-mass logarithms extracted analytically
  - slicing used as a check

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- multi-channel phase space integration

  Berends, Kleiss, Pittau [hep-ph/9904440]

  - adaptive realization using Vegas (e.g. like in Whizard, Sherpa)

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EW+QCD corrections

- consistent photon–jet recombination
  - treat photon like a parton to define jets
  - distinguish W+jet and W+photon by cut on $E_\gamma / E_{\text{jet}}$

    $\Rightarrow$ not collinear save
  - use photon fragmentation function to control non-perturbative collinear physics

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    $\Rightarrow$ not collinear save
  - use photon fragmentation function to control non-perturbative collinear physics
- also full NLO QCD corrections
  - variable (phase-space dependent) scale supported
  - for photon-induced processes

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Monte-Carlo programs

two completely independent calculations

- in mutual agreement
- MPI: FeynArts 1.0 [Böhm, Denner, Küblbeck]
  in-house Mathematica Routines
  loop integral library: DD [Dittmaier]
  Vegas integration
- PSI: FeynArts 3.2,FormCalc 3.1 [Hahn]
  loop integral library: Coli [Denner]
  Pole [Meier,AM]
  - using Weyl-van der Waerden formalism
    Dittmaier [hep-ph/9805445]
  - automatic generation of subtraction/slicing terms
  - automatic multi-channeling using Lusifer
    Dittmaier,Roth [hep-ph/0206070]

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Setup

basic cuts

- $p_{\mathrm{T},l/\mathrm{miss/jet}} > 25$ GeV
- $|y_{l/\mathrm{jet}}| < 2.5$
- lepton isolation: $R_{l,\mathrm{jet}} > 0.5$

recombination

- do not recombine photons and muons (bare μ^+)
- photons and electrons: $R_{\gamma,l} < 0.1$ (γ rec.)
- photons and partons: $R_{\gamma,\mathrm{jet}} < 0.5$

renormalization and factorization scale

- fixed scala ($\mu = M_{\mathrm{W}}$)
- variable scale: $\mu = \sqrt{M_{\mathrm{W}}^2 + p_{\mathrm{T}}^{\mathrm{had}}}$

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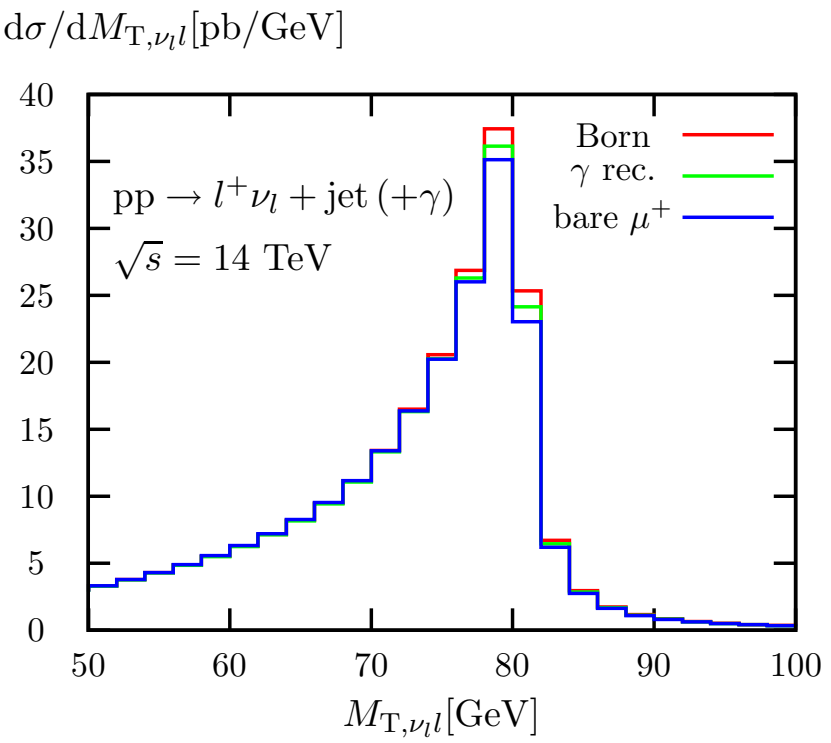
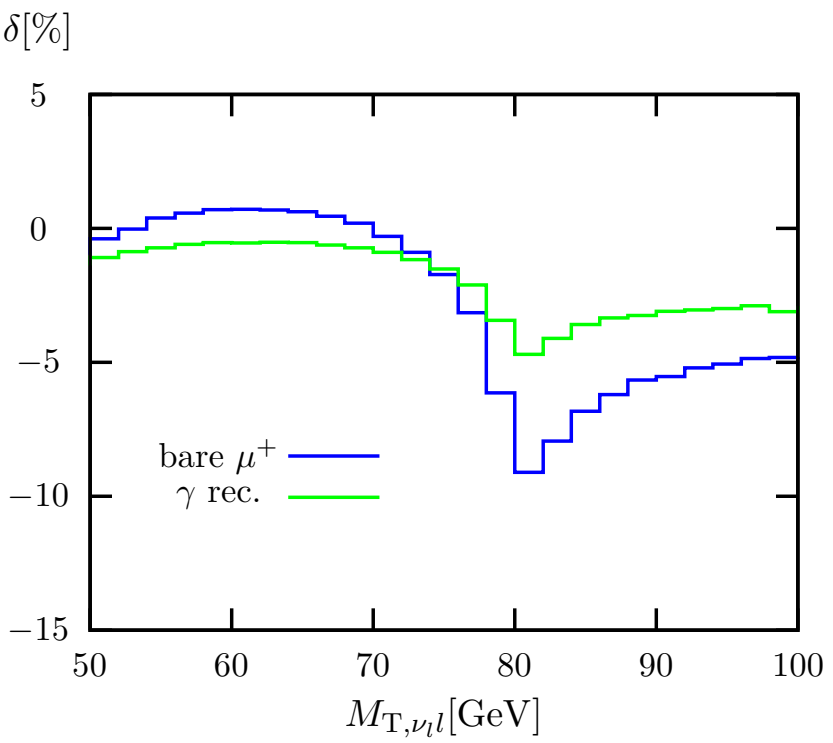
EW corrections

M_{T} distribution for the LHC:

$$M_{\mathrm{T},l\nu_l} = \sqrt{2\, p_{\mathrm{T},l}\, p_{\mathrm{T}}^{\mathrm{miss}}\, (1 - \cos\phi_{\nu_l l})}$$





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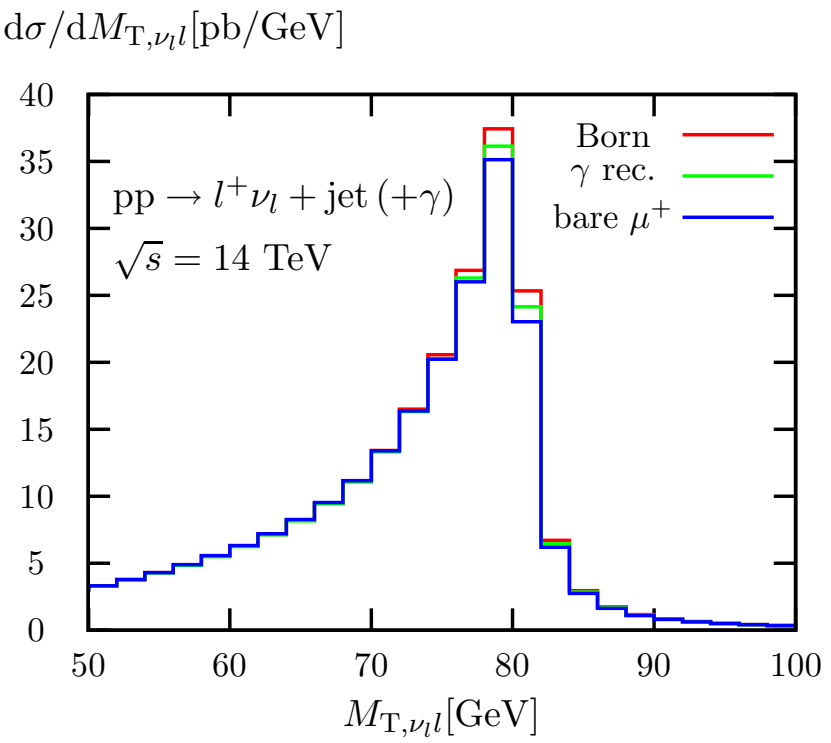

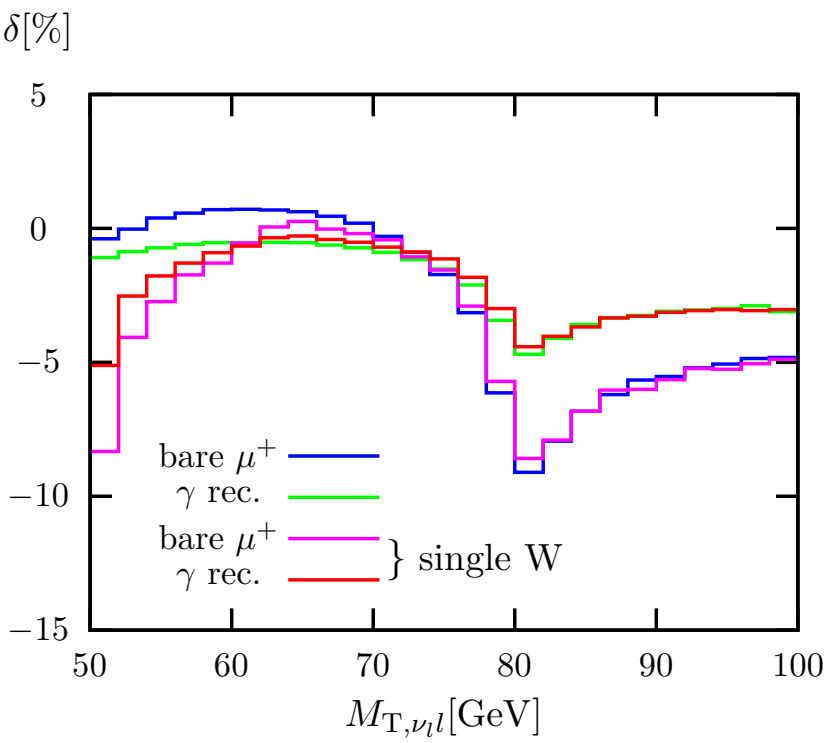

corrections very similar to single W production

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EW corrections

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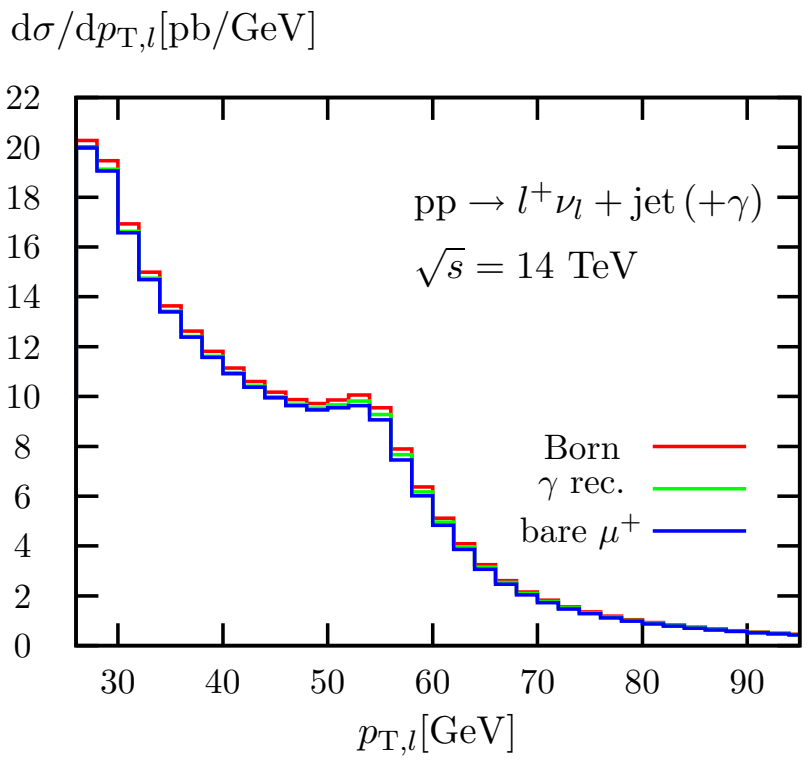

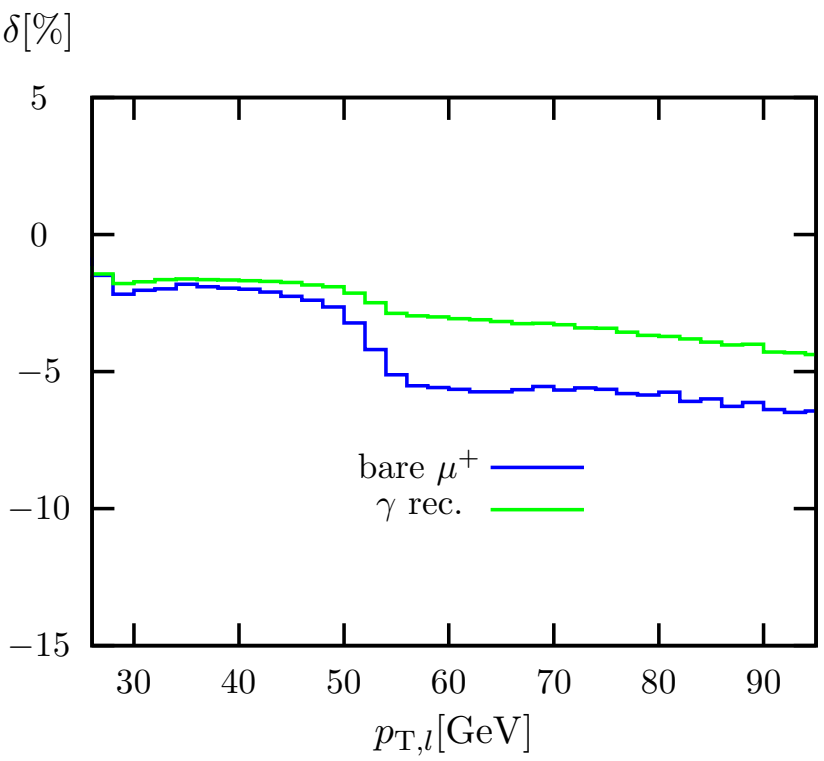

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EW corrections

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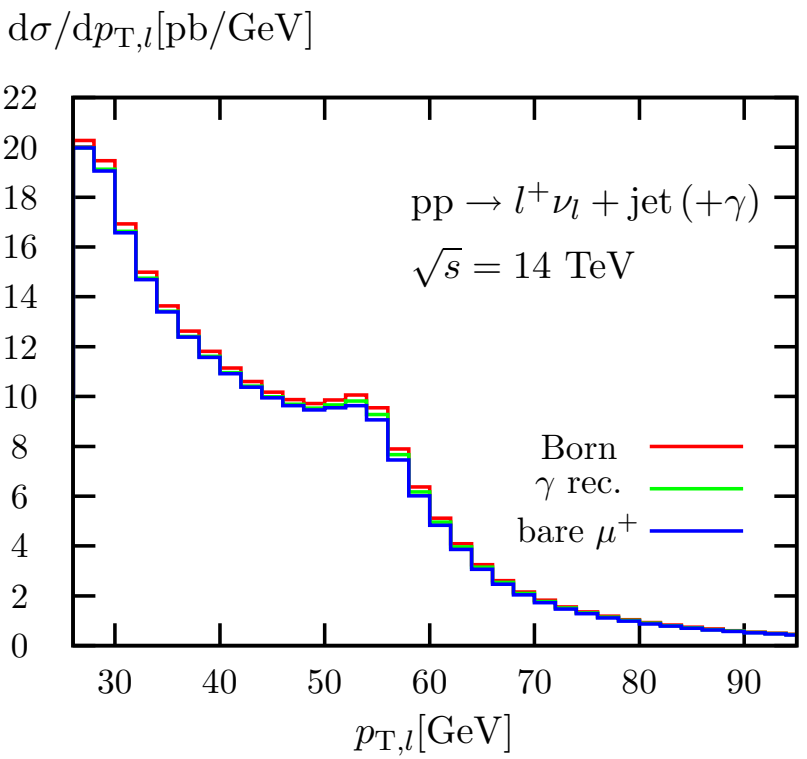
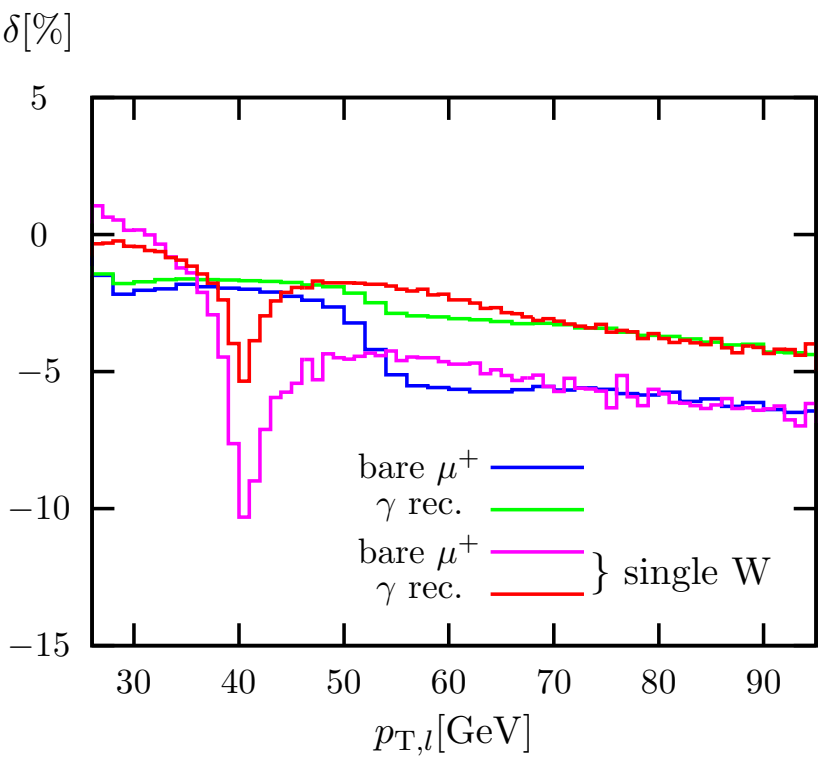
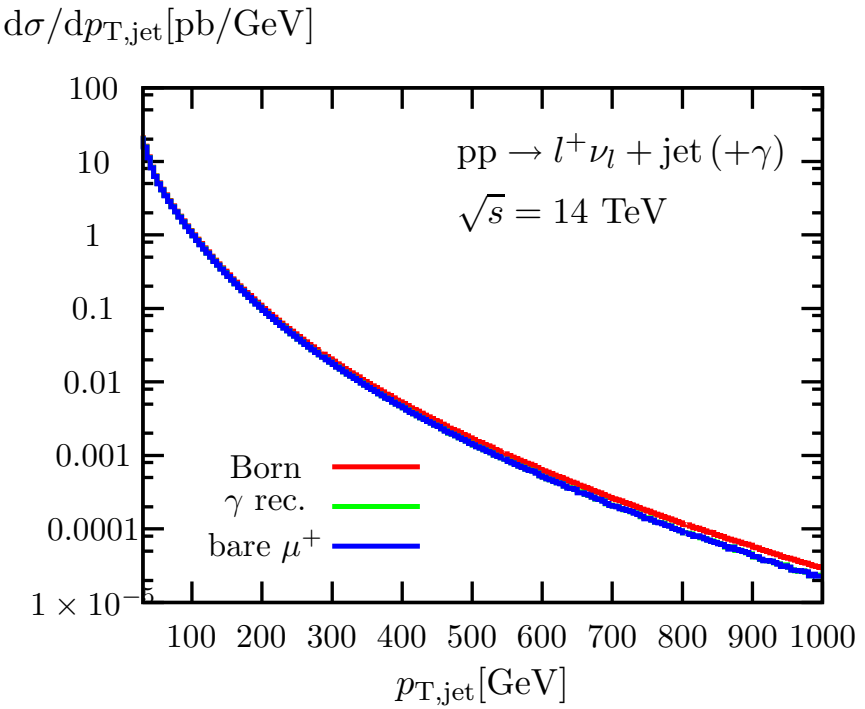


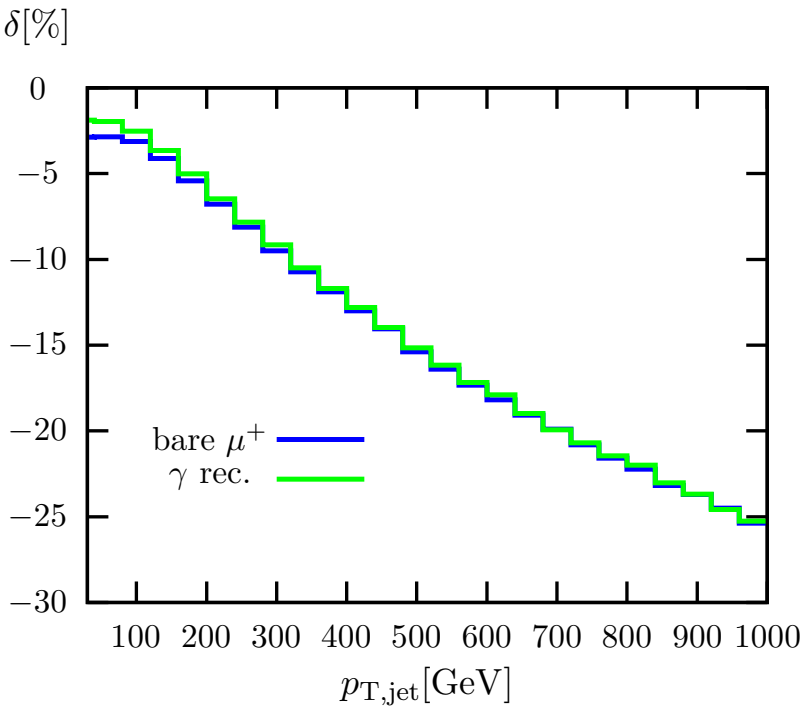


no similarity to single W production

EW corrections

$p_{\mathrm{T,jet}}$ distribution for the LHC:





large corrections at large energies (Sudakov logs)
(on-shell W good approximation)

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QCD corrections

$p_{\mathrm{T,jet}}$ distribution for the LHC:

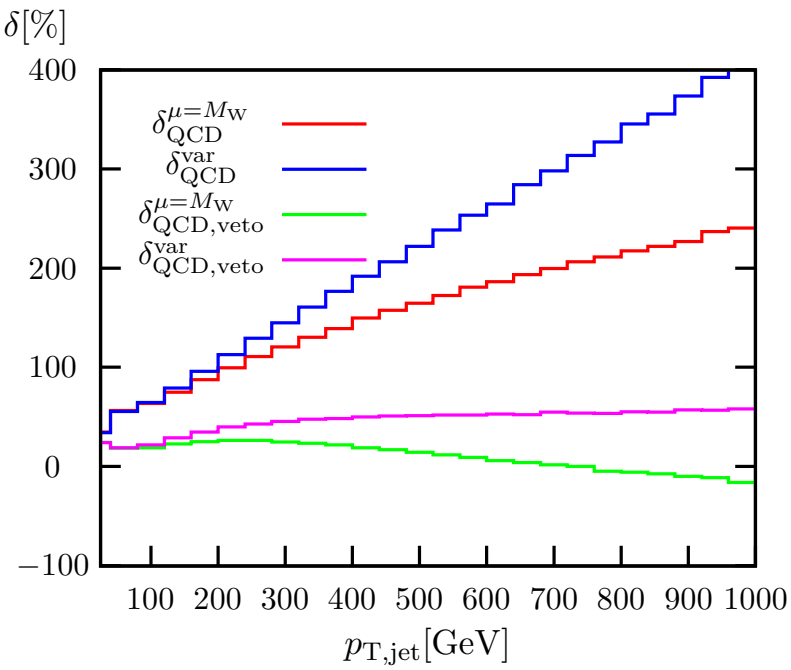
huge NLO QCD corrections:

new kinematical configuration:

back-to-back jets balance p_{T}

$\Rightarrow$ 2 jet events with W emission

$\Rightarrow$ no genuine QCD correction for W+jet



use simple jet veto:

veto second jet with $p_{\mathrm{T}} > \frac{1}{2}\, p_{\mathrm{T}}^{\mathrm{lead.}}$

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Summary

- W+jet is a SM benchmark process
- our calculation
  - flexible Monte Carlo program
  - recalculation of NLO QCD corrections
  - complete EW corrections
  - physical final state (all off-shell effects included)
- EW corrections
  - typically at the percent level
  - distributions affected by final state radiation
  - growing with energy ($\sim -25\%$ at $p_{\mathrm{T,jet}} = 1$ TeV)
- outlook
  - investigate $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ for inclusive W production
  - Z+jet production

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Back-up slides

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non collinear-safe subtract.

- usual subtraction procedure:

$$\int \mathrm{d}\Phi_{n+1}\,|\mathcal{M}|^2 = \int \mathrm{d}\Phi_{n+1}\left(|\mathcal{M}|^2 - |\mathcal{M}_{\mathrm{Sub}}|^2\right) + \int \mathrm{d}\Phi_n \int \mathrm{d}k_\gamma\,|\mathcal{M}_{\mathrm{Sub}}|^2$$

  - clever choice of $|\mathcal{M}_{\mathrm{Sub}}|^2$

    $\Rightarrow \left(|\mathcal{M}|^2 - |\mathcal{M}_{\mathrm{Sub}}|^2\right)$ is integrable

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- non-collinear safe implementation:
  - $\mathcal{M}_{\mathrm{Sub}} = \sum_f \mathcal{M}_{\mathrm{Sub}}(z_f)$ where $z_f \to p_f^0/(p_f^0 + p_\gamma^0)$ for coll. events
  - no recombination: cut on p_f (not $p_f + p_\gamma$) and z_f
  - integrate over z_f in $\mathrm{d}k_\gamma$ numerically

(soft divergence treated via Plus-distribution)

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photon–jet recombination

Treat photon like another parton?

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The problem: W+jet $\Leftrightarrow$ W+photon

- do not distinguish (also calculate W+γ and its NLO QCD corr.)
- or cut on photon energy fraction z_γ inside a jet

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- or cut on photon energy fraction z_γ inside a jet

  $\Rightarrow$ this is what we want

  But: not infrared safe

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photon–jet recombination

What does z_γ cut imply?

- sensitivity to $q \to q\gamma$ splitting
- non-perturbative corrections to be included

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photon–jet recombination

What does z_γ cut imply?

- sensitivity to $q \to q\gamma$ splitting
- non-perturbative corrections to be included
- introduce quark-to-photon fragmentation function $D_{q\to\gamma}(z_\gamma, \mu_F)$
  - measured in hadronic Z decays at LEP ($Z \to q\bar{q} \to q\bar{q}\gamma$)
  - using ALEPH fit:

$$D_{q\to\gamma}(z_\gamma,\mu_F) = \frac{\alpha Q_q^2}{2\pi} P_{q\to\gamma}(z_\gamma)\left(\ln\frac{m_q^2}{\mu_F^2} + 2\ln z_\gamma + 1\right) + D_{q\to\gamma}^{\mathrm{ALEPH}}(z_\gamma,\mu_F),$$

where

$$D_{q\to\gamma}^{\mathrm{ALEPH}}(z_\gamma,\mu_F) = \frac{\alpha Q_q^2}{2\pi}\left(P_{q\to\gamma}(z_\gamma)\ln\frac{\mu_F^2}{(1-z_\gamma)^2\mu_0^2} + C\right)$$

$$P_{q\to\gamma}(z_\gamma) = \frac{1+(1-z_\gamma)^2}{z_\gamma}$$

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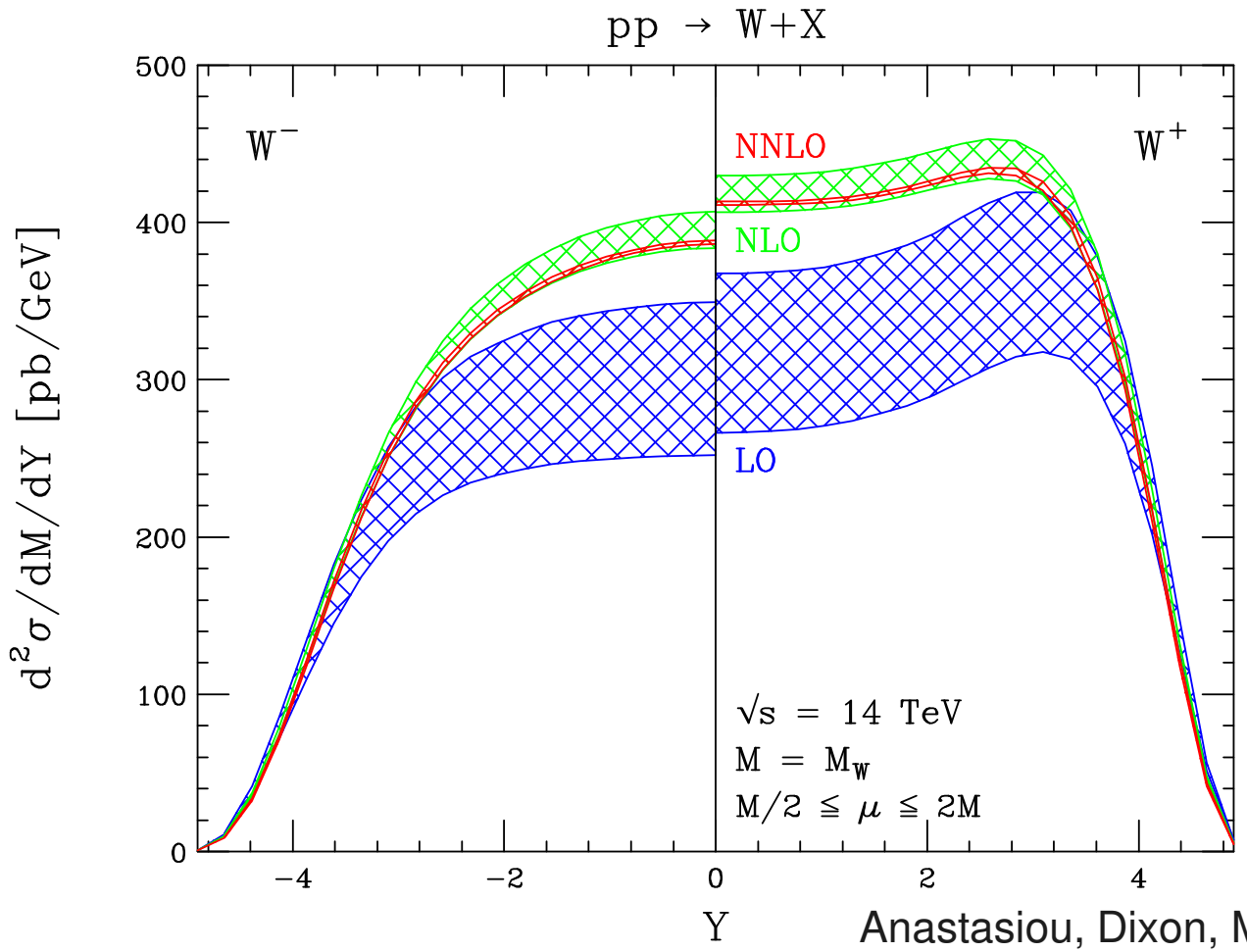
QCD Predictions: single W

NNLO QCD:

- total cross section — v.Neerven, Zijlstra [NPB **382** (1992) 11]; Harlander, Kilgore [hep-ph/0201206]
- rapidity distributions — Anastasiou et al. [hep-ph/0312266]
- fully differential cross sections — Melnikov, Petriello [hep-ph/0609070]; Catani et al. [arXiv:0903.2120]

QCD Predictions: single W

Rapidity distribution: 1% uncertainty at NNLO



Anastasiou, Dixon, Melnikov, Petriello [hep-ph/0312266]

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further QCD improvements:

- NNNLO in soft + virtual approximation Moch, Vogt [hep-ph/0508265]
- soft gluon resummation for $p_{\mathrm{T},W}$ distribution

  Balasz, Yuan [hep-ph/9704258]; Ellis, Veseli [hep-ph/9706526]; Cao,Yuan [hep-ph/0401026]
- NLO plus parton shower (MC@NLO)

  Frixione, Nason, Webber [hep-ph/0305252]

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EW corrections: single W

EW corrections distort shapes:

- in particular due to final state photon radiation
- also for M_{T} distribution
- strong dependence on lepton-photon recombination

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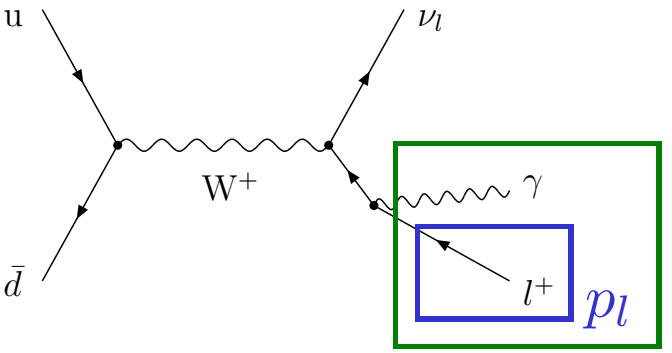


$p_l = p_l + p_\gamma$
(for collinear photons)

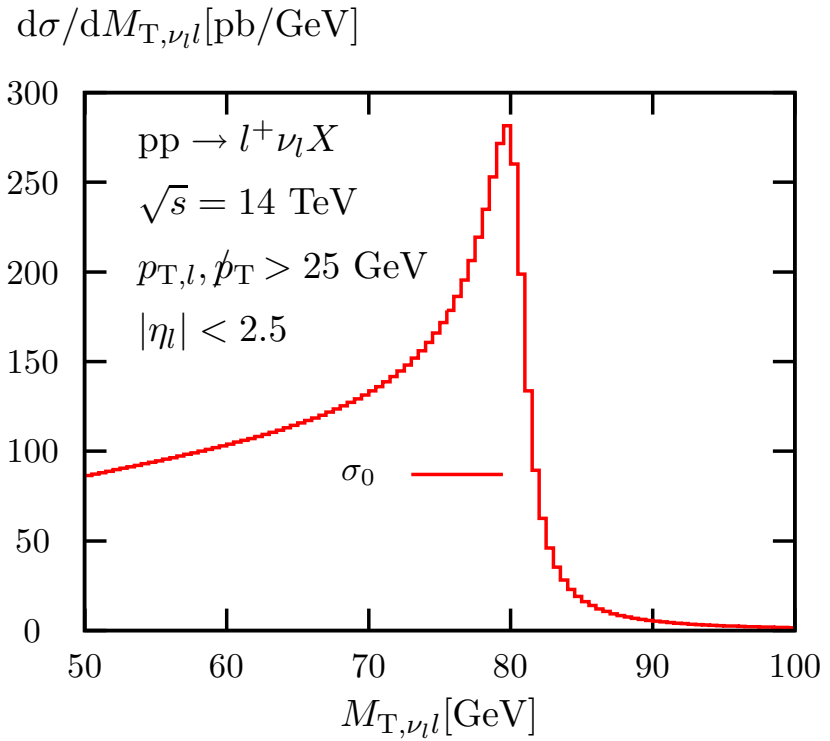
exclusive (bare) leptons (muons): $\alpha \log(M_W^2/M_l^2)$ corrections

inclusive leptons (electrons): no large logs (KLN theorem)

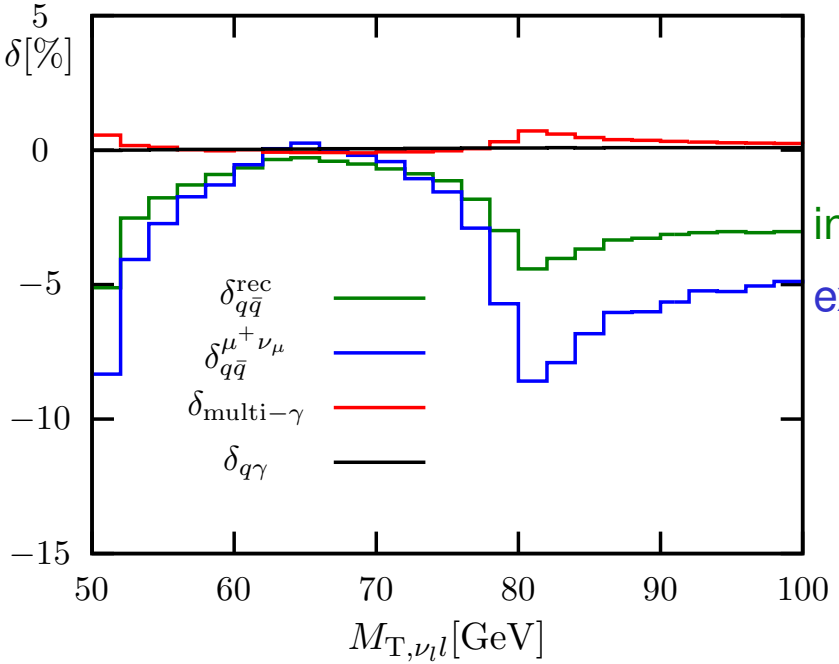
EW corrections: single W

EW corrections distort shapes:

- in particular due to final state photon radiation
- also for M_{T} distribution
- strong dependence on lepton-photon recombination



Brensing, Dittmaier, Krämer, AM [arXiv:0710.3309]



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EW corrections: single W

available EW corrections:

- $\mathcal{O}(\alpha)$ corrections to resonant W production

  Hollik, Wackeroth [hep-ph/9606398]
  Baur, Keller, Wackeroth [hep-ph/9807417]

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  $\Rightarrow \sim$ 170 (65) MeV shift for M_W for $\mu^{\pm}$ ($e^{\pm}$) channel from final state radiation

  $\Rightarrow \sim$ 10 MeV shift for M_W beyond final state radiation

CDF [hep-ex/0007044]

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- complete $\mathcal{O}(\alpha)$ corrections

Dittmaier, Krämer [hep-ph/0109062]
Zykunov [hep-ph/0107059]
Baur, Wackeroth [hep-ph/0405191]
Arbuzov et. al [hep-ph/0506110]
Carloni Calame et. al [hep-ph/0609170]

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- multi-photon final-state radiation

  $\Rightarrow$ additional $\sim$ 10 MeV shift in M_W