Quark masses from low-energy moments of heavy-quark current correlators

Peter Marquard

Institut für Theoretische Teilchenphysik
Universität Karlsruhe

in collaboration with

Y. Kiyo, A. Maier, P. Maierhöfer, A.V. Smirnov
K. Chetyrkin, J.H. Kühn, M. Steinhauser, C. Sturm

Karlsruhe Institute of Technology

Universität Karlsruhe (TH)
Forschungsuniversität • gegründet 1825

SFB TR9

EPS HEP 2009, Krakow

Introduction

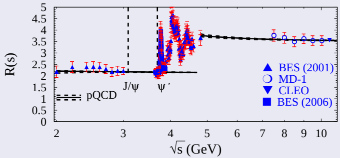
- charm and bottom quark masses are important input parameters of the Standard Model
  - $\Gamma(b \to X_u l \nu) \sim m_b^5 |V_{ub}|^2$
    $\Gamma(b \to X_c l \nu) \sim m_b^5 f(m_c^2/m_b^2)\, |V_{cb}|^2$
  - Higgs decays @ ILC: $\Gamma(H \to b\bar{b}) \sim m_b^2 (1 + \cdots + \mathcal{O}(\alpha_s^5))$
- update of analysis based on R-ratio

$$R(s) = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

  and low-energy moments of the vacuum polarization function [Kühn, Steinhauser, Sturm '07]
- new theory calculations for low-energy moments
- new data for $b\bar{b}$ threshold region (BABAR)

Outline of the method

Experiment



Define experimental moments

$$\mathcal{M}_n^{exp} = \int \frac{R(s)}{s^{n+1}} ds$$

Theory

$$\Pi(q^2) = \frac{1}{12\pi^2} \int \frac{R(s)}{s - q^2} ds$$

Taylor expand $\Pi(q^2)$ around $q^2 = 0$:

$$\Pi(q^2) = \frac{1}{16\pi^2} \sum C_n \left(\frac{q^2}{4m_Q^2} \right)^n$$

$\Rightarrow$ theoretical moments C_n

$$m_Q \propto \left(\frac{\mathcal{M}_n}{C_n} \right)^{\frac{1}{2n}}$$

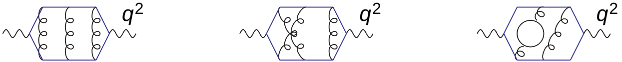
Theory calculation

- calculate Taylor expansion of $\Pi(q^2)$ around $q^2 = 0$ in perturbative QCD up to NNNLO i.e. four loops
- at one and two loops $\Pi(q^2)$ is known analytically [Kallen '55]
- at three loops 30 terms are known in the low-energy expansion [Chetyrkin et al; Boughezal et al; Maier et al]
- at four loops first three terms are known [Chetyrkin et al; Boughezal et al; Maier et al]
- at three and four loops vacuum polarization $\Pi(q^2)$ can be reconstructed using Padé approximations [Chetyrkin et al; Hoang et al; Masjuan et al; Kiyo et al]

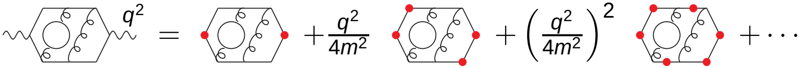
  $\longrightarrow$ more low-energy moments can be obtained

Calculation

700 four-loop Feynman diagrams of the form



expansion around $q^2 = 0$



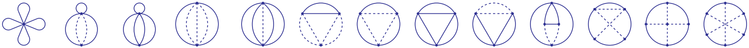
results in $\mathcal{O}(4 \cdot 10^6)$ four-loop vacuum integrals which have to be calculated

Calculation cont'd

- direct calculation of all these integrals not feasible
- but integrals J_k are not independent (IBP) [Chetyrkin, Tkachov '81]

$$J_k = \sum_i R_i^k(d) M_i$$

- M_i are so-called master integrals
- in this case there are only 13 four-loop master integrals, all are known analytically



[Chetyrkin et al; Laporta; Kniehl et al; Schröder et al]

- first three low-energy moments calculated

[Maier, Maierhöfer, PM, Smirnov '09]

Beyond the third moment

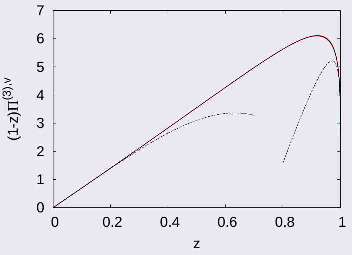
- direct calculation of further moments very time-consuming $\longrightarrow$ use different approach
- collect information from low-energy, threshold and high-energy regions

$$
\begin{array}{lcl}
\Pi(q^2)|_{q^2=0} & : & \text{3 constants} \\
\Pi(q^2)|_{q^2=4m_Q^2} & : & \text{2 constants + logs} \\
\Pi(q^2)|_{q^2=\infty} & : & \text{2 constants + logs}
\end{array}
$$

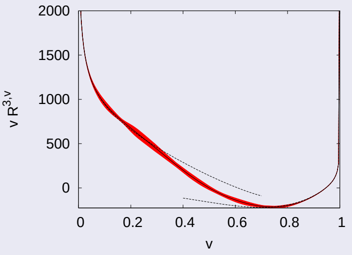
- construct Padé approximations for $\Pi(q^2)$ over the whole energy range $\longrightarrow$ re-expand in small q^2

Results of the Padé Approximation

below threshold: $(1-z)\Pi(z)$



above threshold: $v\,R(v)$



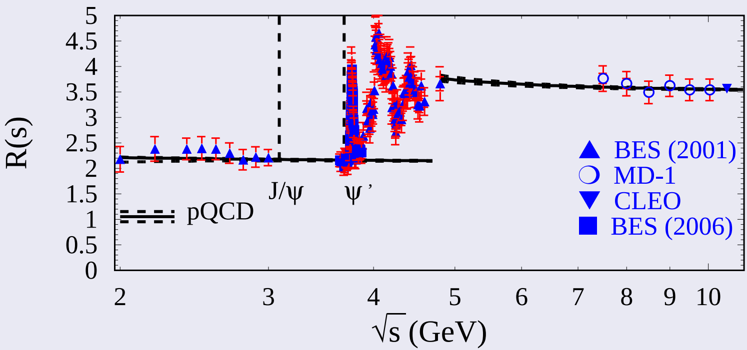
NNNLO low-energy moments

	1	2	3	4	5
$n_l = 3$	-5.6404	-3.4937	-2.8395	-3.349(11)	-3.737(32)
$n_l = 4$	-7.7624	-2.6438	-1.1745	-1.386(10)	-1.754(32)

[Kiyo, Maier, Maierhöfer, PM '09]

Charm data

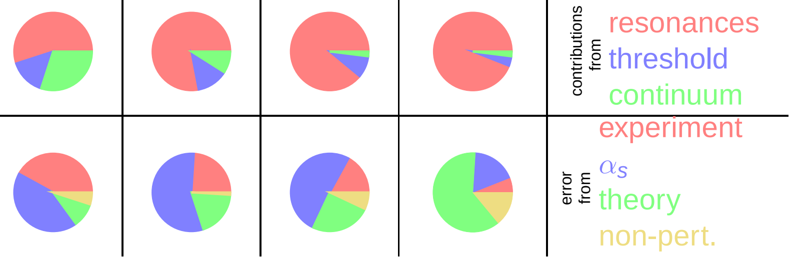
R(s) @ $c\bar{c}$ threshold



Charm quark mass

$m_c(3\,\text{GeV})[\text{MeV}]$ extracted using the lowest four moments

1	2	3	4
986(13)	976(16)	978(17)	1004(33)

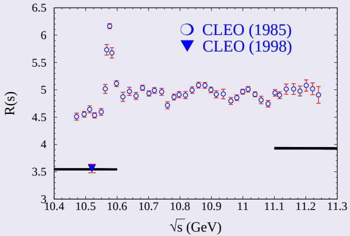


$m_c(3\text{GeV}) = 986(13)\text{MeV}$

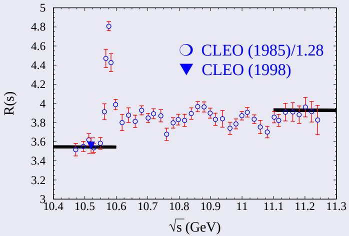
compare with:
$m_c(3\text{GeV}) = 986(10)\text{MeV}$
from LQCD + PQCD [Allison et al]

Bottom data from CLEO

R(s) @ $b\bar{b}$ threshold



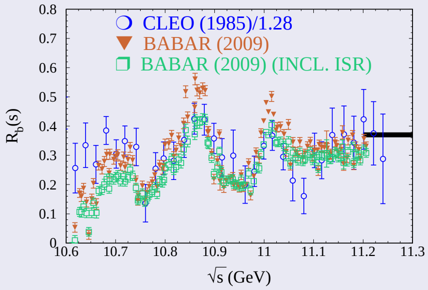
R(s) @ $b\bar{b}$ threshold



- mismatch between old (1985) CLEO data and pQCD
- 1998 CLEO data point in agreement with pQCD predictions
- $\longrightarrow$ 1985 data manually rescaled to fit pQCD predictions
  $\longrightarrow$ 10% error

Bottom data from BABAR

R(s) @ $b\bar{b}$ threshold

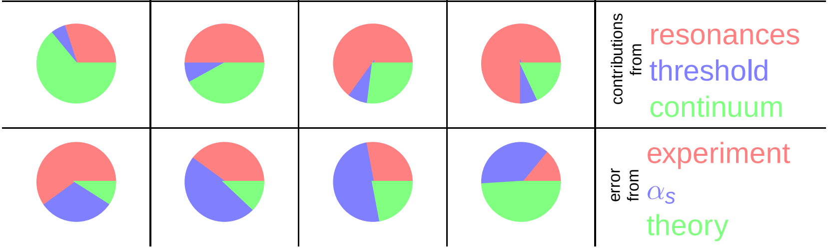


- systematic error ≈ 3%
- + ISR deconvolution
- → total error ≈ 4%

Bottom quark mass

$m_b(10\,\text{GeV})[\text{MeV}]$ extracted using the lowest four moments

1	2	3	4
3597(16)	3610(16)	3619(18)	3631(26)



old:
$m_b(10\text{GeV}) = 3609(25)\text{MeV}$
$m_b(m_b) = 4164(25)\text{MeV}$

new:
$m_b(10\text{GeV}) = 3610(16)\text{MeV}$
$m_b(m_b) = 4163(16)\text{MeV}$

Conclusion

- calculated first three low-energy moments of $\Pi(q^2)$ at NNNLO [Maier, Maierköfer, PM, Smirnov '09]
- reconstructed $\Pi(q^2)$ over the whole energy range using Padé approximations [Kiyo, Maier, Maierköfer, PM '09]
- update of the charm and bottom quark mass analysis including new BABAR data

[Chetyrkin, Kühn, Maier, Maierhöfer, PM, Steinhauser, Sturm '09]

$$m_c(3\text{GeV}) = 986(13)\text{MeV}$$

$$\begin{aligned} m_b(10\text{GeV}) &= 3611(16)\text{MeV} \\ m_b(m_b) &= 4163(16)\text{MeV} \end{aligned}$$