Traveling wave solution of the Reggeon Field Theory [a]

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- Reggeon Field Theory (1967-1975)

  The "supercritical Pomeron"

- From RFT to non-linear Langevin

  Fisher-Kolmogorov-Piscounov-Petrovsky Eq. in 2d

- Traveling Waves: Method and Solutions

  Deterministic *vs*. Stochastic

- The Phase Diagram of the RFT

  The Pomeron as a "Radial Traveling Wave"

- Conclusions and Outlook

  "Nonlinear Universality"

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[a] arXiv:0903.3373 [hep-ph] and PRD (2009)

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REGGEON FIELD THEORY

- Action (non-hermitian $g \to ig$) Gribov, 1967, Abarbanel, 1975

$$S[\bar\varphi, \varphi] = \frac{1}{\alpha'} \int d^2b\, dY \ \left\{ \bar\varphi \left[\partial_Y - \alpha' \nabla^2 \right] \varphi - \mu \bar\varphi \varphi + g \left(\bar\varphi \varphi^2 - \bar\varphi^2 \varphi \right) \right\}$$

- Observables:

$$\langle \mathcal{O} \rangle \propto \int \mathcal{D}\bar\varphi \mathcal{D}\varphi \ \ \mathcal{O}(\bar\varphi, \varphi) \exp\left(\frac{i}{\alpha'} S[\bar\varphi, \varphi] \right)$$

- "Supercritical Pomeron"

$$\mathcal{P}(Y, \vec{b}) = \exp \mu Y - \frac{b^2}{4\alpha' Y} \ ; \ \mu > 0$$

- Problems: (from 1970's)

"Critical" Pomeron:
$\mu \sim 4 - d \Rightarrow \sigma_{tot} \sim Y^{(4-d)/12 \sim .17 \to .32 \text{ at order } (4-d)^2}$
Not phenomenologically acceptable

"Supercritical" Pomeron:
Phenomenologically preferred
Field Theory Not Under Control

From RFT to sFKPP

- RFT $\longrightarrow$ Langevin

$$\int \mathcal{D}\bar{\varphi}\mathcal{D}\varphi \;\exp\left(\frac{i}{\alpha'}S[\bar{\varphi},\varphi]\right) \Rightarrow \int \mathcal{D}\bar{\varphi}\;\exp\left(\frac{i}{\alpha'}\bar{\varphi}\times F(\varphi)\right) \equiv \;\delta\left[F(\varphi)\right]$$

- Stratonovitch transformation (i for each $\vec{b}_i$ cell)

$$\exp\left(\bar{\varphi}_i^2(\alpha\varphi_i - \beta\varphi_i^2)dY\right) \sim \int \;d\eta_i\;\exp\left(-\frac{1}{2}\eta_i^2 - \bar{\varphi}_i\sqrt{2(\alpha\varphi_i - \beta\varphi_i^2)}\eta_i\sqrt{dY}\right)$$

- Langevin equation ($\int d\eta_i =$ Wiener Integral)

$$\partial_Y\varphi = \alpha'\nabla^2\varphi + \mu\varphi - g\varphi^2 + \sqrt{2(\alpha'\varphi - \beta\varphi^2)}\;\nu\left(Y,\vec{b}\right)$$

$$\left\langle \nu\left(Y,\vec{b}\right),\nu\left(Y',\vec{b}'\right)\right\rangle = \delta(Y-Y')\delta^2(\vec{b}-\vec{b}')$$

- RFT $\sim$ Statistical Physics of non-equilibrium processes
  Diffusion, Creation, Merging and Splitting

$RFT \Leftrightarrow sFKPP$

- After Rescaling: RFT $\kappa = 1$; Generalized RFT $\kappa \neq 1$;

$$\partial_Y T(Y,\vec{b}) = \alpha' \nabla_b^2 T + \mu\left(T - T^2\right) + \sqrt{2\alpha'\kappa\,(T - T^2)}\;\nu\left(Y,\vec{b}\right)$$

$\kappa :=$ {Splitting / Merging}

- Dictionnary $RFT \Leftrightarrow 2d - sFKPP$

$$Time: \; t = \mu(Y - Y_0)\;.$$

$$Space: \; \vec{r} = \frac{\mu}{\alpha'}\vec{b}$$

$$Noise: \; \epsilon = \sqrt{2\mu\kappa}$$

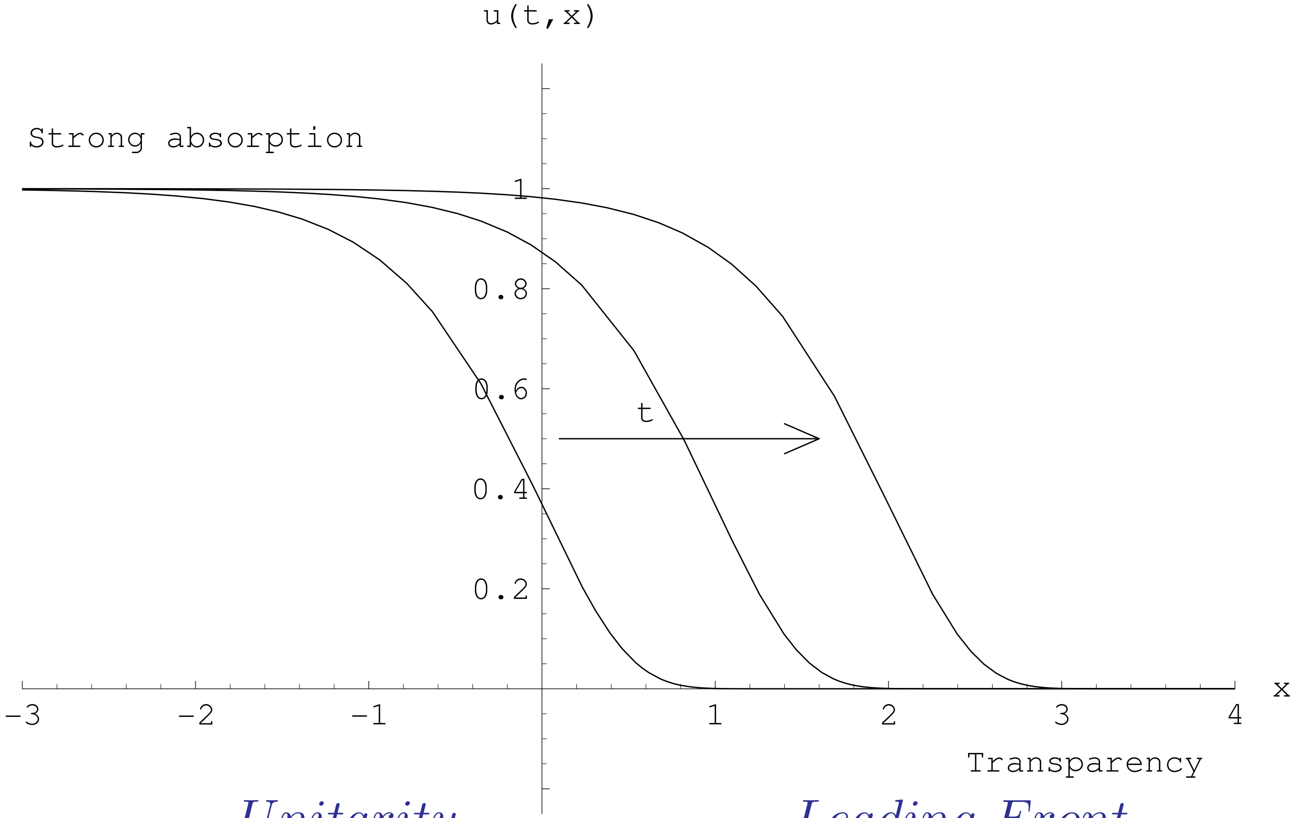
- Further Rescaling: RFT $\equiv$ 1-parameter, 2-dimensional sFKPP

$$\partial_t U(t,\vec{r}) = \nabla_r^2 U + U - U^2 + \epsilon\sqrt{U\,(1 - U)}\;\nu\,(t,\vec{r})$$

$$\epsilon = \sqrt{2\mu\kappa}$$

1d-FKPP: Traveling wave solutions

Kolmogorov,Bramson,Van Sarloos...



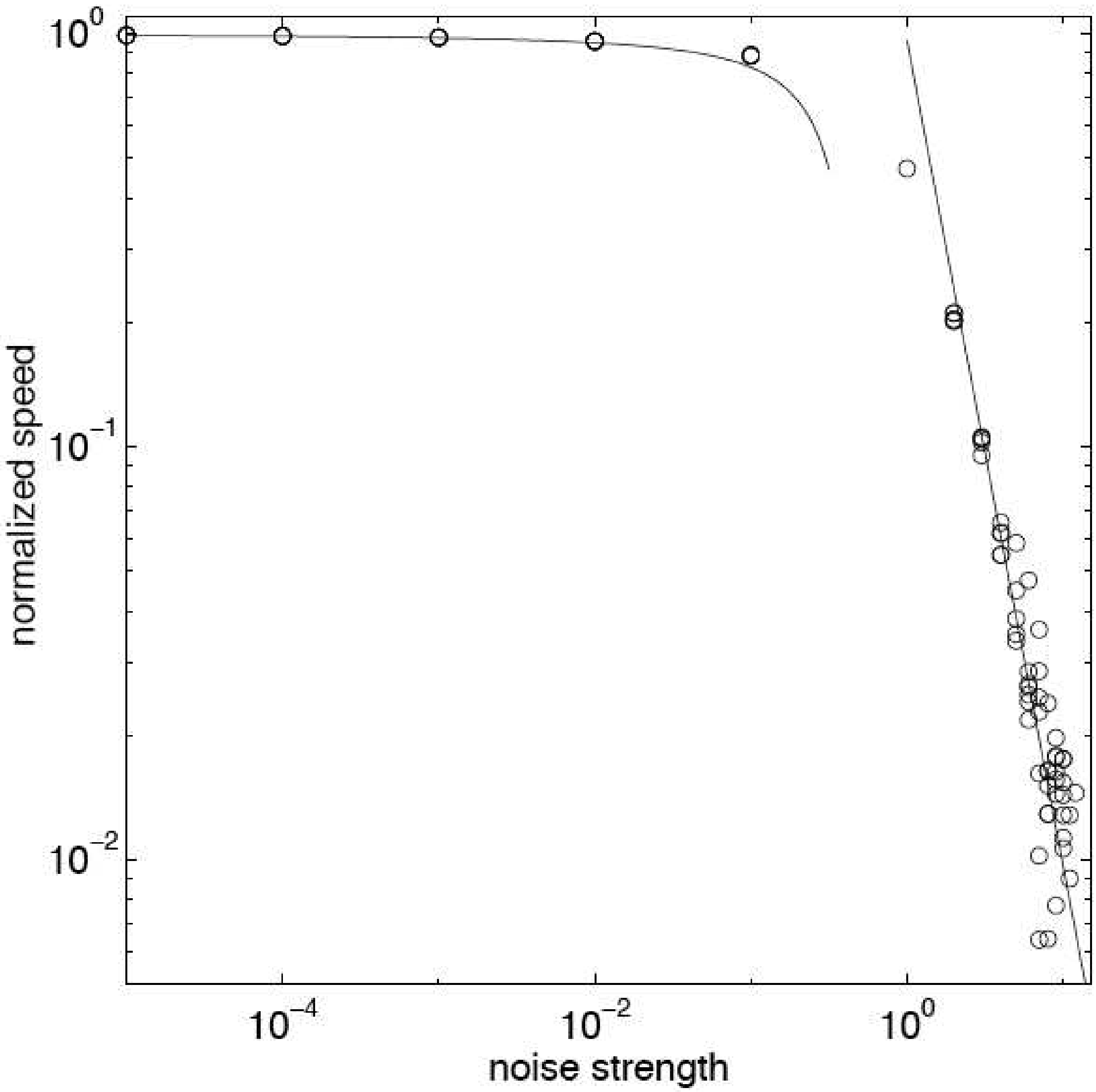
Unitarity *Leading Front*

$$v = \frac{dx}{dt} \equiv v_0 = 2$$

$$\boxed{\partial_t U(t,x) = \partial_{xx} U + U - U^2}$$

1d *Stochastic* FKPP:

$\frac{v}{v_0}$



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Weak Noise Strong Noise

$$\partial_t U(t,x) = \partial_{xx} U + U - U^2 + \epsilon \sqrt{U\,(1-U)}\,\nu\,(t,x)$$

RFT $\equiv$ 2d RADIAL sFKPP:

- RADIAL sFKPP

$$\partial_t U(t,r) = \partial_{rr} U + \frac{1}{r}\partial_r U + U - U^2 + \epsilon\sqrt{\frac{U\,(1-U)}{2\pi r}}\;\nu\,(t,\vec{r})$$

- An energy-decreasing noise:

$$\zeta^2 = \frac{\epsilon^2}{2\pi r} \sim \;\sqrt{\alpha' \mu}\;\frac{\kappa}{2\pi b_s(Y)}$$

The Pomeron as a Radial Traveling Wave

- The zero noise regime: $\kappa \approx 0$, splitting $\ll$ merging

$$\begin{aligned} \sigma_{tot} \sim b_s^2(Y) &= \left\{ b_0 + 2\sqrt{\alpha'\mu} \left[Y - Y_0 - \frac{1}{\mu} \log \frac{Y}{Y_0} + \cdots \right] \right\}^2 \\ T(Y,b) &\sim T(b - b_s(Y)) \end{aligned} \quad (1)$$

- The weak noise regime: $\kappa \approx \mathcal{O}(1)$, splitting $\sim$ merging

$$\begin{aligned} \sigma_{tot} \sim b_s^2(Y) &= \left\{ b_0 + 2\sqrt{\alpha'\mu}\,(Y - Y_0) \left[1 - \frac{\pi^2}{2\log^2\left(2\zeta^{-2}\right)} + \cdots \right] \right\}^2 \\ T(Y,b) &\sim T(b - b_s(Y)) \to T\left(\frac{b - b_s(Y)}{D\sqrt{\alpha'(Y - Y_0)}} \right) \end{aligned} \quad (2)$$

- The strong noise regime: $\kappa \approx \mathcal{O}(100)$, splitting $\gg$ merging

$$\begin{aligned} \sigma_{tot} \sim b_s^2(Y) &= b_s^2(Y_0)\ \exp\left[\frac{8\pi}{\kappa}(Y - Y_0) \right] \\ T(Y,b) &\sim \operatorname{erfc}\left(\frac{b - b_s(Y)}{D\sqrt{\alpha'(Y - Y_0)}} \right) \end{aligned} \quad (3)$$

Conclusions

- RFT + Supercritical Pomeron
  Equivalence with 2d sFKPP Langevin
- Radial Traveling waves
  Universality for supercritical Pomerons!
- Phase Diagram
  Order parameter: $\frac{\text{Splitting}}{\text{Merging}} \propto$ Pomeron loop strength

Outlook

- A Field Theory without Diagrams
  Connection with QCD?
- Full 2-dimensional Theory
  Non azimuthal noise?
- The last word
  Phenomenology?

Note the energy dependence from strong to weak noise:
From "DL Pomeron" $\kappa = 8\pi/.08$, to $\kappa = \mathcal{O}(1)$: Froissart Bound

EXTRA SLIDES

The sFKPP Phase Diagram

- The radial SFKPP

$$\boxed{\partial_t U(t,r) = \partial_r r U + \frac{1}{r}\partial_r U + U - U^2 + \epsilon\sqrt{\frac{U(1-U)}{2\pi r}}\;\nu(t,\vec{r})}$$

- Radial noise strength

$$\boxed{\zeta^2 \sim \frac{\epsilon^2}{2\pi r_s} = \frac{\kappa}{2\pi b_s}\sqrt{\alpha' \mu}}$$

- RFT "Phase Diagram"

$$\boxed{\frac{v_\kappa}{v_0} \equiv \frac{1}{v_0}\,\frac{dr}{dt} = \left\{\frac{\kappa}{4\pi}\;\frac{d\log\{2\pi b_s^2\}}{dY}\right\}\;\frac{1}{\zeta^2}}$$

$$\mathcal{A} = \pi b_s^2 \;\mathrm{Impact-Parameter\ Disk} \tag{4}$$

$$\delta \equiv \frac{d\log\mathcal{A}}{dY} \lesssim \frac{d\log\sigma_{tot}}{dY} \tag{5}$$

The sFKPP Phase Diagram

$$\delta \equiv \frac{d \log \mathcal{A}}{dY}$$

- The zero noise regime:

$$\zeta \lll 1 \quad \frac{v_\kappa}{v_c} \sim 1 \qquad \frac{\kappa}{4\pi} \sim \ 0 \quad 2\sqrt{\mu\alpha'} \sim \ \frac{1}{2}\ \delta b_s = \ \frac{db_s}{dY}\ .$$

- The weak noise regime:

$$\zeta \leq 10^{-1} \quad .9 \leq \frac{v_\kappa}{v_c} \leq 1. \quad \frac{\kappa}{4\pi} \leq \frac{10^{-2}}{\delta} \quad 2\sqrt{\mu\alpha'} \leq \ \frac{1}{2}\ \delta b_s = \ \frac{db_s}{dY}\ .$$

- The middle noise regime:

$$.1 \leq \zeta \leq 1.5 \quad .2 \leq \ \frac{v_\kappa}{v_c} \ \leq .9 \qquad \frac{10^{-2}}{\delta} \leq \frac{\kappa}{4\pi} \leq \frac{1}{\delta} \qquad \frac{db_s}{dY} \leq 2\sqrt{\mu\alpha'} \leq \ 2\frac{db_s}{dY}\ .$$

- The strong noise regime:

$$\zeta \gtrsim 1.4 \quad \frac{v_\kappa}{v_c} \ \lesssim .5 \quad \frac{\kappa}{4\pi} = \frac{1}{\delta} \quad 2\sqrt{\mu\alpha'} \sim \frac{\zeta^2}{2}\ \delta b_s = \frac{\zeta^2}{2}\ 2\frac{db_s}{dY} \gtrsim \frac{db_s}{dY}\ ,$$