Heterotic String Phenomenology.

James Gray - University of Oxford

Collaborators: Lara Anderson, Yang-Hui He, Andre Lukas and Burt Ovrut.
Introduction to Heterotic M-theory

Horava Witten - the strongly coupled limit of heterotic string

- Bulk is eleven dimensional supergravity.
- Boundaries support ten dimensional E8 SYM.
- M5 world volume actions for central branes.
• One dimension out of eleven is already compact. So that gets us down to ten dimensions.

• In order to get an $\mathcal{N} = 1$ supersymmetric 4D theory we compactify the remaining six on a manifold of SU(3) structure.

• Often we specialize to cases with a perturbative, Minkowski supersymmetric ground state (i.e a nice supersymmetric vacuum). This leads us to a Calabi-Yau threefold.

Bianchi Identity in ten dimensions:

$$dH = -\frac{3\alpha'}{\sqrt{2}} \left( \text{tr} F^{(1)} \wedge F^{(1)} + \text{tr} F^{(2)} \wedge F^{(2)} - \text{tr} R \wedge R \right)$$

integrate both sides over a non-trivial 4 cycle in the CY...
\[ \int_{C_4} \text{tr} F^{(1)} \wedge F^{(1)} + \int_{C_4} \text{tr} F^{(2)} \wedge F^{(2)} = \int_{C_4} \text{tr} R \wedge R \]

- The right hand side of this expression is non zero for a Calabi-Yau manifold.
- We see that we must have non-trivial gauge field vevs in the internal dimensions in our vacuum.
- We want to pick these gauge field vevs such that they too preserve $\mathcal{N} = 1$ supersymmetry in the 4D EFT.

The Calabi-Yau metric and gauge fields are not known explicitly.

Use of algebraic geometry to describe the compactification.
Matter in 4D theory:

- If gauge field vevs on standard model orbifold fixed plane are valued in a group $G$ then 4D visible sector gauge group is commutant of $G$ in $E_8$.

  Eg, \[ G = \text{SU}(5) \rightarrow \text{SU}(5) \text{ GUT}. \]

- Dimensional reduction of SYM fields gives 4D matter.

- No adjoint Higgs’ are present to break the GUT.

- Therefore Wilson lines are used instead.

**The goal:** Find a Calabi-Yau and bundle (gauge field vevs) such that we get the standard model gauge group, matter content, Yukawa couplings, etc...
An example of model building: The Monad Program

- Approach: algorithmically scan large classes of Calabi-Yau and gauge bundles at a time, in their entirety.
  - Comprehensive - if you don’t find the model you want then it doesn’t exist in that class
  - Can search for whatever you want not just the MSSM
  - The class we start with are the so-called monad bundles over the favourable complete intersection Calabi-Yau threefolds.
• A few results for the positive monads - a well defined sub-class
  - This class of configurations is infinite
  - Imposing that we obtain an N=1 supersymmetric 4D theory restricts us to 7118 examples over 63 Calabi-Yau (started with ~4500 Calabi-Yau).
  - Possibility of a three family model restricts to 559 examples.
  - SU(5) GUT with 3 families before Wilson line breaking - one possible model.
  - We can then algorithmically determine the gauge group, complete particle spectrum, Yukawa couplings of the resulting theories.
Moduli in 4D theory:

(in a Calabi-Yau compactification)

- The size of the eleventh direction
- Energy preserving deformations of the Calabi-Yau
- Energy preserving deformations of the bundle (gauge field vevs)
- Positions of any additional branes ... etc...

All of these fields must be given a mass by some mechanism -- moduli stabilization.

This turns out to be hard in heterotic.
A simple illustration of the problem:

\[ K = - \ln(S + \overline{S}) - 3 \ln(T + \overline{T}) - 3 \ln(Z + \overline{Z}) \]

hard to stabilize everything non-perturbatively, so add flux: \( W_{\text{pert}} = W(Z) \)

Potential is given by usual \( \mathcal{N} = 1 \) formula:

\[ V = e^K \left[ K^{I\overline{J}} F_I \overline{F}_J - 3 |W|^2 \right] \text{ where } F_I = \partial_I W + K_I W \]

\[ F_S = - \frac{1}{S + \overline{S}} W \quad F_T = - \frac{3}{T + \overline{T}} W \]

so perturbatively:

\[ K_{S\overline{S}} = \frac{1}{(S + \overline{S})^2} \quad K_{T\overline{T}} = \frac{3}{(T + \overline{T})^2} \]
Plugging this in:

\[ V \rightarrow e^K \left[ K^{\overline{Z}Z} F_Z \overline{F}_{\overline{Z}} + |W|^2 \right] \sim \frac{1}{(S + \overline{S})(T + \overline{T})^3} \]

- Perturbative run away force on dilaton and Kahler moduli.
- Hard to counter with non-perturbative effects unless very small.
- So we need an (approximate) Minkowski vacuum.

For susy we require:

\[ \partial_Z W = 0 \, , \, W = 0 \] at least approximately.
\[ W = ia + bZ + icZ^2 + dZ^3 \]

where \( a, b, c \) and \( d \) are quantized.

- \( \partial_Z W = 0 \) can be solved easily enough.
- The resulting \( Z \) can then be substituted back into \( W \).
- Then attempt to choose \( a \) so as to make \( W \) small.
- But in heterotic \( a \) is real...

...so we can’t make the perturbative runaway forces small.
• This can be made more rigorous and generalized to arbitrary Calabi-Yau in the large complex structure limit.

• Away from that limit I have not found any Susy Minkowski vacua in the examples where K and W are known.

There are lots of people working on ways to get around this:

• Lots of work on using more general manifolds of SU(3) structure than Calabi-Yau

• The (technical) problem with this is that then all of the known nice model building techniques no longer apply.
Conclusions

• Heterotic M-theory is still one of the most likely ways in which string theory can be linked to phenomenology.

• At the model building level the theory is very successful, in particular in incorporating desirable aspects of GUT physics.

• Moduli stabilization is more difficult in heterotic than in the type II superstring theories, certainly without loosing the model building technology that has been developed.