

# Effect of High Mass $t'$ on $\sin 2\Phi_{B_s}$

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# Introduction

Recent measurements  $2\Phi_{B_s}$  (also called  $-2\beta_s$  or  $\phi_s$ )

$$2\Phi_{B_s} \text{ (CDF)} \in - [0.32, 2.82] \quad (68\% \text{ C.L.})$$

$$2\Phi_{B_s} \text{ (DØ)} = - 0.57 \pm_{0.30}^{0.24} \text{ (stat)} \pm_{0.02}^{0.07} \text{ (syst)}$$

$$\text{However, } 2\Phi_{B_s}^{\text{SM}} \sim - 0.04$$

4<sup>th</sup> generation (SM4) introduces another heavy quark  $t'$  to interfere with  $t$  in the  $B_s$ - $B_s$  mixing box diagrams

# Large $\sin 2\Phi_{Bs}$

SM4 makes SM triangle into quadrangle

$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} + V_{t's}^* V_{t'b} = 0$$

Define

$$V_{t's}^* V_{t'b} \equiv r_{sb} \exp(i\phi_{sb})$$

Combine  $\Delta m_{Bs}$  [box diagram] and

$\mathcal{B}(b \rightarrow sl\ell)$  [Z penguin diagram]

to determine  $r_{sb}$  and  $\phi_{sb}$

# Use parametrization matrix [Hou, Soni and Steger PLB 192 (1987)]

$V_{CKM}^4 =$

$c_{12} c_{13} c_{14}$

$$-c_{13} s_{12} s_{14} s_{24} \exp[-i(\phi_{db} - \phi_{sb})]$$

$$-c_{24} s_{13} s_{14} s_{34} \exp[-i(\phi_{db} + \phi_{ub})]$$

$-c_{14} c_{23} s_{12}$

$$-c_{12} c_{14} s_{13} s_{23} \exp[i\phi_{ub}]$$

$$-c_{12} c_{23} s_{14} s_{24} \exp[-i(\phi_{db} - \phi_{sb})]$$

$$+s_{12} s_{13} s_{14} s_{23} s_{24} \exp[-i(\phi_{db} - \phi_{sb} - i\phi_{ub})]$$

$$-c_{13} c_{24} s_{14} s_{23} s_{34} \exp[-i\phi_{db}]$$

$-c_{12} c_{14} c_{23} s_{13} \exp[i\phi_{ub}]$

$$+c_{14} s_{12} s_{23}$$

$$+c_{23} s_{12} s_{13} s_{14} s_{24} \exp[-i(\phi_{db} - \phi_{sb} - i\phi_{ub})]$$

$$+c_{12} s_{14} s_{23} s_{24} \exp[-i(\phi_{db} - \phi_{sb})]$$

$$-c_{13} c_{23} c_{24} s_{14} s_{34} \exp[-i\phi_{db}]$$

$c_{13} c_{24} s_{12}$

$$-s_{13} s_{24} s_{34} \exp[-i(\phi_{sb} + \phi_{ub})]$$

$c_{12} c_{23} c_{24}$

$$-c_{24} s_{12} s_{13} s_{23} \exp[i\phi_{ub}]$$

$$-c_{13} s_{23} s_{24} s_{34} \exp[-i\phi_{sb}]$$

$-c_{23} c_{24} s_{12} s_{13} \exp[i\phi_{ub}]$

$$-c_{12} c_{24} s_{23}$$

$$-c_{13} c_{23} s_{24} s_{34} \exp[i\phi_{sb}]$$

$c_{34} s_{13} \exp[-i\phi_{ub}]$

$c_{13} c_{34} s_{23}$

$c_{13} c_{23} c_{34}$

$-s_{34}$

$c_{12} c_{13} s_{14} \exp[i\phi_{db}]$

$$+c_{13} c_{14} s_{12} s_{24} \exp[i\phi_{sb}]$$

$$+c_{14} c_{24} s_{13} s_{34} \exp[-i\phi_{ub}]$$

$-c_{23} s_{12} s_{14} \exp[i\phi_{db}]$

$$-c_{12} s_{13} s_{14} s_{23} \exp[i(\phi_{db} + \phi_{ub})]$$

$$+c_{12} c_{14} c_{23} s_{24} \exp[i\phi_{sb}]$$

$$-c_{14} s_{12} s_{13} s_{23} s_{24} \exp[i(\phi_{sb} + \phi_{ub})]$$

$$+c_{13} c_{14} c_{24} s_{23} s_{34}$$

$-c_{12} c_{23} s_{13} s_{14} \exp[i(\phi_{db} + \phi_{ub})]$

$$+s_{12} s_{14} s_{23} \exp[i\phi_{db}]$$

$$-c_{14} c_{23} s_{12} s_{13} s_{24} \exp[i(\phi_{sb} + \phi_{ub})]$$

$$-c_{12} c_{14} s_{23} s_{24} \exp[i\phi_{sb}]$$

$$+c_{13} c_{14} c_{23} c_{24} s_{34}$$

$-c_{24} c_{34} s_{14} \exp[-i\phi_{db}]$

$-c_{34} s_{24} \exp[-i\phi_{sb}]$

$c_{14} c_{24} c_{34}$

# Use parametrization matrix [Hou, Soni and Steger PLB 192 (1987)]

$$V_{ud} = -c_{13} s_{14} s_{12} s_{24} e^{i\Phi_{sb} - i\Phi_d} - s_{13} c_{24} s_{14} s_{34} e^{-i\Phi_d - i\phi_3} + c_{14} c_{13} c_{12}$$

$$V_{us} = c_{13} c_{24} s_{12} - s_{13} s_{34} s_{24} e^{-i\Phi_{sb} - i\phi_3}$$

$$V_{ub} = s_{13} c_{34} e^{-i\phi_3}$$

$$V_{ub'} = c_{14} c_{13} s_{12} s_{24} e^{i\Phi_{sb}} + c_{13} c_{12} s_{14} e^{i\Phi_d} + s_{13} c_{14} c_{24} e^{-i\phi_3} s_{34}$$

$$V_{t'd} = -c_{34} c_{24} s_{14} e^{-i\Phi_d}$$

$$V_{t's} = -c_{34} s_{24} e^{-i\Phi_{sb}}$$

$$V_{t'b} = -s_{34}$$

$$V_{t'b'} = c_{14} c_{34} c_{24}$$

$$V_{cd} = s_{14} s_{13} s_{12} s_{24} s_{23} e^{i\Phi_{sb} - i\Phi_d + i\phi_3} - c_{12} c_{23} s_{14} s_{24} e^{i\Phi_{sb} - i\Phi_d} - c_{13} c_{24} s_{14} s_{34} s_{23} e^{-i\Phi_d} - c_{14} c_{12} e^{i\phi_3} s_{13} s_{23} - c_{14} c_{23} s_{12}$$

$$V_{cs} = -c_{13} s_{34} s_{24} s_{23} e^{-i\Phi_{sb}} - c_{24} e^{i\phi_3} s_{13} s_{12} s_{23} + c_{12} c_{24} c_{23}$$

$$V_{cb} = c_{13} c_{34} s_{23}$$

$$V_{cb'} = -c_{14} s_{13} s_{12} s_{24} s_{23} e^{i\Phi_{sb} + i\phi_3} + c_{14} c_{12} c_{23} s_{24} e^{i\Phi_{sb}} - c_{12} s_{14} s_{13} s_{23} e^{i\Phi_d + i\phi_3} - c_{23} s_{14} s_{12} e^{i\Phi_d} + c_{14} c_{13} c_{24} s_{34} s_{23}$$

$$V_{td} = c_{23} s_{14} s_{13} s_{12} s_{24} e^{i\Phi_{sb} - i\Phi_d + i\phi_3} + c_{12} s_{14} s_{24} s_{23} e^{i\Phi_{sb} - i\Phi_d} - c_{13} c_{24} c_{23} s_{14} s_{34} e^{-i\Phi_d} + c_{14} s_{12} s_{23} - c_{14} c_{12} c_{23} e^{i\phi_3} s_{13}$$

$$V_{ts} = -c_{13} c_{23} s_{34} s_{24} e^{-i\Phi_{sb}} - c_{12} c_{24} s_{23} - c_{24} c_{23} e^{i\phi_3} s_{13} s_{12}$$

$$V_{tb} = c_{13} c_{34} c_{23}$$

$$V_{tb'} = -c_{14} c_{23} s_{13} s_{12} s_{24} e^{i\Phi_{sb} + i\phi_3} - c_{14} c_{12} s_{24} s_{23} e^{i\Phi_{sb}} - c_{12} c_{23} s_{14} s_{13} e^{i\Phi_d + i\phi_3} + s_{14} s_{12} s_{23} e^{i\Phi_d} + c_{14} c_{13} c_{24} c_{23} s_{34}$$

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[PRD72(2005),PRD76(2007)]

explored the  $m_{t'} = 300$  GEV case for

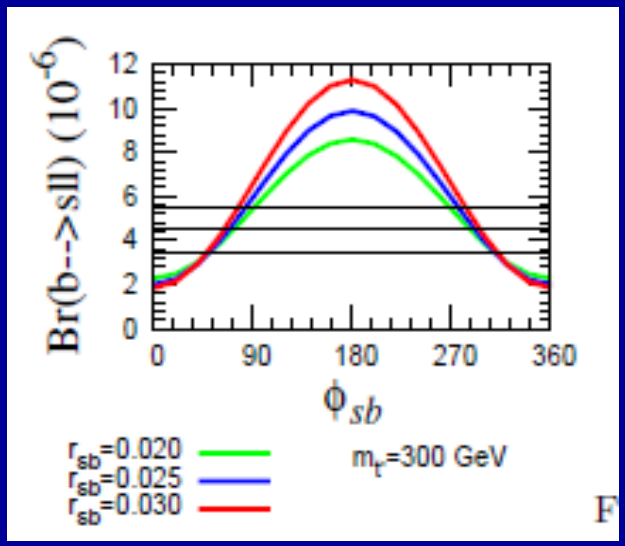
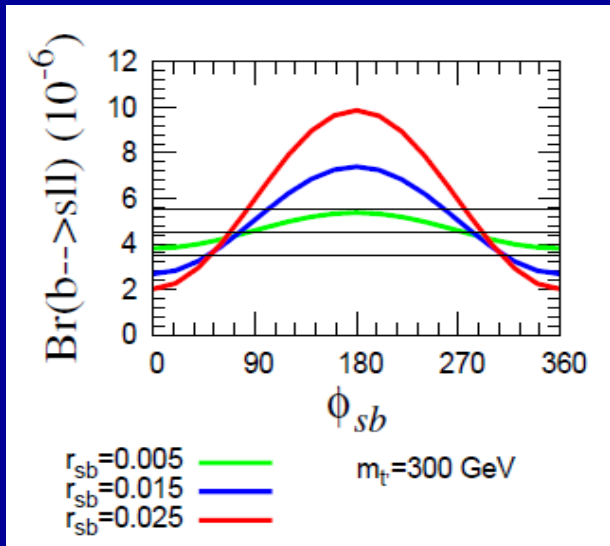
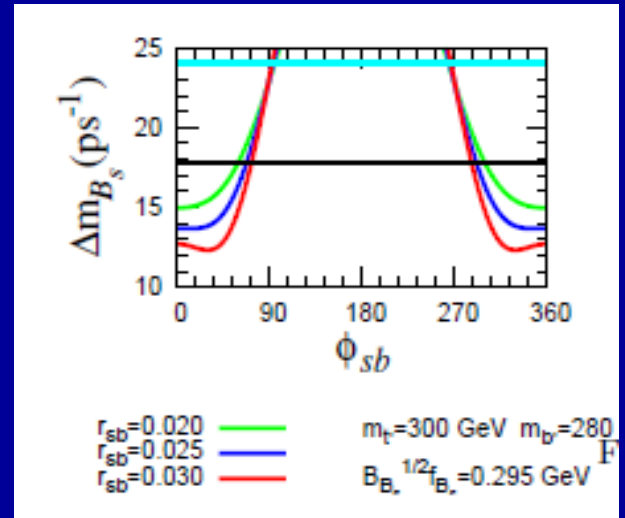
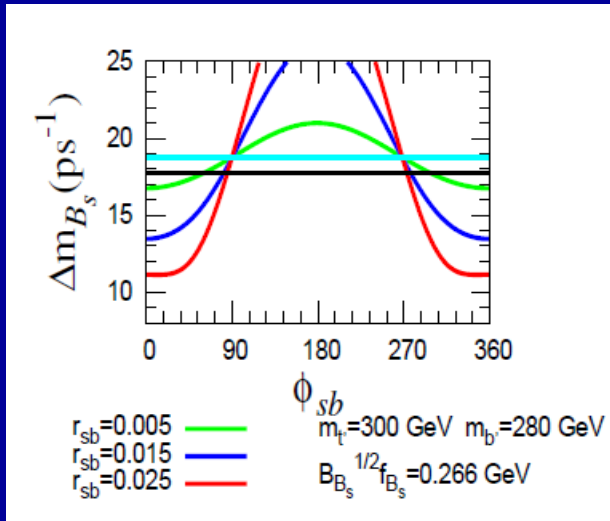
$$f_{B_s} \sqrt{B_{B_s}} = 295 \text{ MEV}$$

Revisit  $m_{t'} = 300$  GEV with  $f_{B_s} \sqrt{B_{B_s}} = 266(18)$  MEV

[HPQCD Collabor. arXiv:0902.1815]

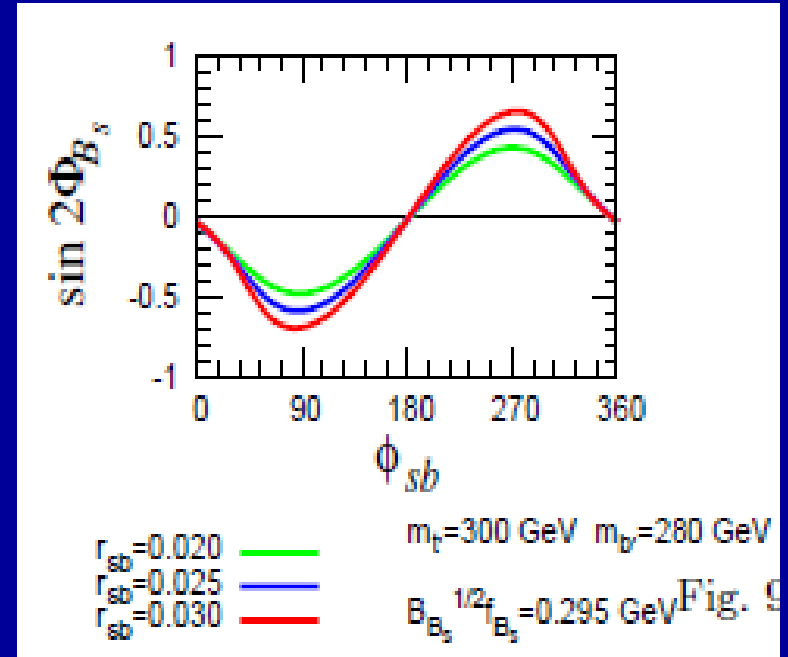
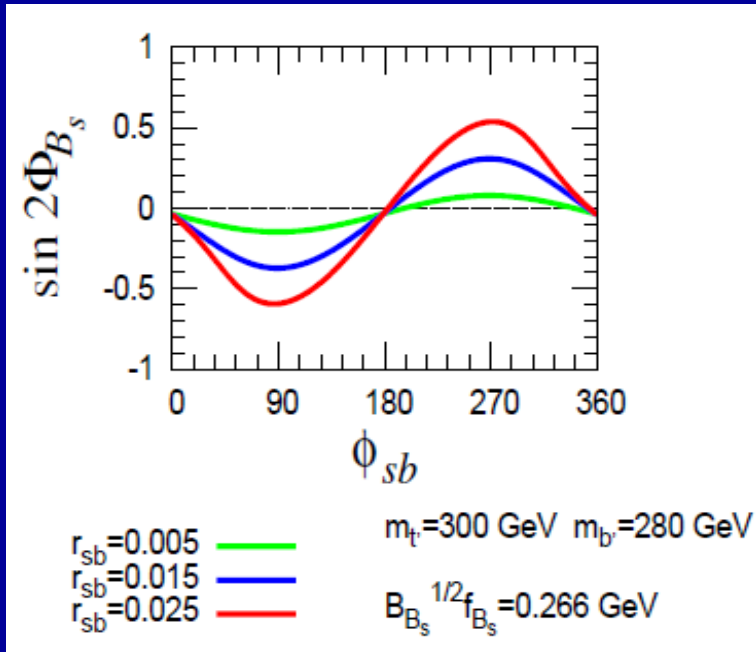
$$f_{B_s} \sqrt{B_{B_s}} = 266 \text{ MEV} \quad V_{t's}^* V_{t'b} \equiv r_{sb} \exp(i\phi_{sb})$$

$$f_{B_s} \sqrt{B_{B_s}} = 295 \text{ MEV}$$



$$f_{B_s} \sqrt{B_{B_s}} = 266 \text{ MEV} \quad V_{t's}^* V_{t'b} \equiv r_{sb} \exp(i\phi_{sb})$$

$$f_{B_s} \sqrt{B_{B_s}} = 295 \text{ MEV}$$



$$r_{sb} = 0.005 \sim 0.025,$$

$$\phi_{sb} = 60^\circ \sim 85^\circ,$$

$$\sin 2\Phi_{B_s} = -0.13 \sim -0.59$$

$$r_{sb} = 0.020 \sim 0.030,$$

$$\phi_{sb} = 60^\circ \sim 80^\circ,$$

$$\sin 2\Phi_{B_s} = -0.5 \sim -0.7$$



$$f_{B_s} \sqrt{B_{B_s}} = 266 \text{ MEV} \quad |V_{t's}^* V_{t'b}| \equiv r_{sb} \exp(i\phi_{sb}) \quad f_{B_s} \sqrt{B_{B_s}} = 295 \text{ MEV}$$

Blue curve (central value)

$$|V_{t's}^* V_{t'b}| = r_{sb} = 0.015,$$

$$\phi_{sb} = 81^\circ,$$

$$\sin 2\Phi_{B_s} = -0.37$$

Blue curve (central value)

$$|V_{t's}^* V_{t'b}| = r_{sb} = 0.025,$$

$$\phi_{sb} = 70^\circ,$$

$$\sin 2\Phi_{B_s} = -0.6$$

What is the range of  $V_{t'b}$  ?

# Upper Bound of $|V_{t'b}|$

$$V_{t'b} = -s_{34}$$

[Yanir JHEP06 (2002)]

$$\Gamma(Z \rightarrow q\bar{q}) = \frac{\alpha}{16\hat{s}_W^2\hat{c}_W^2} m_Z (|a_q|^2 + |v_q|^2) (1 + \delta_q^{(0)}) (1 + \delta_{QED}^q) \times \\ \times (1 + \delta_{QCD}^q) (1 + \delta_\mu^q) (1 + \delta_{tQCD}^q) (1 + \delta_b)$$

$$\delta_b \approx 10^{-2} \left[ \left( -\frac{m_t^2}{2m_Z^2} + 0.2 \right) |V_{tb}|^2 + \left( -\frac{m_{t'}^2}{2m_Z^2} + 0.2 \right) |V_{t'b}|^2 \right]$$

$m_{t'} = 300 \text{ GeV}$	$ V_{tb} ^2 + 3.4 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.24$
$m_{t'} = 350 \text{ GeV}$	$ V_{tb} ^2 + 4.7 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.20$
$m_{t'} = 400 \text{ GeV}$	$ V_{tb} ^2 + 6.1 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.17$
$m_{t'} = 450 \text{ GeV}$	$ V_{tb} ^2 + 7.8 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.14$
$m_{t'} = 500 \text{ GeV}$	$ V_{tb} ^2 + 9.6 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.13$

$$|V_{t'b}| = 0.24$$

Upper Bound

Hou, Nagashima and Soddu

Use

$$m_{t'} = 300 \text{ GeV} \quad |V_{tb}|^2 + 3.4|V_{t'b}|^2 < 1.14 \quad |V_{t'b}| < 0.24$$

$$|V_{t'b}| = 0.22 \quad [\text{PRD72(2005), PRD76(2007)}]$$

“wanted larger  $V_{t'b}$ ”

What is a Lower Bound of  $|V_{t'b}|$  ?

It can be obtained from  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  and  $x_D$

# Lower Bound of $|V_{t'b}|$

$$V_{t'b} = -s_{34}$$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.47_{-0.89}^{+1.30}) \times 10^{-10}$$

[Anisimovsky *et al.* PRL **93** (2004)]

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.73 \pm_{1.05}^{1.15}) \times 10^{-10}$$

[Artamonov *et al.* PRL **101** (2008)]

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) < 3.6 \times 10^{-10} \text{ (90\% C.L.)}$$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa_+ \left| \frac{\lambda_c^{ds}}{|V_{us}|} P_c + \frac{\lambda_t^{ds}}{|V_{us}|^5} \eta_t X_0(x_t) + \frac{\lambda_{t'}^{ds}}{|V_{us}|^5} \eta_{t'} X_0(x_{t'}) \right|^2$$

# Lower Bound of $|V_{t'b}|$

$$V_{t'b} = -s_{34}$$

$$x_D^{\text{exp}} = (9.1 \pm_{2.6}^{2.5}) \times 10^{-3}$$

Barberio *et al.* [HFAG] arXiv:0808.1297

$$M_{12}^{D^0} \propto \lambda_s^2 S_0(x_s) + 2\lambda_s \lambda_b S(x_s, x_b) + \lambda_b^2 S_0(x_b) + LD$$

$$+ 2\lambda_s \lambda_{b'} S(x_s, x_{b'}) + 2\lambda_b \lambda_{b'} S(x_b, x_{b'}) + LD$$

$$+ \lambda_{b'}^2 S_0(x_{b'}),$$

$$\lambda_x^{D^0} = V_{cx} V_{ux}^*$$

Neglect all terms in  $M_{12}^{D^0}$ , except  $M_{12,b'}^{D^0}$  and equate this with  $x_D^{\text{exp}}$  and enhance the possible space by a factor of 3

[Golowich *et al.* PRD76(2007),  
Bobrowski *et al.* arXiv:0902.4893]

$m_{b'} = 280 \text{ GeV}$	$ V_{ub'} V_{cb'}  < 0.0033$
$m_{b'} = 330 \text{ GeV}$	$ V_{ub'} V_{cb'}  < 0.0029$
$m_{b'} = 380 \text{ GeV}$	$ V_{ub'} V_{cb'}  < 0.0026$
$m_{b'} = 430 \text{ GeV}$	$ V_{ub'} V_{cb'}  < 0.0023$
$m_{b'} = 480 \text{ GeV}$	$ V_{ub'} V_{cb'}  < 0.0021$

$$\lambda_x^{D^0} = V_{cx} V_{ux}^*$$

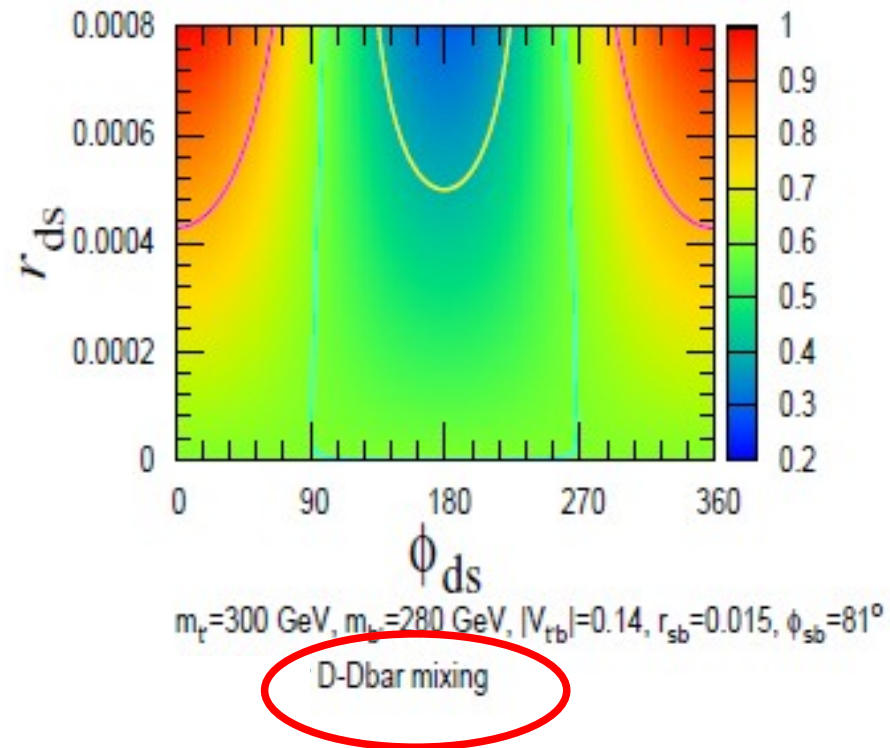
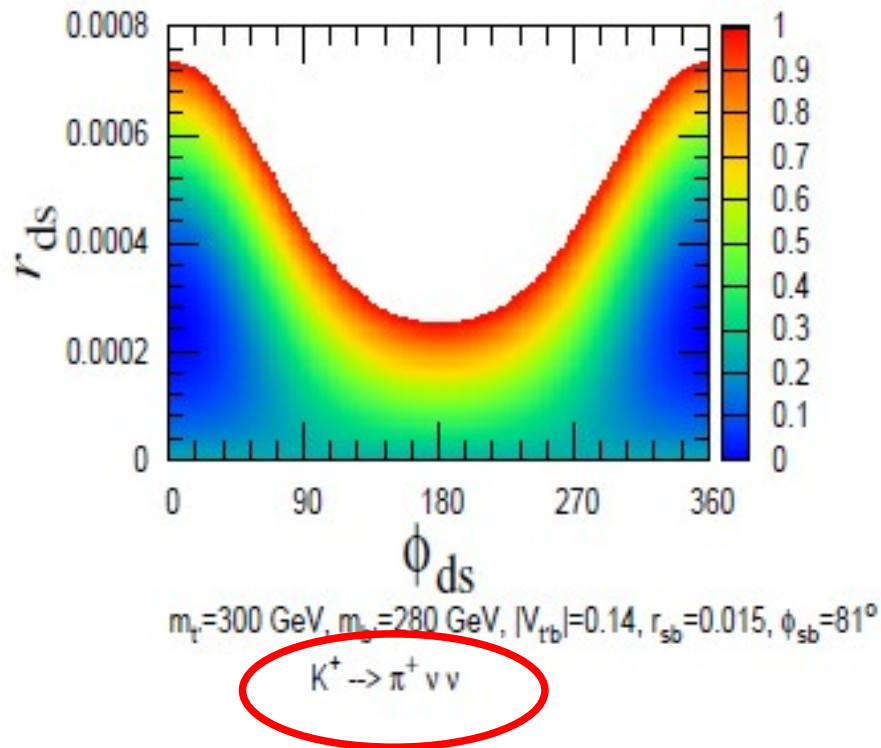
$$V_{ub'} = c_{14} c_{13} s_{12} s_{24} e^{i\Phi_{sb}} + c_{13} c_{12} s_{14} e^{i\Phi_d} + s_{13} c_{14} c_{24} e^{-i\phi_3} s_{34}$$

$$V_{t'd} = -c_{34} c_{24} s_{14} e^{-i\Phi_d}$$

Vanishing  $|V_{t'd}|$  DOES NOT lead to vanishing  $|V_{ub'}|$

# UPDATE OF $m_{t'} = 300$ GEV ( $f_{B_s} \sqrt{B_{B_s}} = 266$ MEV)

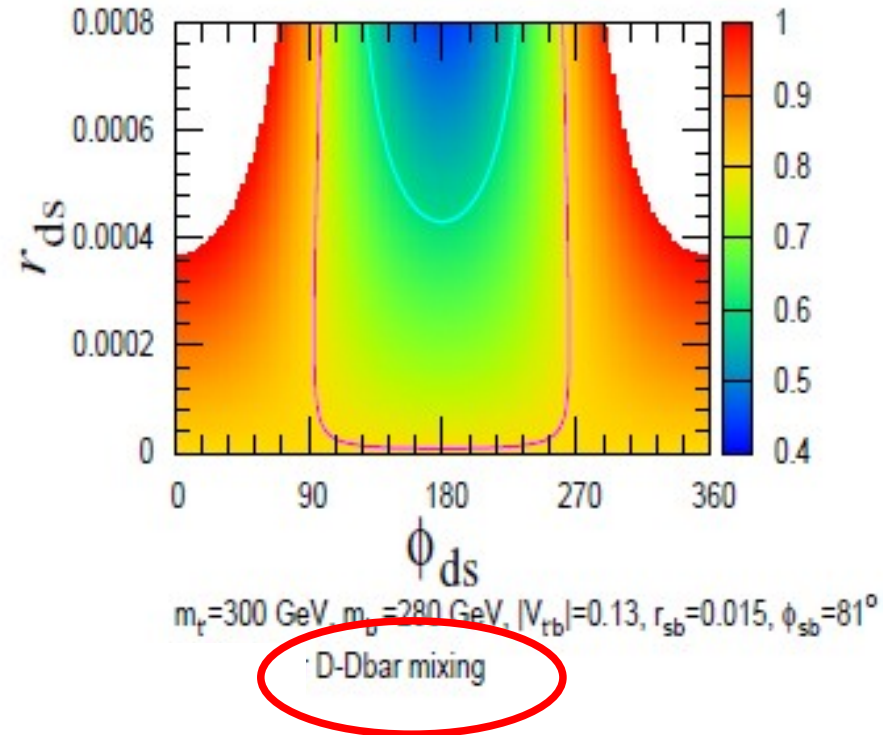
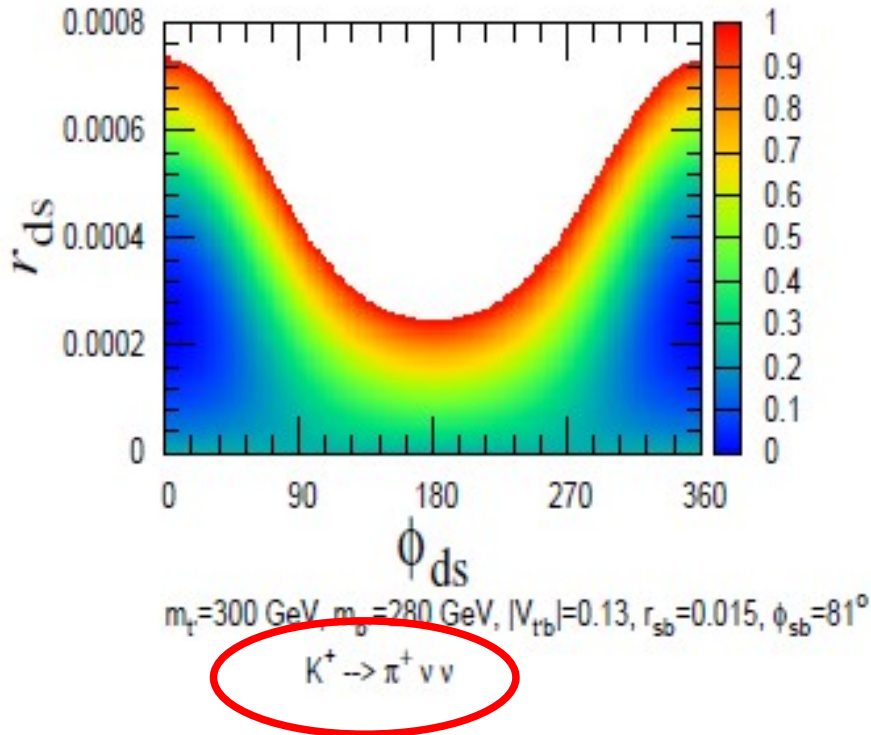
For  $|V_{t's}^* V_{t'b}| = 0.015$  fixed,  $\phi_{sb} = 81^\circ$ ,  $V_{t'd}^* V_{t's} \equiv r_{ds} \exp(i\phi_{ds})$



$$|V_{t'b}| = 0.14$$

# UPDATE OF $m_{t'} = 300$ GEV ( $f_{B_s} \sqrt{B_{B_s}} = 266$ MEV)

For  $|V_{t's}^* V_{t'b}| = 0.015$  fixed,  $\phi_{sb} = 81^\circ$ ,  $V_{t'd}^* V_{t's} \equiv r_{ds} \exp(i\phi_{ds})$

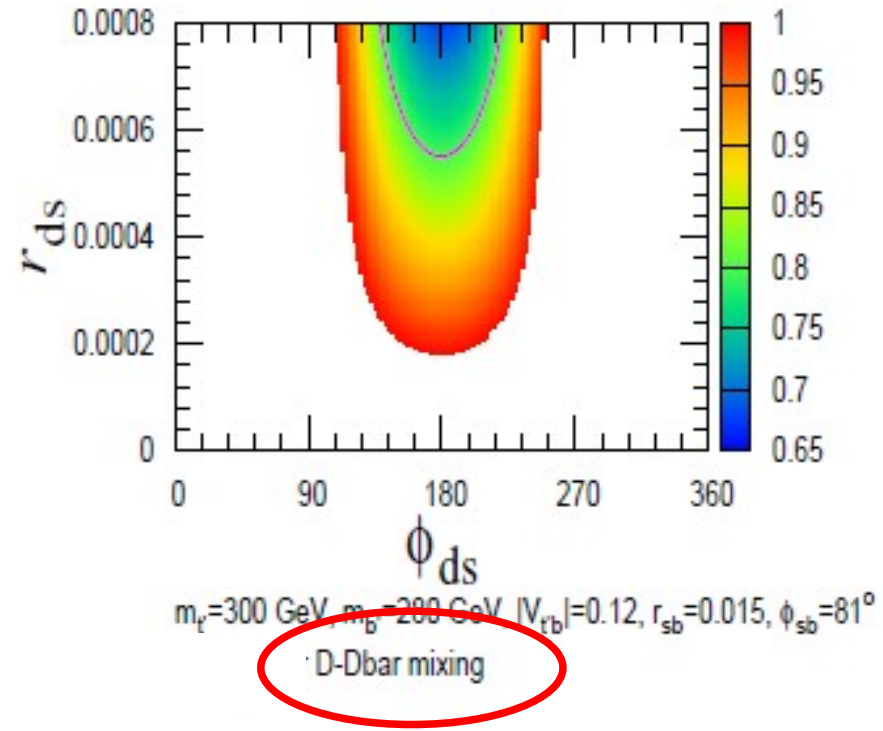
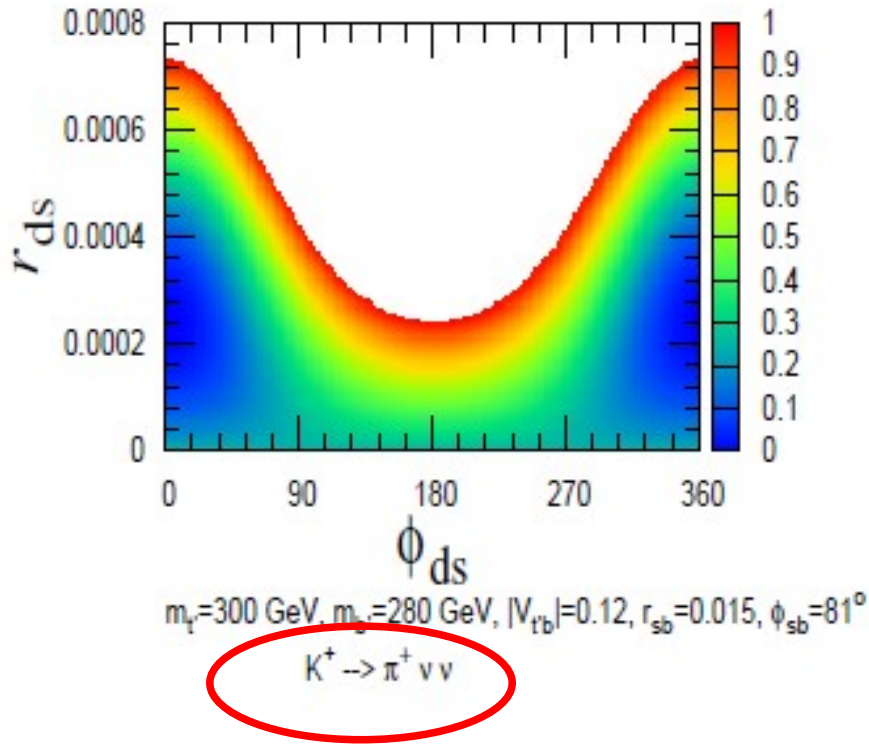


$$|V_{t'b}| = 0.13$$



# UPDATE OF $m_{t'} = 300$ GEV ( $f_{B_s} \sqrt{B_{B_s}} = 266$ MEV)

For  $|V_{t's}^* V_{t'b}| = 0.015$  fixed,  $\phi_{sb} = 81^\circ$ ,  $V_{t'd}^* V_{t's} \equiv r_{ds} \exp(i\phi_{ds})$



$$|V_{t'b}| = 0.12$$

Lower Bound

# Calculate $\text{Re}(\varepsilon'/\varepsilon)$ ?

$$\text{Re} \frac{\varepsilon'}{\varepsilon} = \text{Im} (\lambda_c^{ds}) P_0 + \text{Im} (\lambda_t^{ds}) F(x_t) + \text{Im} (\lambda_{t'}^{ds}) F(x_{t'}),$$

$$F(x) = P_X X_0(x) + P_Y Y_0(x) + P_Z Z_0(x) + P_E E_0(x).$$

$$P_i = r_i^{(0)} + r_i^{(6)} R_6 + r_i^{(8)} R_8.$$

[Buras *et al.* JHEP01 (2004)]

What  $R_6$ ,  $R_8$  really are ?

Try  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$

# Status of $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$

E391a has recently released the results from the second run,

$$\mathcal{B}^{\text{exp}}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 6.7 \times 10^{-8}$$

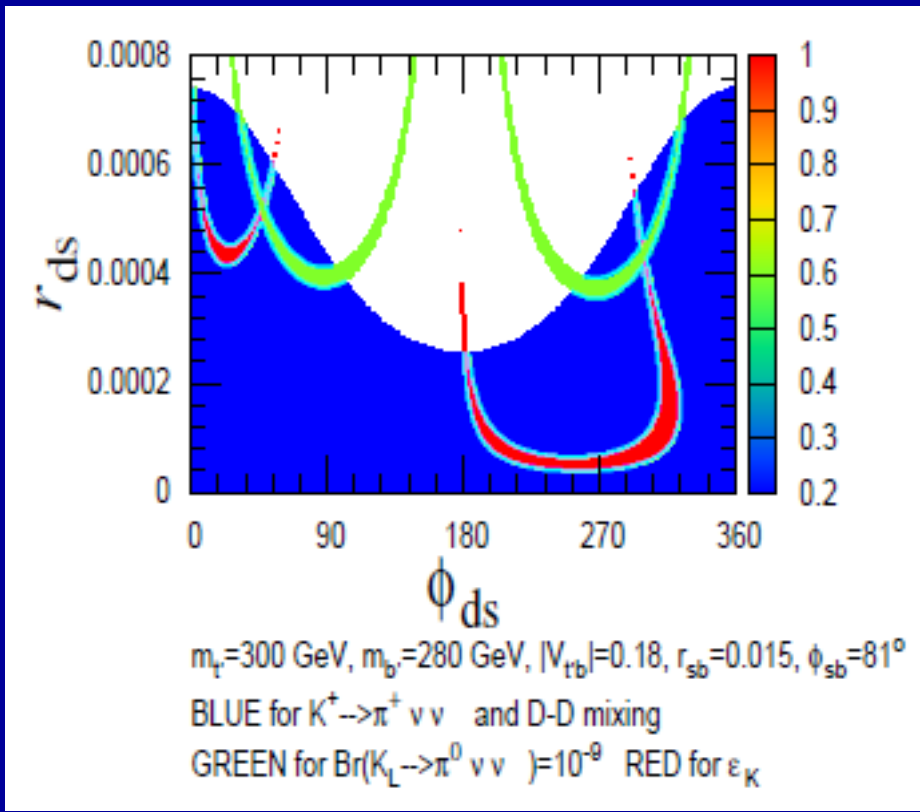
[E391a Collab. PRL 100 (2008)]

E14 will have a three-year physics run beginning in 2011.

$$0.12 < |V_{t'b}| < 0.24$$

Take  $|V_{t'b}| = 0.18$  &  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 10^{-9}$

$$\mathcal{B}^{\text{exp}}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 6.7 \times 10^{-8}$$

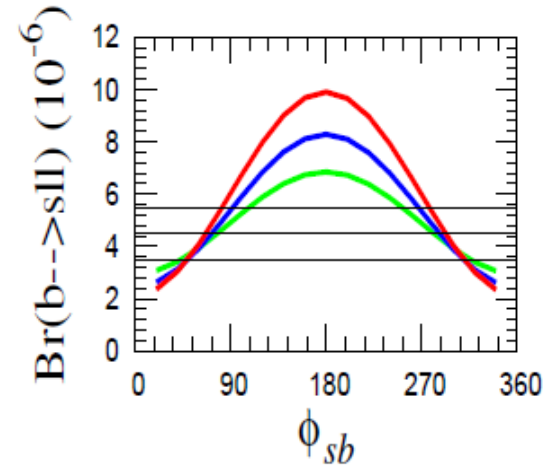
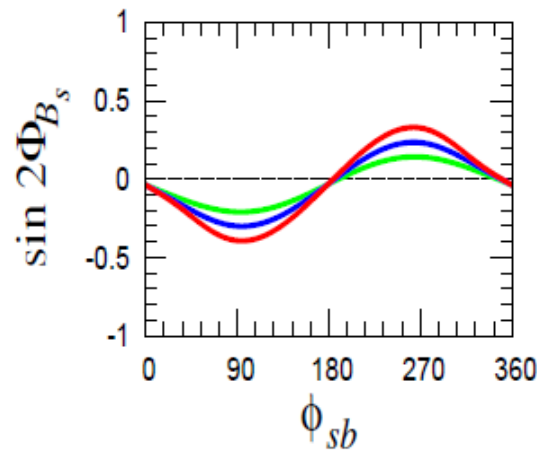
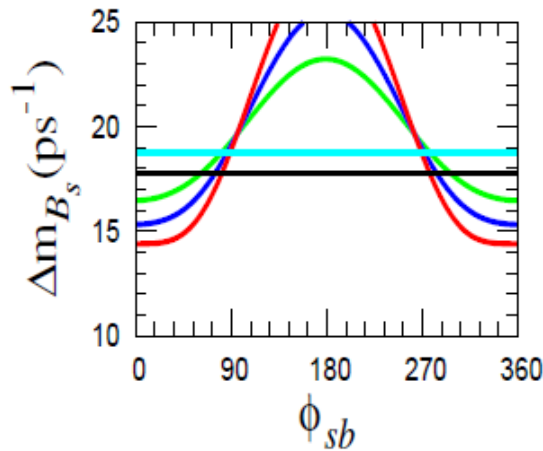


*RED* for  $\epsilon_K$   
*GREEN* for  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$

$$V_{t'd}^* V_{t's} \equiv r_{ds} \exp(i\phi_{ds})$$

$$m_{t'} = 500 \text{ GEV}$$

$$f_{B_s} \sqrt{B_{B_s}} = 266 \text{ MEV}$$



$r_{sb}=0.004$  — green  
 $r_{sb}=0.006$  — blue  
 $r_{sb}=0.008$  — red  
 $m_{t'}=500 \text{ GeV}$   $m_{b'}=480 \text{ GeV}$   
 $B_{B_c}^{1/2} f_{B_c}=0.266 \text{ GeV}$

$r_{sb}=0.004$  — green  
 $r_{sb}=0.006$  — blue  
 $r_{sb}=0.008$  — red  
 $m_{t'}=500 \text{ GeV}$   $m_{b'}=480 \text{ GeV}$   
 $B_{B_s}^{1/2} f_{B_s}=0.266 \text{ GeV}$

$r_{sb}=0.004$  — green  
 $r_{sb}=0.006$  — blue  
 $r_{sb}=0.008$  — red  
 $m_{t'}=500 \text{ GeV}$

Take central value (blue curve)  $r_{sb} = 0.006$ ,  $\phi_{sb} = 75^\circ$ ,  
 $\sin 2\Phi_{B_s} = -0.33$

$$m_{t'} = 500 \text{ GeV}$$

$$\Gamma(Z \rightarrow b\bar{b})$$

$m_{t'} = 300 \text{ GeV}$	$ V_{tb} ^2 + 3.4 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.24$
$m_{t'} = 350 \text{ GeV}$	$ V_{tb} ^2 + 4.7 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.20$
$m_{t'} = 400 \text{ GeV}$	$ V_{tb} ^2 + 6.1 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.17$
$m_{t'} = 450 \text{ GeV}$	$ V_{tb} ^2 + 7.8 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.14$
$m_{t'} = 500 \text{ GeV}$	$ V_{tb} ^2 + 9.6 V_{t'b} ^2 < 1.14$	$ V_{t'b}  < 0.13$

$$|V_{t'b}| = 0.13$$

Upper Bound

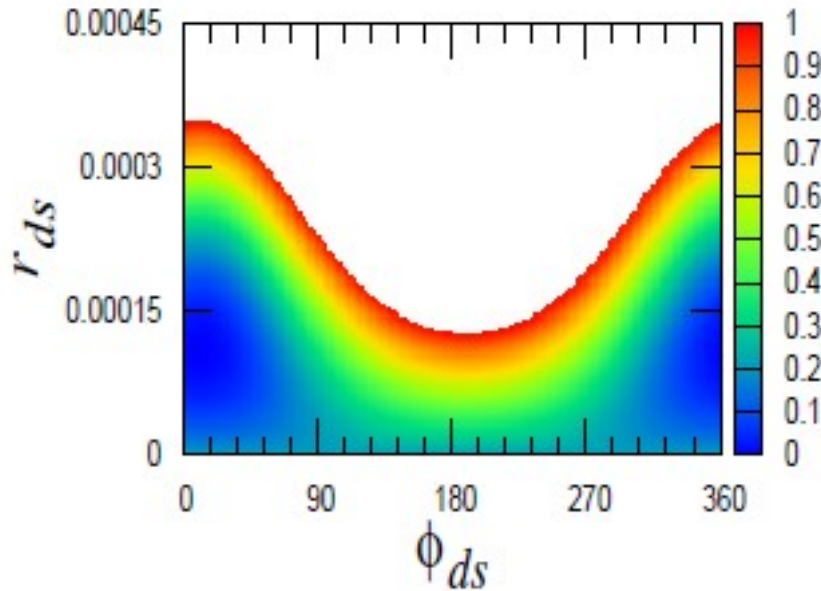
$$m_{t'} = 500 \text{ GEV}$$

Take  $m_{b'} = 480 \text{ GEV}$

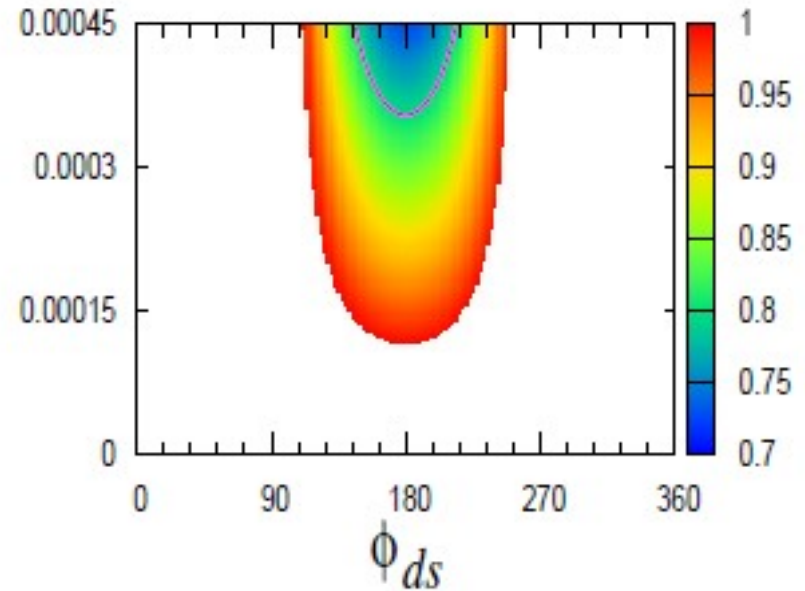
$$m_{b'} = 480 \text{ GEV} \quad |V_{ub'} V_{cb'}| < 0.0021$$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \bar{\nu} \nu) < 3.6 \times 10^{-10}$$

$$|V_{ub'}^* V_{cb'}| < 0.0021$$



$m_{t'}=500 \text{ GeV}, |V_{t'b}|=0.06, r_{sb}=0.006, \phi_{sb}=75^\circ$   
 COLOR for  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



$m_{b'}=480 \text{ GeV}, |V_{t'b}|=0.06, r_{sb}=0.006, \phi_{sb}=75^\circ$   
 COLOR for D-D mixing

$$|V_{t'b}| = 0.06$$

Lower Bound

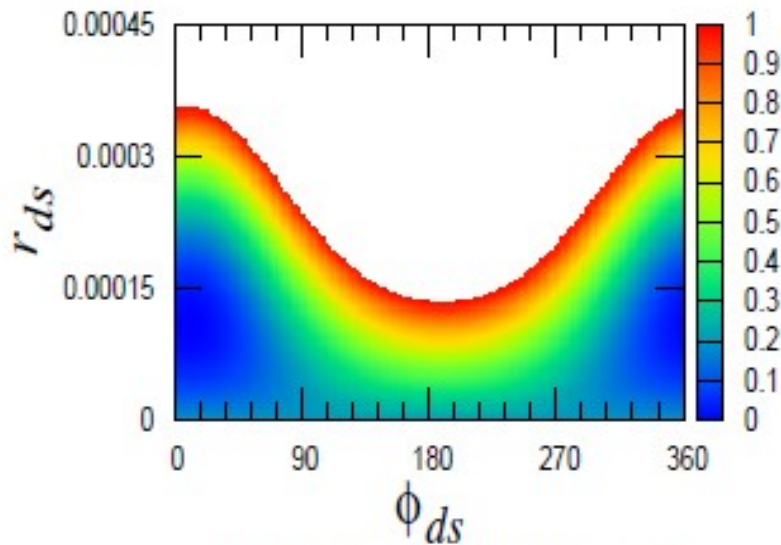
$$m_{t'} = 500 \text{ GeV}, m_{b'} = 480 \text{ GeV}$$

$$0.06 < |V_{t'b}| < 0.13$$

$$\text{Take } |V_{t'b}| = 0.100$$

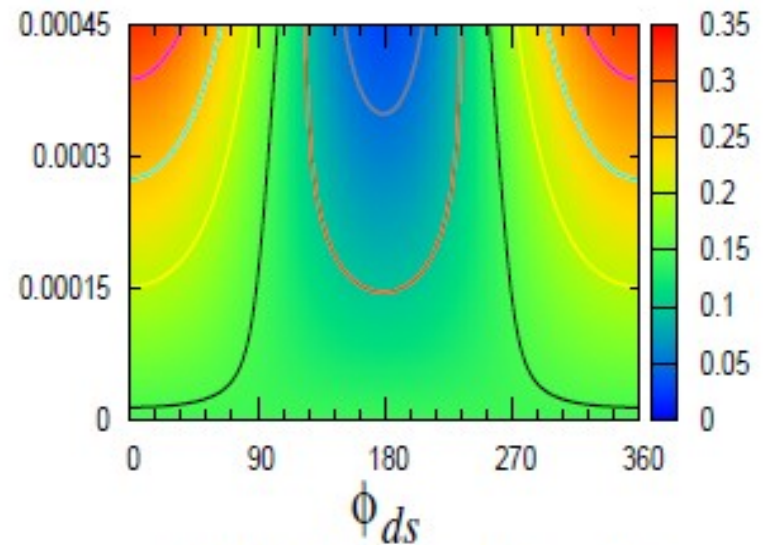
$$\mathcal{B}(K^+ \rightarrow \pi^+ \bar{\nu} \nu) < 3.6 \times 10^{-10}$$

$$|V_{ub'}^* V_{cb'}| < 0.0021$$



$$m_{t'} = 500 \text{ GeV}, |V_{t'b}| = 0.10, r_{sb} = 0.006, \phi_{sb} = 75^\circ$$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$



$$m_{b'} = 480 \text{ GeV}, |V_{t'b}| = 0.10, r_{sb} = 0.006, \phi_{sb} = 75^\circ$$

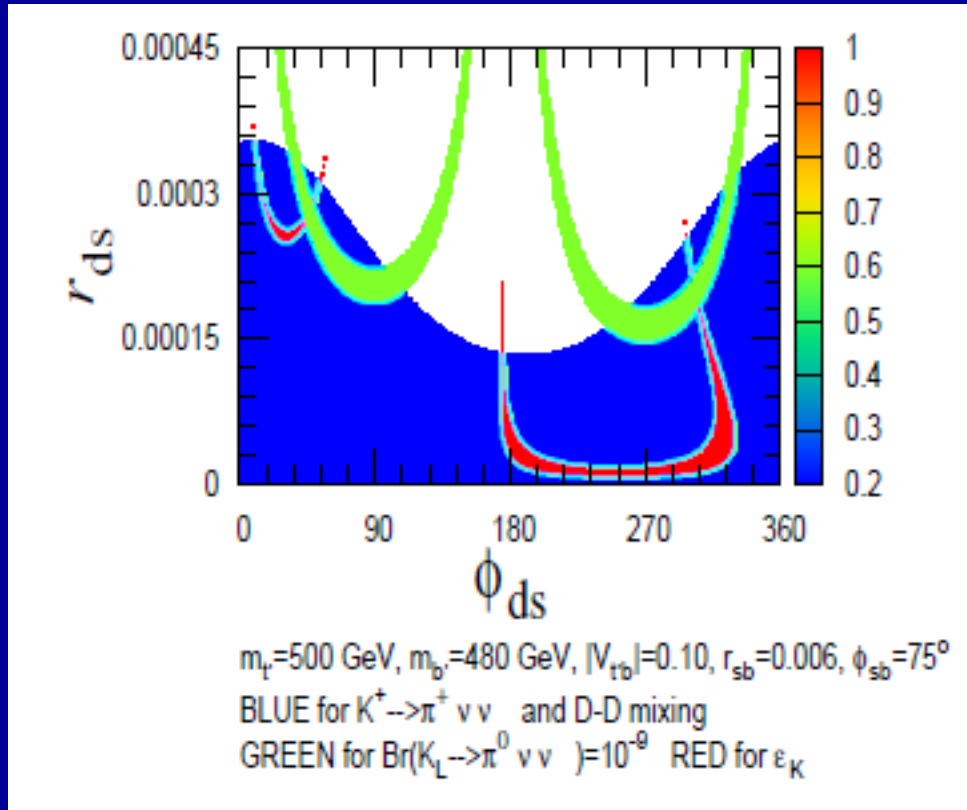
D-D mixing



$$m_{t'} = 500 \text{ GEV}, m_{b'} = 480 \text{ GEV}$$

$$0.06 < |V_{t'b}| < 0.13$$

Take  $|V_{t'b}| = 0.10$  &  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 10^{-9}$



$$\mathcal{B}^{\text{exp}}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 6.7 \times 10^{-8}$$

*RED* for  $\epsilon_K$

*GREEN* for  $\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})$

$$V_{t'd}^* V_{t's} \equiv r_{ds} \exp(i\phi_{ds})$$

# Conclusion

- I. For  $f_{B_s} \sqrt{B_{B_s}} = 266$  MEV, the central value of  $\sin 2\Phi_{B_s} \sim -0.35$ , insensitive of  $m_{t'}$  (if  $f_{B_s} \sqrt{B_{B_s}}$  higher, central  $\sin 2\Phi_{B_s}$  decrease for heavier  $m_{t'}$ )
- II. Upper bound of  $|V_{t'b}|$  can be obtained from  $\Gamma(Z \rightarrow b\bar{b})$
- III. For fixed  $|V_{t's}^* V_{t'b}|$ , lower bound of  $|V_{t'b}|$  can be obtained from  $\mathcal{B}(K^+ \rightarrow \pi^+ \nu\bar{\nu})$  and  $D - \bar{D}$  mixing
- IV. measurement  $\mathcal{B}(K_L \rightarrow \pi^0 \nu\bar{\nu})$  determines  $V_{t'd}^* V_{t's}$