Automatic calculation of one-loop amplitudes

Andreas van Hameren
IFJ-PAN, Kraków

in collaboration with
C.G. Papadopoulos, NCSR “Demokritos”, Athens
R. Pittau, University of Granada

EPS-HEP 2009, Kraków, 16-07-2009

Supported in part by the EU RTN European Programme, MRTN-CT-2006-035505 (HEPTOOLS, Tools and Precision Calculations for Physics Discoveries at Colliders) and by the Polish Ministry of Scientific Research and Information Technology grant No 153/6 PR UE/2007/7 2007-2010.

Motivation

- physics at LHC demands precise qualitative knowledge about signals and backgrounds;
- Monte Carlo programs are a preferred tools to condensate such knowledge;
- multi-leg hard processes need to be included in these. Many interesting signals (Higgs production) include decaying heavy particles.
- NLO corrections have to be included
  - to reduce scale dependence;
  - to get better description of shapes of distributions;
- several groups of researchers are dealing with the problem of calculating multi-leg processes at NLO.

Motivation

Backgrounds

- $pp \to VV + j$ Dittmaier,Kallweit,Uwer; Campbell,Ellis,Zanderighi
- $pp \to t\bar{t}b\bar{b}$ Bredenstein,Denner,Dittmaier,Pozzorini
- $pp \to VVV$ ZZZ:Lazopoulos,Melnikov,Petriello; WWZ:Hankele,Zeppenfeld;
  VVV: Binoth,Ossola,Papadopoulos,Pittau
- $pp \to VV + 2j$ VBF: Jäger,Oleari,Zeppenfeld; Bozzi
- $pp \to t\bar{t}Z$ Lazopoulos,Melnikov,Petriello
- $pp \to t\bar{t} + j$ Dittmaier,Uwer,Weinzierl
- $pp \to W + 3j$ Ellis,Melnikov,Zanderighi
  Berger,Bern,Dixon,Febres Cordero,Forde,Gleisberg,Ita,Kosower,Maître

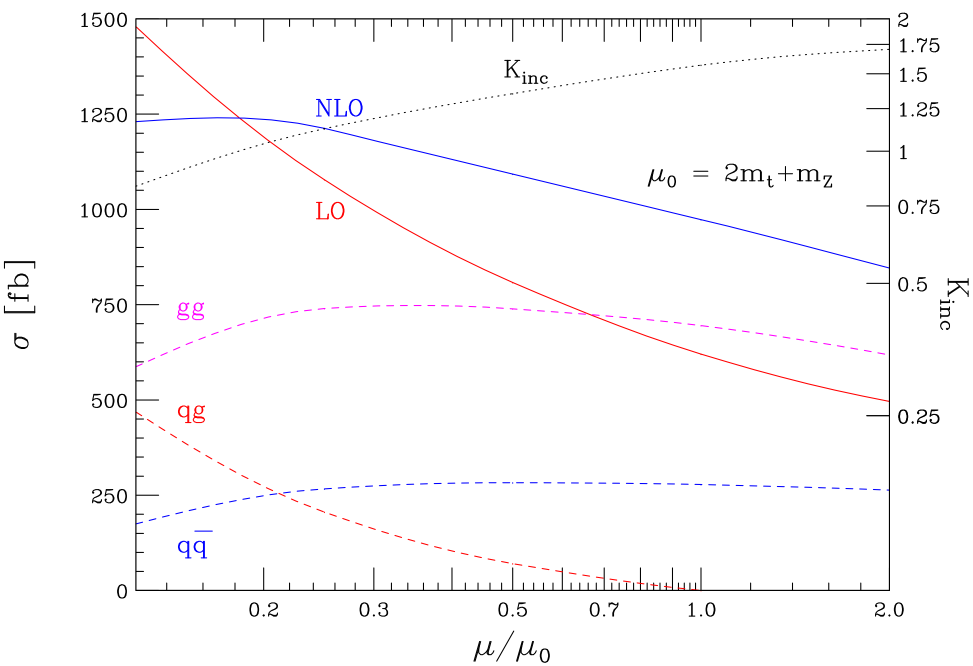
Signals

- $pp \to H + 2j$ Campbell,Ellis,Zanderighi; Ciccolini,Denner,Dittmaier
- $pp \to H + t\bar{t}$ Beenakker,Dittmaier,Krämer,Plümer,Spira,Zerwas;
  Dawson,Jackson,Reina,Wackeroth

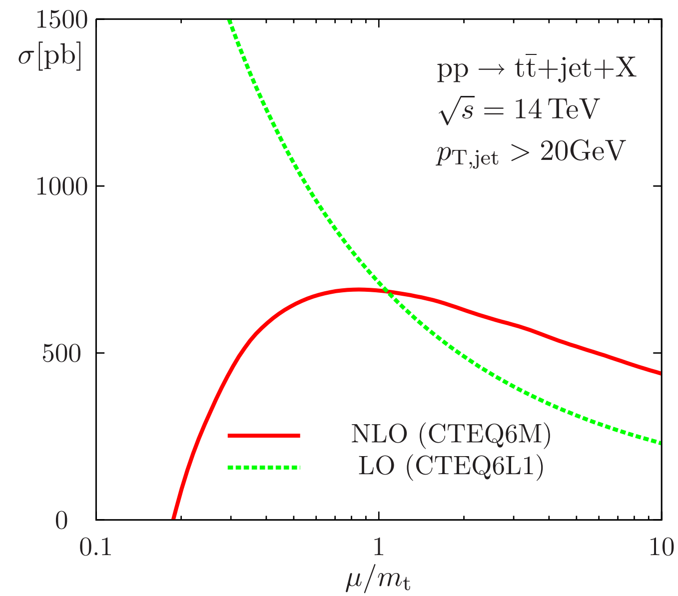
Motivation

Scale dependence ($\mu = \mu_R = \mu_F$)

$pp \to t\bar{t}Z$ Lazopoulos,Melnikov,Petriello



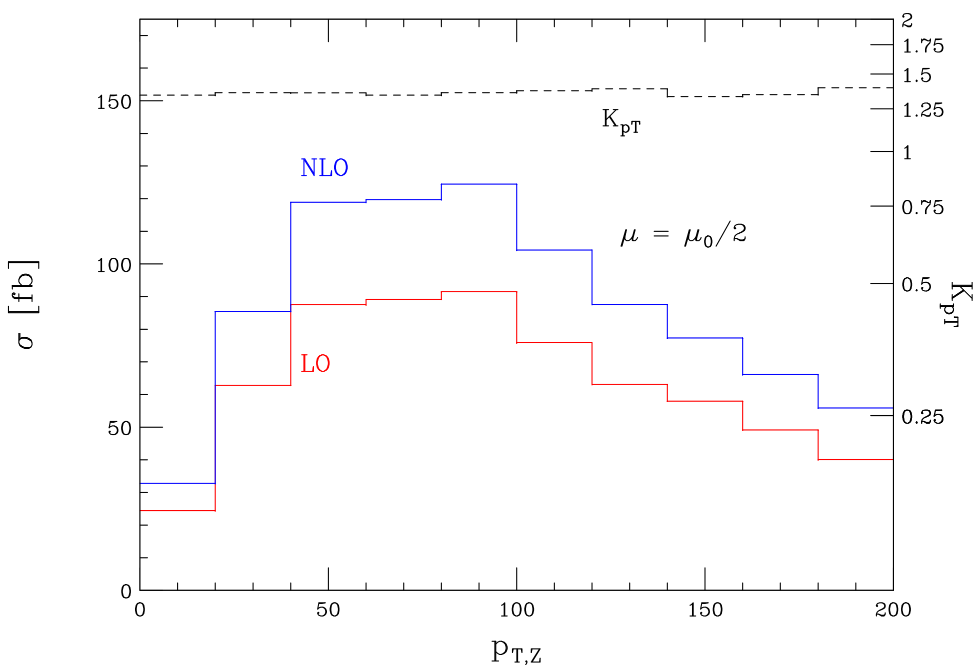
$pp \to t\bar{t} + j$ Dittmaier,Uwer,Weinzierl



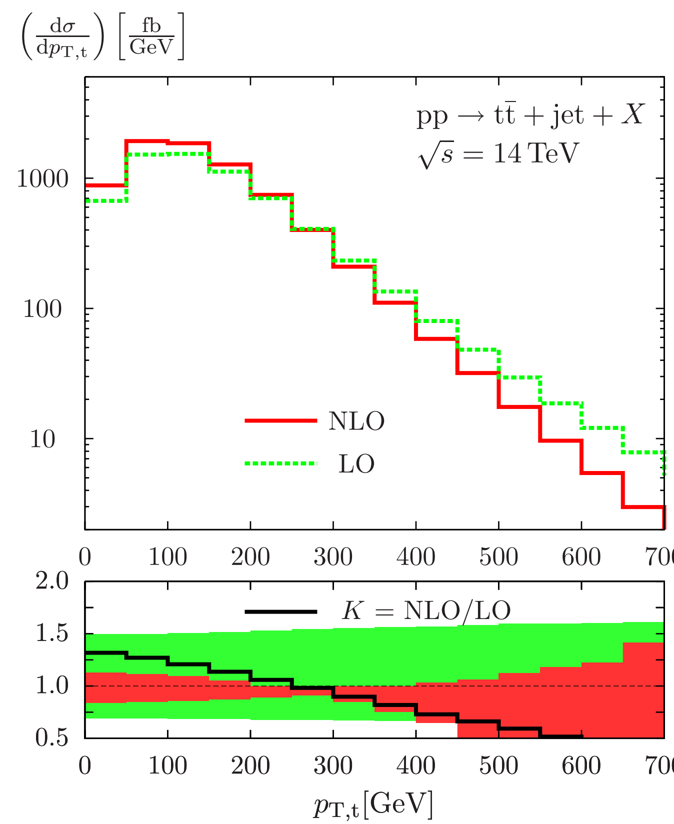
Motivation

Shape p_T-distribution

$pp \to t\bar{t}Z$ Lazopoulos,Melnikov,Petriello



$pp \to t\bar{t} + j$ Dittmaier,Uwer,Weinzierl



Motivation

- so far, mostly dedicated studies applying several computational techniques;
- LO calculations (including partonic phase-space generation) have been completely automatized: ALPGEN, MadGraph, SHERPA, GRACE, HELAC, ...;
- we want to do the same with NLO calculations
  HELAC Czakon,Dragiottis,Garzelli,Ossola,Pittau,Papadopoulos,Worek,AvH
- and we are not the only ones:
  GRACE Fujimoto,Kurihara
  ROCKET Ellis,Giele,Kunszt,Melnikov,Zanderighi
  BLACKHAT/SHERPA Berger,Bern,Dixon,Febres Cordero,Forde,Gleisberg Ita,Kosower,Maître
- one of the bottlenecks is the evaluation of the virtual, one-loop, contribution.
  Automation also by:
  GOLEM Binoth,Guffanti,Guillet,Heinrich,Karg,Kauer,Reiter,Reuter
  D-dim Unitarity Lazopoulos

Ingredients for the calculations

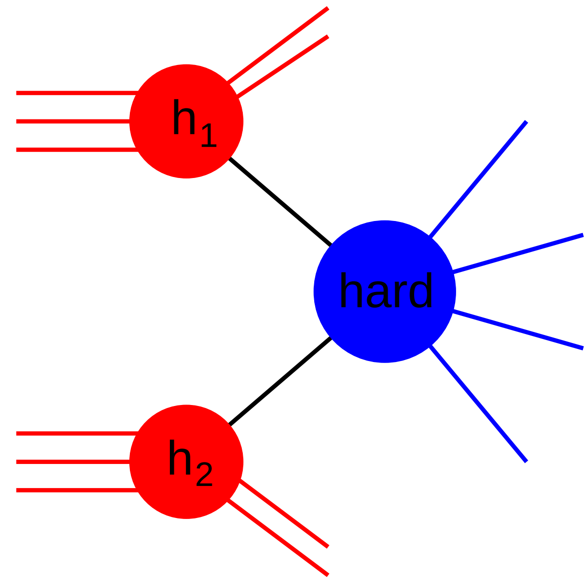
The mathematical framework of calculations in elementary particle physics is Quantum Field Theory. Two important ingredients in the calculations related to LHC physics are:

<u>Factorization</u>

$$d\sigma(\,h_1(p_1)h_2(p_2) \to X\,) =$$

$$\sum_{k,l}\int dx_1\,dx_2\,f_{1,k}(x_1,\mu_F)f_{2,l}(x_2,\mu_F)$$

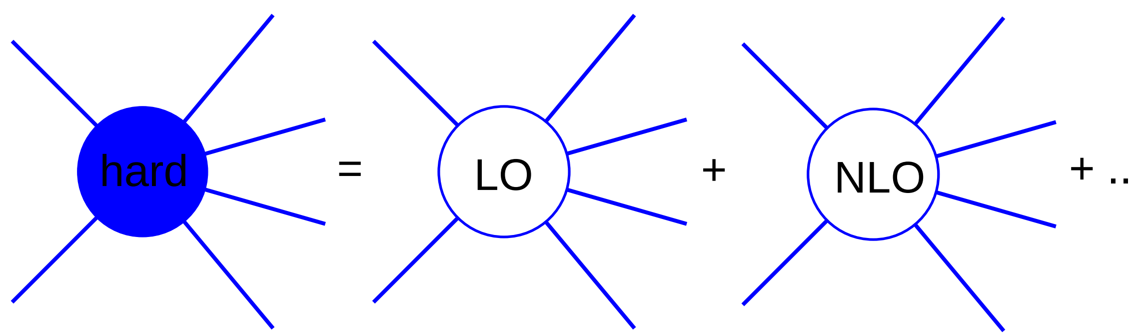
$$\times\, d\sigma_{\text{hard}}(\,\phi_k(x_1p_1)\phi_l(x_2p_2) \to X;\,\mu_F\,)$$



<u>Perturbation theory</u>

$$d\sigma_{\text{hard}} =$$

$$d\sigma^{(0)}_{\text{hard}} + \alpha d\sigma^{(1)}_{\text{hard}} + \ldots$$



Ingredients for the calculations

<u>NLO cross sections</u>

- one order higher in perturbation theory: one more loop or one more leg (squared);
- IR-divergence of integral over phase space for which the extra leg is unobserved cancels against IR-divergence of loop integral KLN.

$$\langle O\rangle^{\text{LO}} = \int d\Phi_n \, |\mathcal{M}_n^{(0)}|^2 \, O_n^{\text{LO}}$$

$$\langle O\rangle^{\text{NLO}} = \int d\Phi_n \left[2\Re\big(\mathcal{M}_n^{(0)}\mathcal{M}_n^{(1)}\big) + \mathcal{C}_n + \int d\Phi_1 \, \mathcal{A}_{n+1} \right] O_n^{\text{LO}} + \int d\Phi_{n+1} \left[\, |\mathcal{M}_{n+1}^{(0)}|^2 \, O_{n+1}^{\text{NLO}} - \mathcal{A}_{n+1} O_n^{\text{LO}} \, \right]$$

Eg. dipole subtraction Catani, Seymour '97

Ingredients for the calculations

Phase space integration

$$\langle O\rangle = \int d\Phi_n(P;\{p\}_n)\,|\mathcal{M}_n(\{p\}_n)|^2\,O_n(\{p\}_n)$$

In practice, PS integration has to be, and *can* be, done by Monte Carlo.

Summation over helicities

$$\langle O\rangle = \int d\Phi_n(P;\{p\}_n)\sum_{\{\lambda\}_n}|\mathcal{M}_n(\{p\}_n,\{\lambda\}_n)|^2\,O_n(\{p\}_n)$$

Calculate helicity amplitudes and perform sum over helicities explicitly, maybe even by MC.

Summation over colors

$$\langle O\rangle = \int d\Phi_n(P;\{p\}_n)\sum_{\{\lambda\}_n}\sum_{\{a\}_n}|\mathcal{M}_n(\{p\}_n,\{\lambda\}_n,\{a\}_n)|^2\,O_n(\{p\}_n)$$

$$\mathcal{M}_n(\{p\}_n,\{\lambda\}_n,\{a\}_n) = \sum_{\text{perm}}\mathcal{C}(\{a\}_n)\,\mathcal{A}_n(\{p\}_n,\{\lambda\}_n)$$

Ingredients for the calculations

Phase space integration

$$\langle O\rangle = \int d\Phi_n(P;\{p\}_n)\,|\mathcal{M}_n(\{p\}_n)|^2\,O_n(\{p\}_n)$$

In practice, PS integration has to be, and *can* be, done by Monte Carlo.

Summation over helicities

$$\langle O\rangle = \int d\Phi_n(P;\{p\}_n)\sum_{\{\lambda\}_n}|\mathcal{M}_n(\{p\}_n,\{\lambda\}_n)|^2\,O_n(\{p\}_n)$$

Calculate helicity amplitudes and perform sum over helicities explicitly, maybe even by MC.

Summation over colors

$$\langle O\rangle = \int d\Phi_n(P;\{p\}_n)\sum_{\{\lambda\}_n}\sum_{\{a\}_n}|\mathcal{M}_n(\{p\}_n,\{\lambda\}_n,\{a\}_n)|^2\,O_n(\{p\}_n)$$

Perform sum over colors numerically, maybe even by MC

Draggiotis,Kleiss,Papadopoulos ’98; Caravaglios,Mangano,Moretti,Pittau ’99.

Aim

We want to design a program to evaluate $\mathcal{M}_n^{(1)}(\{p\}_n, \{\lambda\}_n, \{a\}_n)$ as functions of its input as efficiently as possible.

The program should be highly automatic.

Philosophy

We are not particularly interested in algebraic/analytic expressions.

One-loop amplitude with O_{ssola}P_{apadopoulos}P_{ittau}

Identify a set of n_{tot} denominators and write

$$\mathcal{M}^{(1)} = \sum_{I \subset \{0,1,2,\dots,n_{\text{tot}}-1\}} \int d^{\text{Dim}}q \, \frac{N_I(q)}{\prod_{i\in I} D_i} \quad , \quad D_i = (q+p_i)^2 - m_i^2$$

One-loop amplitude with O_{ssola}P_{apadopoulos}P_{ittau}

Identify a set of n_{tot} denominators and write

$$\mathcal{M}^{(1)} = \sum_{I \subset \{0,1,2,\dots,n_{\text{tot}}-1\}} \int d^{\text{Dim}}q \, \frac{N_I(q)}{\prod_{i \in I} D_i} \quad , \quad D_i = (q+p_i)^2 - m_i^2$$

Integrals can be expressed in terms of universal scalar-functions:

$$\int \frac{d^{\text{Dim}}q \; N(q)}{D_0 D_1 \cdots D_{n-1}} = \sum_{i_1,i_2,i_3,i_4} d_{i_1 i_2 i_3 i_4} \int \frac{d^{\text{Dim}}q}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1,i_2,i_3} c_{i_1 i_2 i_3} \int \frac{d^{\text{Dim}}q}{D_{i_1} D_{i_2} D_{i_3}}$$

$$+ \sum_{i_1,i_2} b_{i_1 i_2} \int \frac{d^{\text{Dim}}q}{D_{i_1} D_{i_2}} + \sum_{i_1} a_{i_1} \int \frac{d^{\text{Dim}}q}{D_{i_1}} + \text{rational terms} + \mathcal{O}(\text{Dim} - 4)$$

One-loop amplitude with O

Identify a set of n_{tot} denominators and write

$$\mathcal{M}^{(1)} = \sum_{I\subset\{0,1,2,\dots,n_{\text{tot}}-1\}} \int d^{\text{Dim}}q\, \frac{N_I(q)}{\prod_{i\in I} D_i} \quad , \quad D_i = (q+p_i)^2 - m_i^2$$

Integrals can be expressed in terms of universal scalar-functions:

$$\int \frac{d^{\text{Dim}}q\; N(q)}{D_0 D_1 \cdots D_{n-1}} = \sum_{i_1,i_2,i_3,i_4} d_{i_1 i_2 i_3 i_4} \int \frac{d^{\text{Dim}}q}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1,i_2,i_3} c_{i_1 i_2 i_3} \int \frac{d^{\text{Dim}}q}{D_{i_1} D_{i_2} D_{i_3}}$$
$$+ \sum_{i_1,i_2} b_{i_1 i_2} \int \frac{d^{\text{Dim}}q}{D_{i_1} D_{i_2}} + \sum_{i_1} a_{i_1} \int \frac{d^{\text{Dim}}q}{D_{i_1}} + \text{rational terms} + \mathcal{O}(\text{Dim}-4)$$

Coefficients can be determined from polynomial equations involving few more coefficients

$$\frac{N(q)}{D_0 D_1 \cdots D_{n-1}} = \sum_{i_1,i_2,i_3,i_4} \frac{d_{i_1 i_2 i_3 i_4} + \tilde{d}_{i_1 i_2 i_3 i_4}(q)}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1,i_2,i_3} \frac{c_{i_1 i_2 i_3} + \tilde{c}_{i_1 i_2 i_3}(q)}{D_{i_1} D_{i_2} D_{i_3}}$$
$$+ \sum_{i_1,i_2} \frac{b_{i_1 i_2} + \tilde{b}_{i_1 i_2}(q)}{D_{i_1} D_{i_2}} + \sum_{i_1} \frac{a_{i_1} + \tilde{a}_{i_1}(q)}{D_{i_1}}$$

1 extra coefficient for $\tilde{d}$, 6 for $\tilde{c}$, 8 for $\tilde{b}$, 4 for $\tilde{a}$

One-loop amplitude with OPP

$$\int \frac{d^{\mathrm{Dim}}q\, N(q)}{D_0 D_1 \cdots D_{n-1}} = \sum_{i_1,i_2,i_3,i_4} d_{i_1 i_2 i_3 i_4} \int \frac{d^{\mathrm{Dim}}q}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1,i_2,i_3} c_{i_1 i_2 i_3} \int \frac{d^{\mathrm{Dim}}q}{D_{i_1} D_{i_2} D_{i_3}}$$
$$+ \sum_{i_1,i_2} b_{i_1 i_2} \int \frac{d^{\mathrm{Dim}}q}{D_{i_1} D_{i_2}} + \sum_{i_1} a_{i_1} \int \frac{d^{\mathrm{Dim}}q}{D_{i_1}} + \text{rational terms} + O(\mathrm{Dim}-4)$$

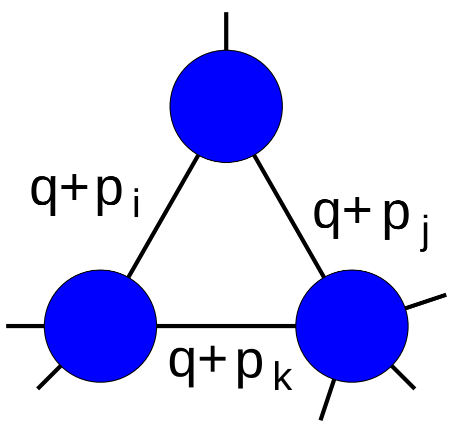
- universal set of scalar-functions can be coded once and for all eg. QCDloop Ellis,Zanderighi, OneLOop AvH;
- coefficients d, c, b, a can be determined from polynomial equations in 4 dimensions.
- to NLO we are not interested in $O(\mathrm{Dim}-4)$.
- rational terms can be written in terms of
  - simple universal integrals with already determined coefficients (R_1, coming from denominators for Dim $\neq 4$),
  - plus a finite counterterm, with extra Feynman rules Draggiotis, Garzelli,Papadopoulos,Pittau (R_2, coming from numerator for Dim $\neq 4$).

Evaluation of the numerator

$$\frac{N(q)}{D_0 D_1 \cdots D_{n-1}} = \sum_{i_1,i_2,i_3,i_4} \frac{d_{i_1 i_2 i_3 i_4} + \tilde{d}_{i_1 i_2 i_3 i_4}(q)}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1,i_2,i_3} \frac{c_{i_1 i_2 i_3} + \tilde{c}_{i_1 i_2 i_3}(q)}{D_{i_1} D_{i_2} D_{i_3}}$$
$$+ \sum_{i_1,i_2} \frac{b_{i_1 i_2} + \tilde{b}_{i_1 i_2}(q)}{D_{i_1} D_{i_2}} + \sum_{i_1} \frac{a_{i_1} + \tilde{a}_{i_1}(q)}{D_{i_1}}$$

- set of equations triangulizes if q is chosen such that some $D_j = 0$
- for such q, $N(q)$ only contains contributions from Feynman graphs containing *at least* these denominators.

For example: suppose q is such that $D_i = D_j = D_k = 0$:



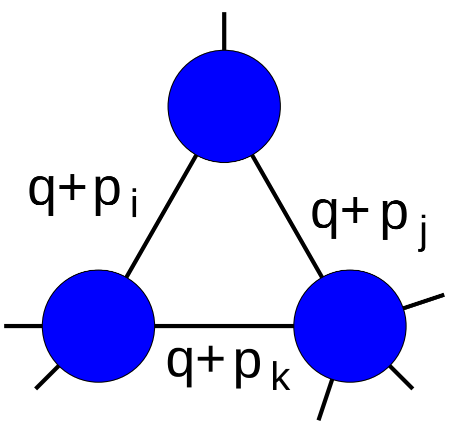
- the external momenta into the blobs, and thus the external particles into the blobs, are determined by $p_j - p_i$, $p_k - p_j$, $p_i - p_k$;
- o.s.-currents without q already calculated;
- the blobs are tree-like.

Evaluation of the numerator

$$\frac{N(q)}{D_0 D_1 \cdots D_{n-1}} = \sum_{i_1,i_2,i_3,i_4} \frac{d_{i_1 i_2 i_3 i_4} + \tilde{d}_{i_1 i_2 i_3 i_4}(q)}{D_{i_1} D_{i_2} D_{i_3} D_{i_4}} + \sum_{i_1,i_2,i_3} \frac{c_{i_1 i_2 i_3} + \tilde{c}_{i_1 i_2 i_3}(q)}{D_{i_1} D_{i_2} D_{i_3}}$$
$$+ \sum_{i_1,i_2} \frac{b_{i_1 i_2} + \tilde{b}_{i_1 i_2}(q)}{D_{i_1} D_{i_2}} + \sum_{i_1} \frac{a_{i_1} + \tilde{a}_{i_1}(q)}{D_{i_1}}$$

- set of equations triangulizes if q is chosen such that some $D_j = 0$
- for such q, $N(q)$ only contains contributions from Feynman graphs containing *at least* these denominators.

For example: suppose q is such that $D_i = D_j = D_k = 0$:



- We can use the tree-level machinery to calculate the one-loop integrand.
- Purely numerical analogue of "unitarity-cut method" for ordered amplitudes. Bern,Dixon,Dunbar,Kosower '94; Bern,Dixon,Kosower '97; Britto,Cachazo,Feng '04

Summary

- NLO precision is needed for LHC;
- preferably obtained with the help of automatic tools;
- OPP is a good method to automatize the calculation of the one-loop amplitude, neccesary for the virtual part in the NLO contribution;
- HELAC in combination with CutTools is able so far to deal with 6-leg one-loop amplitudes, eg $\mathrm{pp} \to \mathrm{t}\bar{\mathrm{t}}\,\mathrm{b}\bar{\mathrm{b}}$, $\mathrm{pp} \to W^+W^-\,\mathrm{b}\bar{\mathrm{b}}$, $\mathrm{pp} \to \mathrm{b}\bar{\mathrm{b}}\mathrm{b}\bar{\mathrm{b}}$, $\mathrm{pp} \to \mathrm{V}\mathrm{ggg}$, $\mathrm{pp} \to \mathrm{t}\bar{\mathrm{t}}\mathrm{gg}$.