NNLL Electroweak Corrections to Gauge Boson Pair Production at LHC

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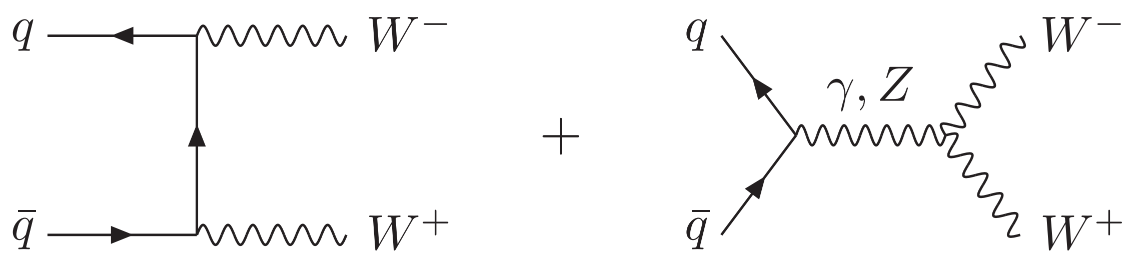
EPS HEP 2009 – Krakow – July 16-22

W pair production

- Important test of the vector boson trilinear couplings of the SM
- Background to the Higgs discovery channel $p\,p \to H \to W^+\,W^-$

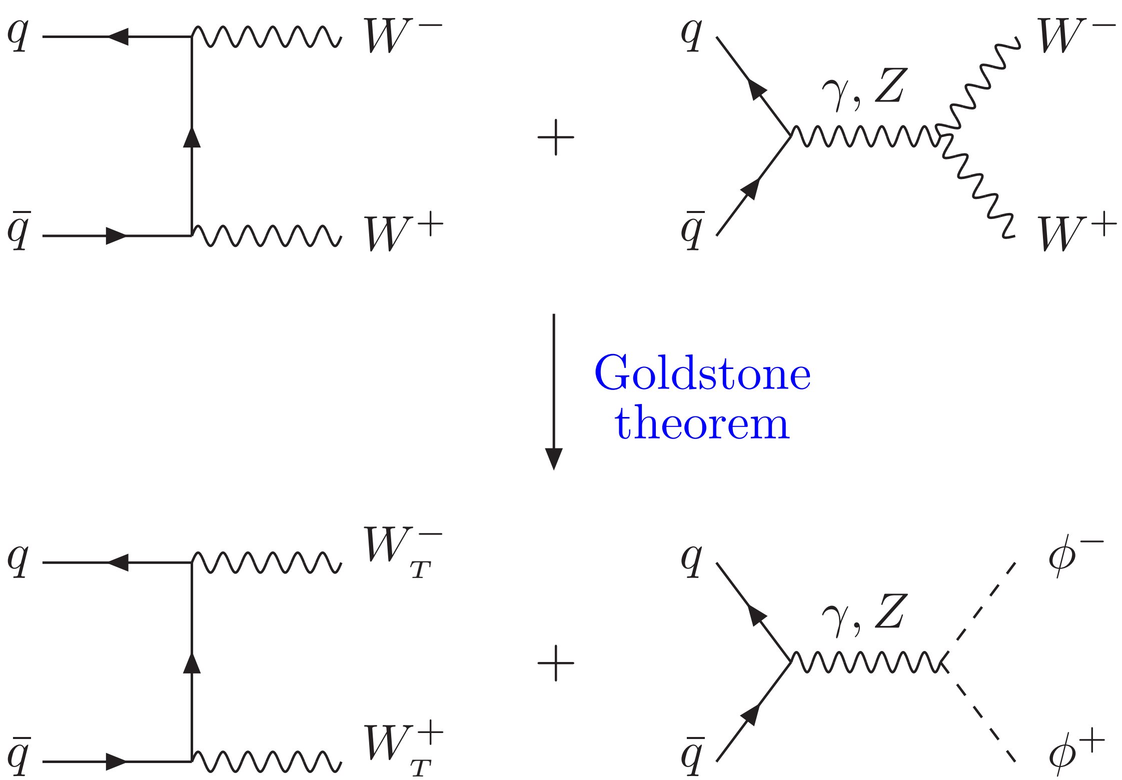
Partonic processes

- $g\,g \to W^+\,W^-$ $\rightsquigarrow$ 5% of the total cross section
- $q\,\bar{q} \to W^+\,W^-$



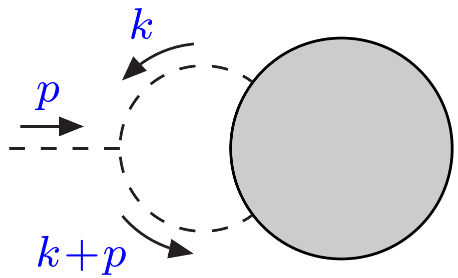
High energy limit

$$s \sim |t| \sim |u| \gg M_W^2$$



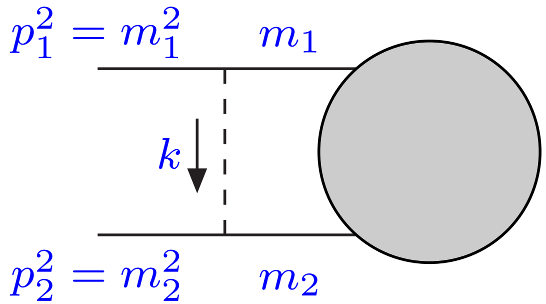
Infrared singularities

Collinear singularities



$k \longrightarrow \alpha\, p$

Soft singularities



$k \longrightarrow 0$

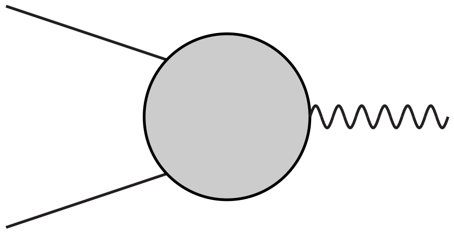
- If m_1 (and/or m_2) is massless $\longrightarrow$ soft-collinear singularities
- Singularity regularized by a mass M $\longrightarrow \ln^n\left(\frac{s}{M^2}\right)$

Large logarithms in SU(N)

Collinear logs depend just on the external legs $\longrightarrow$ Factorization

One can study a simpler problem:

Scattering in an external abelian field

$\mathcal{F} \quad =$  $\qquad \mathcal{F} = \mathcal{Z}\mathcal{F}_{\mathrm{Born}}$

$$\frac{\partial}{\partial \log Q^2}\mathcal{Z} = \left\{ \int_{\mu^2}^{Q^2} \frac{dx}{x}\gamma(\alpha(x)) + \zeta(\alpha(Q^2)) + \xi(\alpha(\mu^2)) \right\}\mathcal{Z} \qquad Q^2 = -s$$

- γ and ζ $\longrightarrow$ universal
- ξ and $\mathcal{Z}_0 \equiv \mathcal{Z}(Q^2 = \mu^2)$ $\longrightarrow$ initial conditions
  They depend on the IR structure and on the definition of μ

$$\mathcal{Z} = \mathcal{Z}_0 \exp\left\{ \int_{\mu^2}^{Q^2} \frac{dx}{x} \left[\int_{\mu^2}^{x} \frac{dx'}{x'} \gamma(\alpha(x')) + \zeta(\alpha(x)) + \xi(\alpha(\mu^2)) \right] \right\}$$

Loop expansion

$$f = \sum_{n=0}^{\infty} \left(\frac{\alpha}{4\pi} \right)^n f^{(n)}, \qquad \gamma^{(0)} = \zeta^{(0)} = \xi^{(0)} = 0 \qquad \mathcal{Z}_0^{(0)} = 1 \qquad L = \log \frac{Q^2}{\mu^2}$$

$$\mathcal{Z}^{(1)} = \frac{1}{2} \gamma^{(1)} L^2 + \left(\zeta^{(1)} + \xi^{(1)} \right) L + \mathcal{Z}_0^{(1)}$$

- By comparing with explicit calculation, we get the 1-loop coefficients

$$\begin{aligned} \mathcal{Z}^{(2)} &= \frac{1}{8} \left[\gamma^{(1)} \right]^2 L^4 + \frac{1}{2} \left[\zeta^{(1)} + \xi^{(1)} - \frac{1}{3} \beta_0 \right] \gamma^{(1)} L^3 \\ &+ \frac{1}{2} \left[\gamma^{(2)} + \left(\zeta^{(1)} + \xi^{(1)} \right)^2 - \beta_0 \zeta^{(1)} + \mathcal{Z}_0^{(1)} \gamma^{(1)} \right] L^2 \end{aligned}$$

- Just $\gamma^{(2)}$ has to be computed at 2-loop level

Large logarithms in a $2 \to 2$ process

Important the "color" structure

- Process $2_F \to 2_F$: $\mathcal{A} = \mathcal{A}_1\, \boldsymbol{T_a} \times \boldsymbol{T_a} + \mathcal{A}_2\, \mathbf{1} \times \mathbf{1}$
- Process $2_F \to 2_A$: $\mathcal{A} = \mathcal{A}_1\, \boldsymbol{T_a T_b} + \mathcal{A}_2\, \boldsymbol{T_b T_a} + \mathcal{A}_3\, \delta_{ab} \mathbf{1}$

F, A = fudamental, adjoint representation of SU(N)

$$\mathcal{A}_i = \mathcal{Z} \tilde{\mathcal{A}}_i \qquad \to \qquad \mathcal{Z} \text{ contains the collinear logs already computed}$$

$$\frac{\partial}{\partial \log Q^2} \tilde{\mathcal{A}}_i = \sum_j \chi_{ij}(\alpha(Q^2)) \tilde{\mathcal{A}}_j \qquad \tilde{\boldsymbol{\mathcal{A}}} = \mathrm{P}\left[\exp\left\{\int_{\mu^2}^{Q^2} \frac{dx}{x} \boldsymbol{\chi}(\alpha(x))\right\}\right] \tilde{\boldsymbol{\mathcal{A}}}_0(\alpha(\mu^2))$$

Loop expansion

$$\tilde{\boldsymbol{\mathcal{A}}}^{(1)} = \boldsymbol{\chi}^{(1)} \tilde{\boldsymbol{\mathcal{A}}}_0^{(0)} L + \tilde{\boldsymbol{\mathcal{A}}}_0^{(1)}$$

$$\tilde{\boldsymbol{\mathcal{A}}}^{(2)} = \frac{1}{2}\left[\left(\boldsymbol{\chi}^{(1)}\right)^2 - \beta_0 \boldsymbol{\chi}^{(1)}\right] \tilde{\boldsymbol{\mathcal{A}}}_0^{(0)} L^2 + \left[\boldsymbol{\chi}^{(1)} \tilde{\boldsymbol{\mathcal{A}}}_0^{(1)} + \boldsymbol{\chi}^{(2)} \tilde{\boldsymbol{\mathcal{A}}}_0^{(0)}\right] L + \tilde{\boldsymbol{\mathcal{A}}}_0^{(2)}$$

- 2-loop coefficients up to NNLL are determined by 1-loop coefficients

Evolution equations in the electroweak SM

Massless fermions, photon mass λ as infrared regulator

In the high energy limit we have two soft scales:

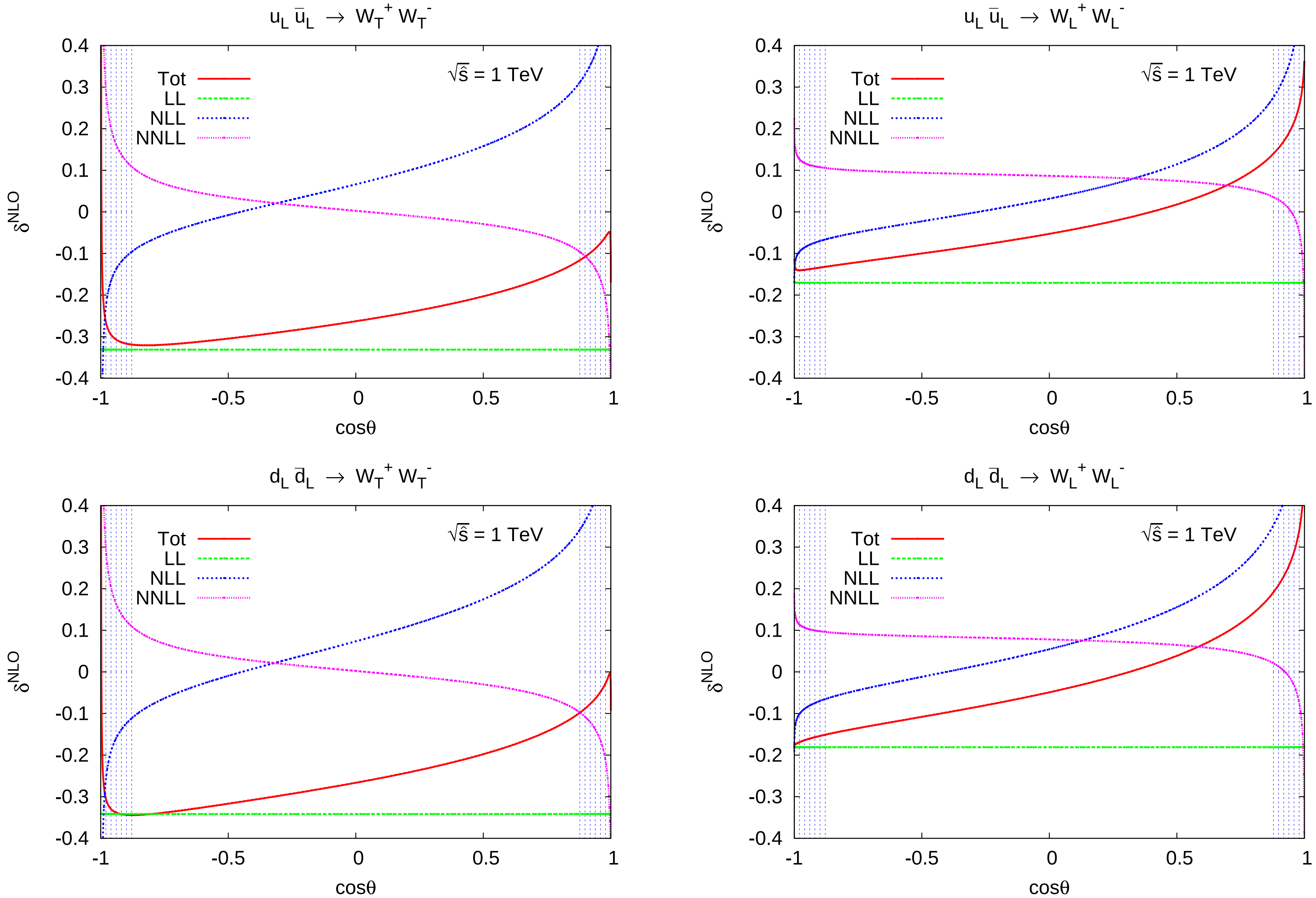
- Electrweak scale: $M_W \sim M_Z \sim M_H \sim M$
- Photon mass scale: λ

We have to analyze two regimes

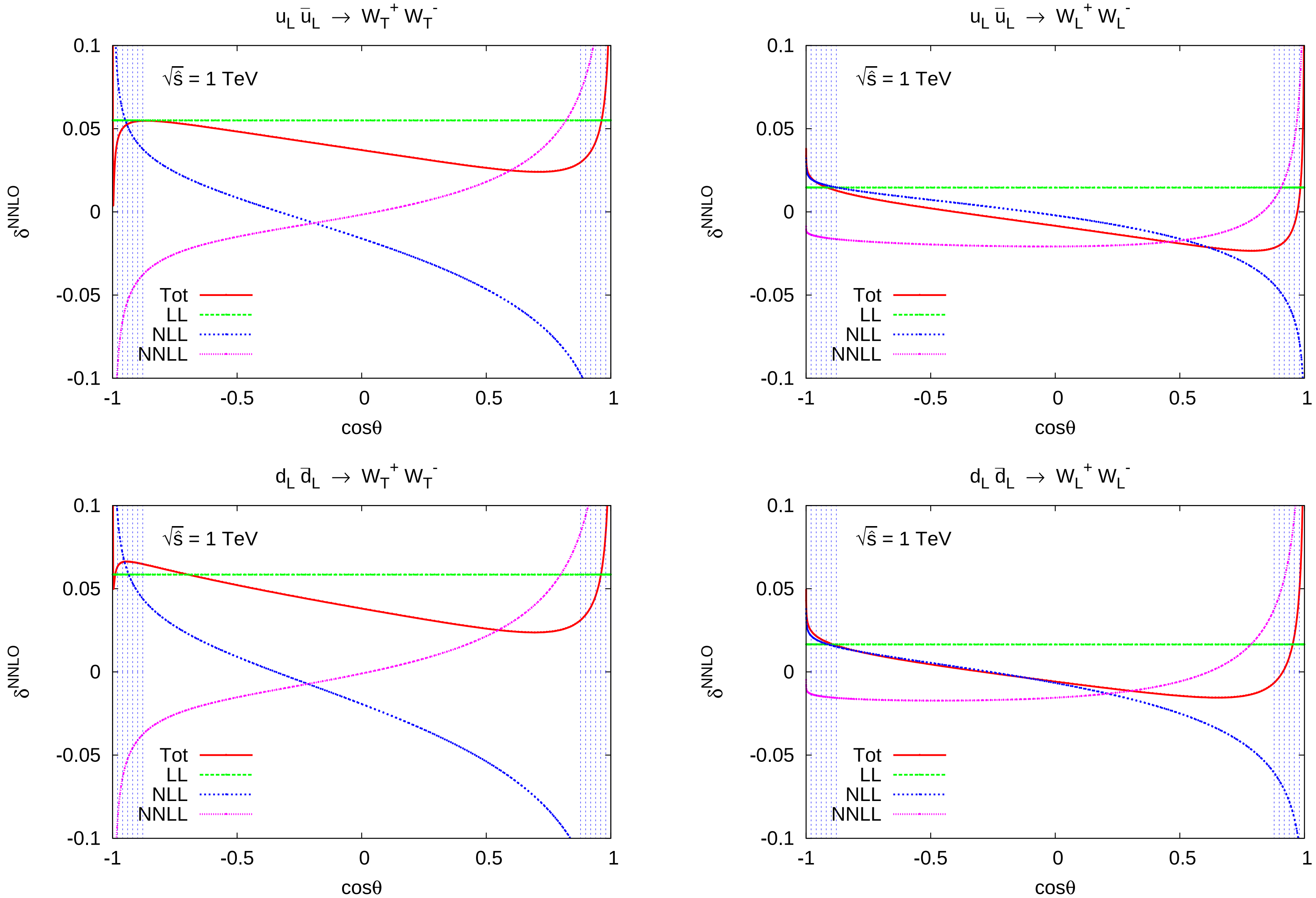
$I)$ $\sqrt{s} \gg \mu \gg M \gg \lambda$
All gauge bosons lighter than the cut-off $\rightsquigarrow$ unbroken SU(2)×U(1) theory
Matching condition: $\mathcal{A}_I(\mu = \sqrt{s}) = \mathcal{A}_{\text{Born}}$

$II)$ $\sqrt{s} \gg M \gg \mu \gg \lambda$
Just the photon is light $\rightsquigarrow$ only photonic contribution to Sudakow logs.
Matching condition: $\mathcal{A}_{II}(\mu = M) = \mathcal{A}_I(\mu = M)$

$$\delta^{\rm NLO} = \frac{d\sigma_{\rm NLO}}{d\sigma_{\rm LO}}$$



$$\delta^{\rm NNLO} = \frac{d\sigma_{\rm NNLO}}{d\sigma_{\rm LO}}$$



Hadronic distrbutions

$$\frac{d\sigma}{dp_T} = \frac{1}{N_c^2}\sum_{ij}\int_0^1 dx_1 \int_0^1 dx_2\, f_{h_1,i}(x_1,\mu_F^2)\, f_{h_2,j}(x_2,\mu_F^2)\,\theta(x_1 x_2 - \tau_{\min})\,\frac{d\hat{\sigma}_{ij}}{dp_T}$$

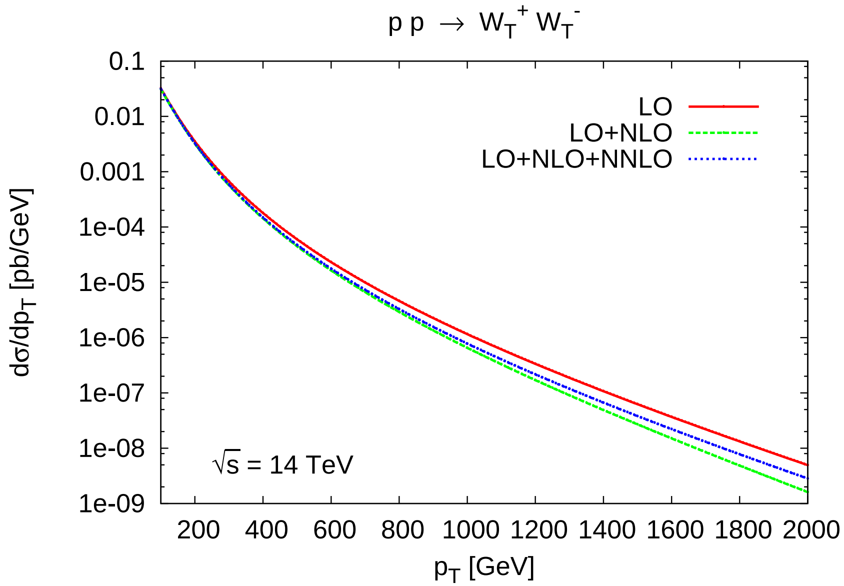
$$\frac{d\hat{\sigma}_{ij}}{dp_T} = \frac{4p_T}{\sqrt{\hat{s}-4M_W^2}\sqrt{\hat{s}-s\,\tau_{\min}}}\left[\frac{d\hat{\sigma}_{ij}}{d\cos\theta} + (\hat{t}\leftrightarrow\hat{u})\right]$$

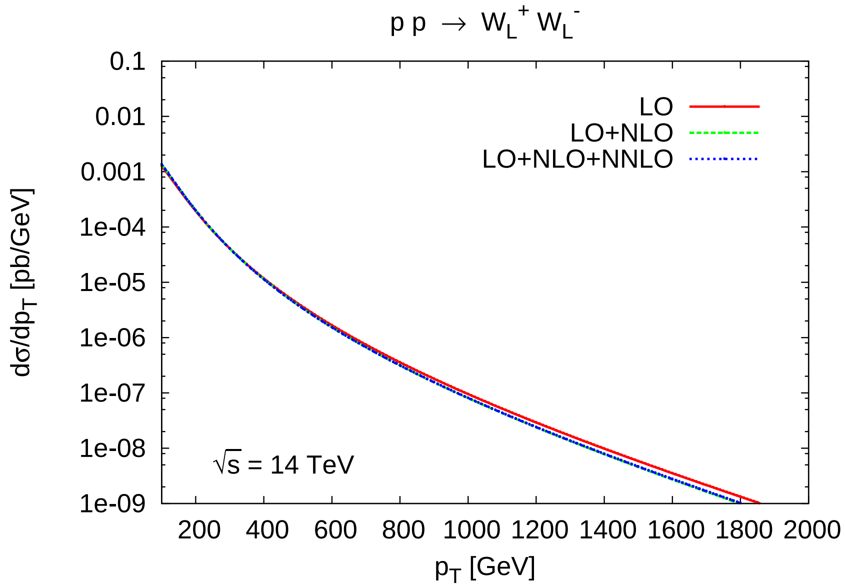
$$\hat{s} = x_1\, x_2\, s \qquad\qquad \tau_{\min} = \frac{4(p_T^2+M_W^2)}{s}$$

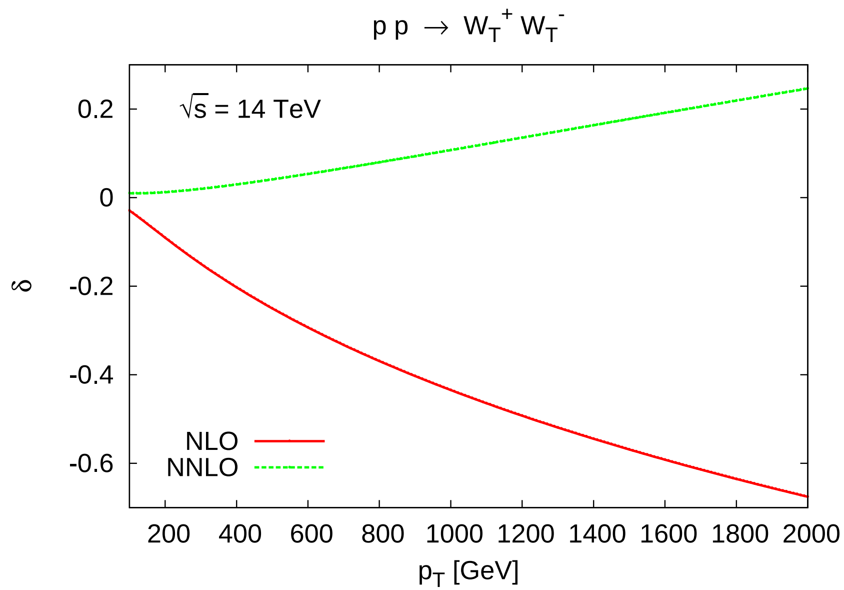
$$\frac{d\sigma}{dM_{WW}} = \frac{1}{N_c^2}\sum_{ij}\int_0^1 dx_1 \int_0^1 dx_2\, f_{h_1,i}(x_1,\mu_F^2)\, f_{h_2,j}(x_2,\mu_F^2)\,\frac{d\hat{\sigma}_{ij}}{\mathrm{d}M_{WW}}$$

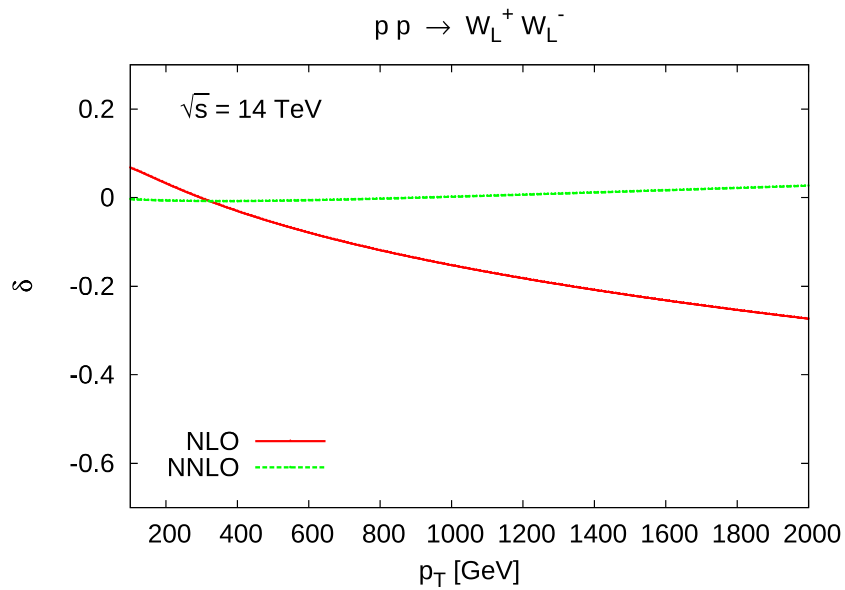
$$\frac{d\hat{\sigma}_{ij}}{dM_{WW}} = \int_{-\alpha}^{\alpha} d\cos\theta\,\frac{d\hat{\sigma}_{ij}}{d\cos\theta}\,\delta(\sqrt{\hat{s}}-M_{WW}) \qquad \alpha = \cos\theta_{\mathrm{cut}} \qquad \theta_{\mathrm{cut}} \geq 30^o$$

p_T distribution

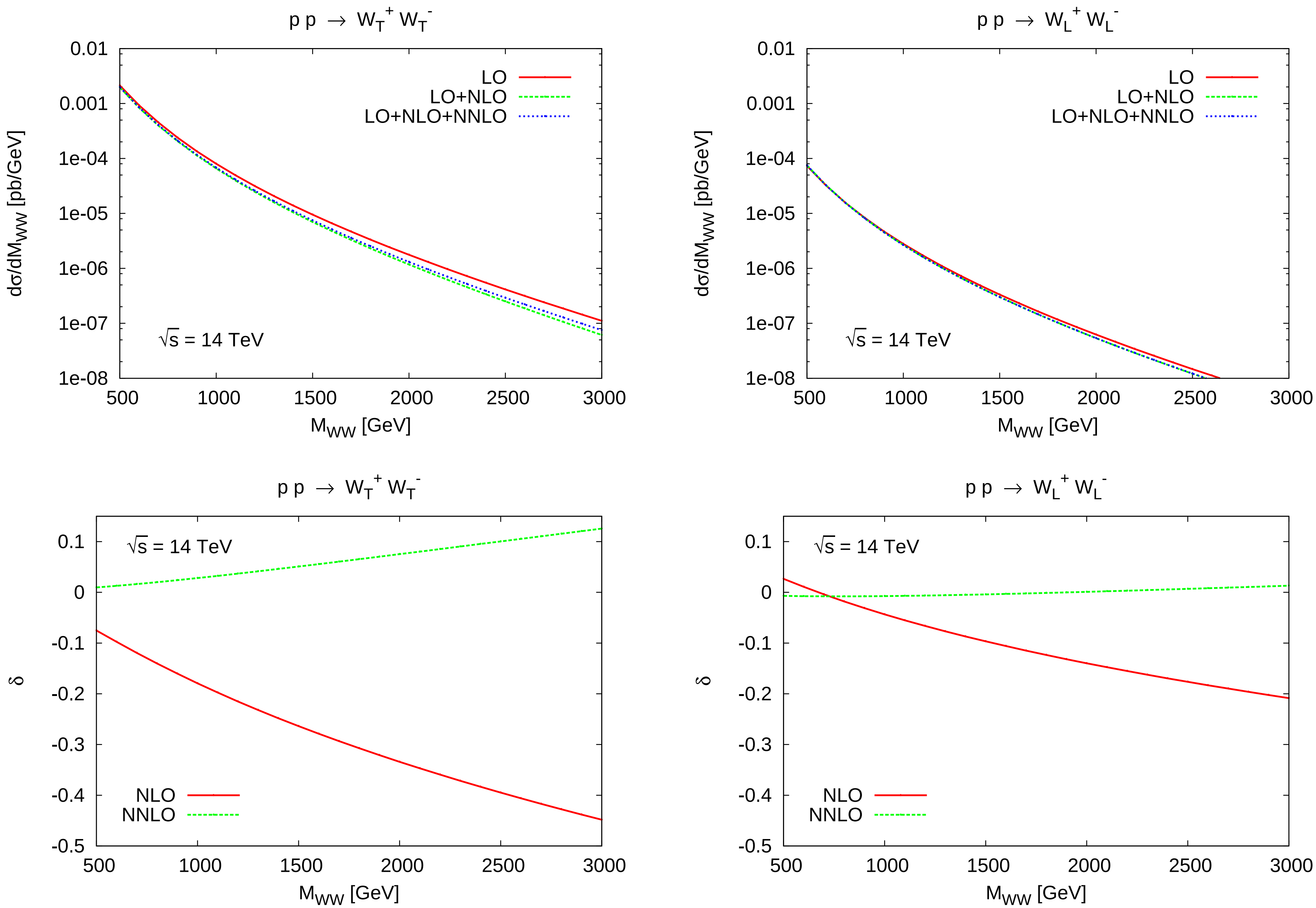








Invariant mass distribution ($\theta_{\text{cut}} = 30^o$)



Conclusions

- NNLL are not negligible with respect to LL and NLL
- NNLO corrections at high energies are important for the production of transversely polarized W's (10% at 1 TeV for the p_T distribution).
- Next project: ZZ and WZ production.