The Discrepancy Between τ and e^+e^- Spectral Functions Revisited and the Consequences for the Muon Magnetic Anomaly

Preliminary result in arXiv: 0906.5443 [hep-ph]

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Outline

- Introduction
- Situation at ICHEP06
- New analysis
  - New tau data from Belle
  - Isospin breaking corrections revisited
- Summary and future prospects

Muon Magnetic Anomaly a_μ

(1) **Measurements:** $\delta(a_\mu)_{exp} = 6.3 \cdot 10^{-10}$ [i.e. 0.54 ppm]

(2) **Predictions:** a_μ receives contributions from all SM sectors:

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm had} + a_\mu^{\rm Weak}$$

- Needs comparable precision to compare (1) and (2)

  Discrepancy between (1) & (2)? ➔ Hint for New Physics

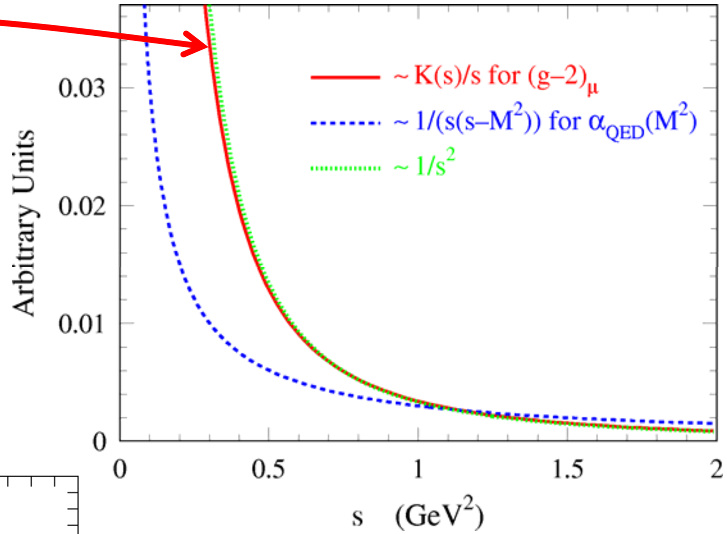
- $\delta(a_\mu^{had})$ has the largest uncertainty

➔ Improve a_μ^{had} from dominant $\pi^+\pi^-$ channel [~73% of a_μ^{had} & ~82% of $\delta(a_\mu^{had})$]

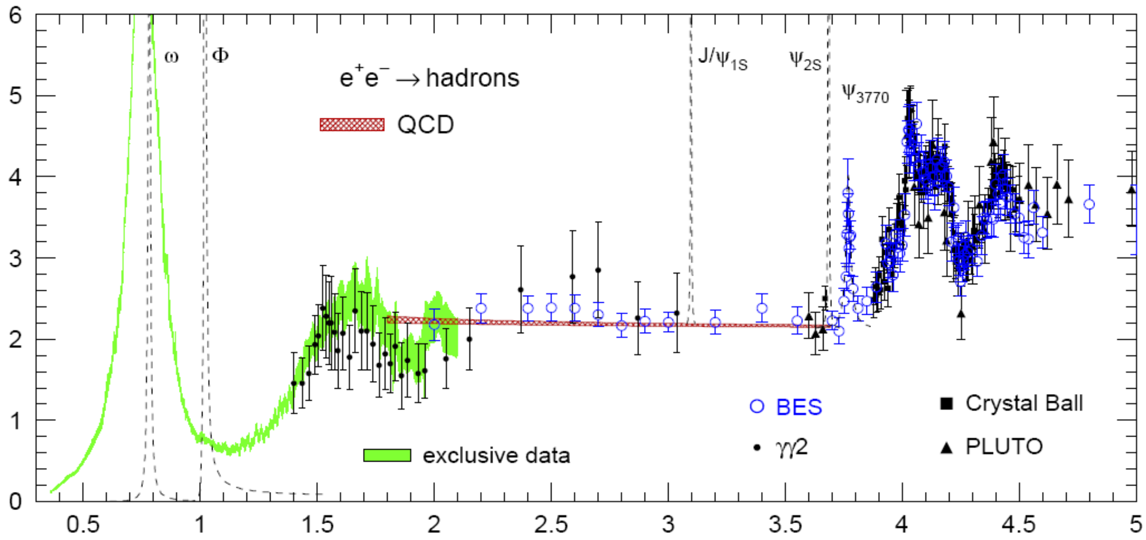
How is the LO Hadronic Contribution Calculated?

Could not predict from 1st principles but can be rigorously calculated using ee annihilation data via Dispersion Relation

$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} R(s)$$



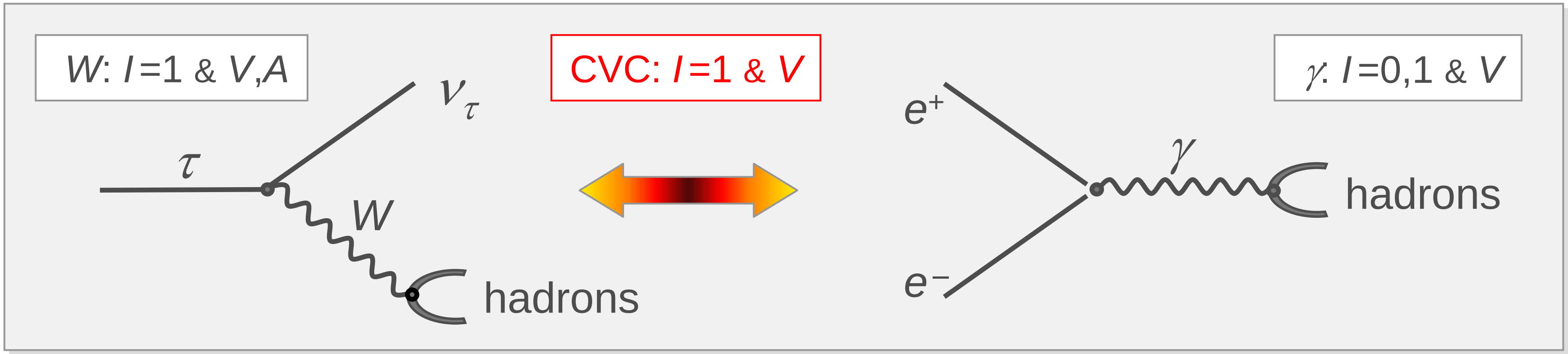
$$R(s) = \frac{\sigma_0(e^+e^- \to \text{hadrons}\,(\gamma))}{\sigma_{pt}(e^+e^- \to \mu^+\mu^-)}$$



➔ Data driven prediction

➔ Precision depends on quality of ee data

Connect τ and e^+e^- Data through CVC – SU(2)



Hadronic physics factorizes in Spectral Functions :

Isospin symmetry connects I=1 e^+e^- cross section to vector τ spectral functions:

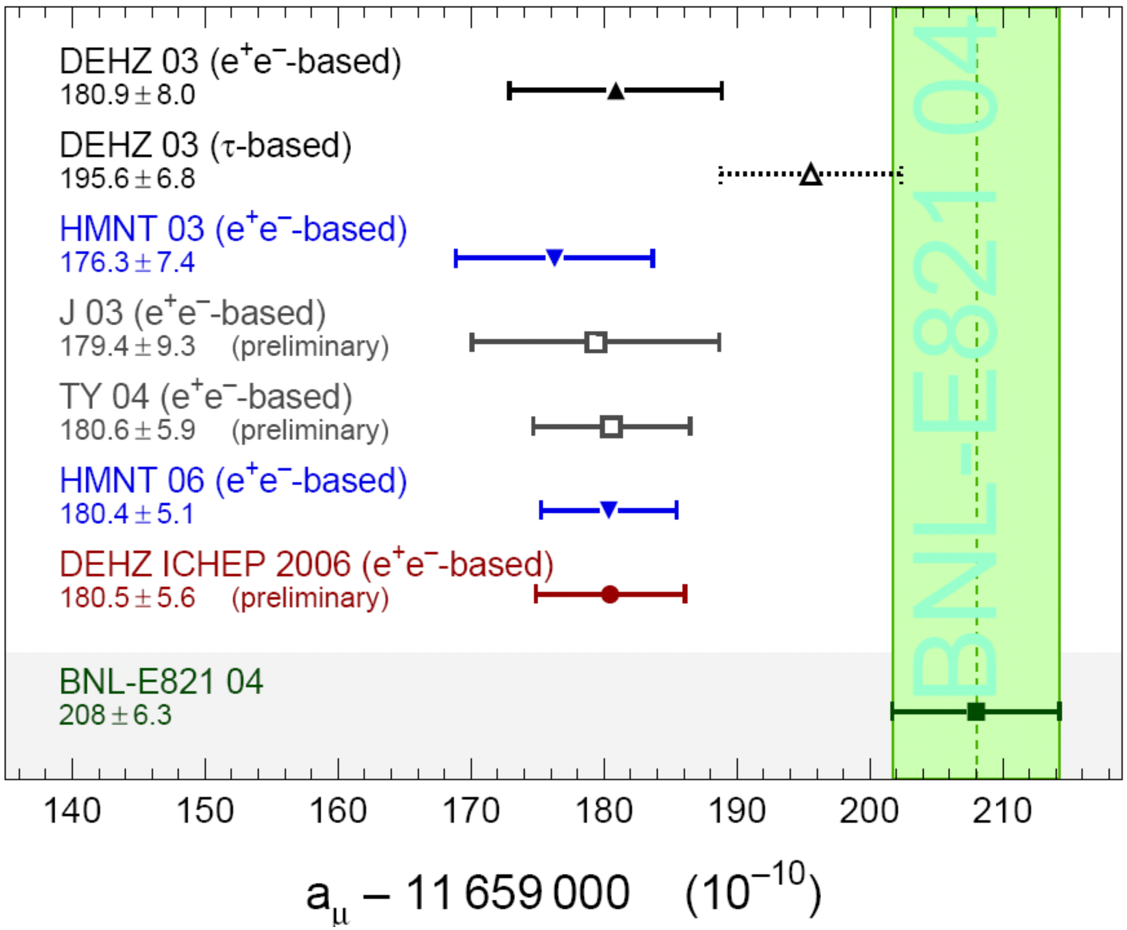
$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\,\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]}\;\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\;\frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+2s/m_\tau^2\right)}$$

branching fractions | mass spectrum | kinematic factor (PS)

Open Issues (Situation at ICHEP 2006)



- Over 3σ between ee-based prediction with the measurement
- Disagreement between ee- and τ-based predictions
  (disagreement more pronounced in SFs and B_CVC comparisons)

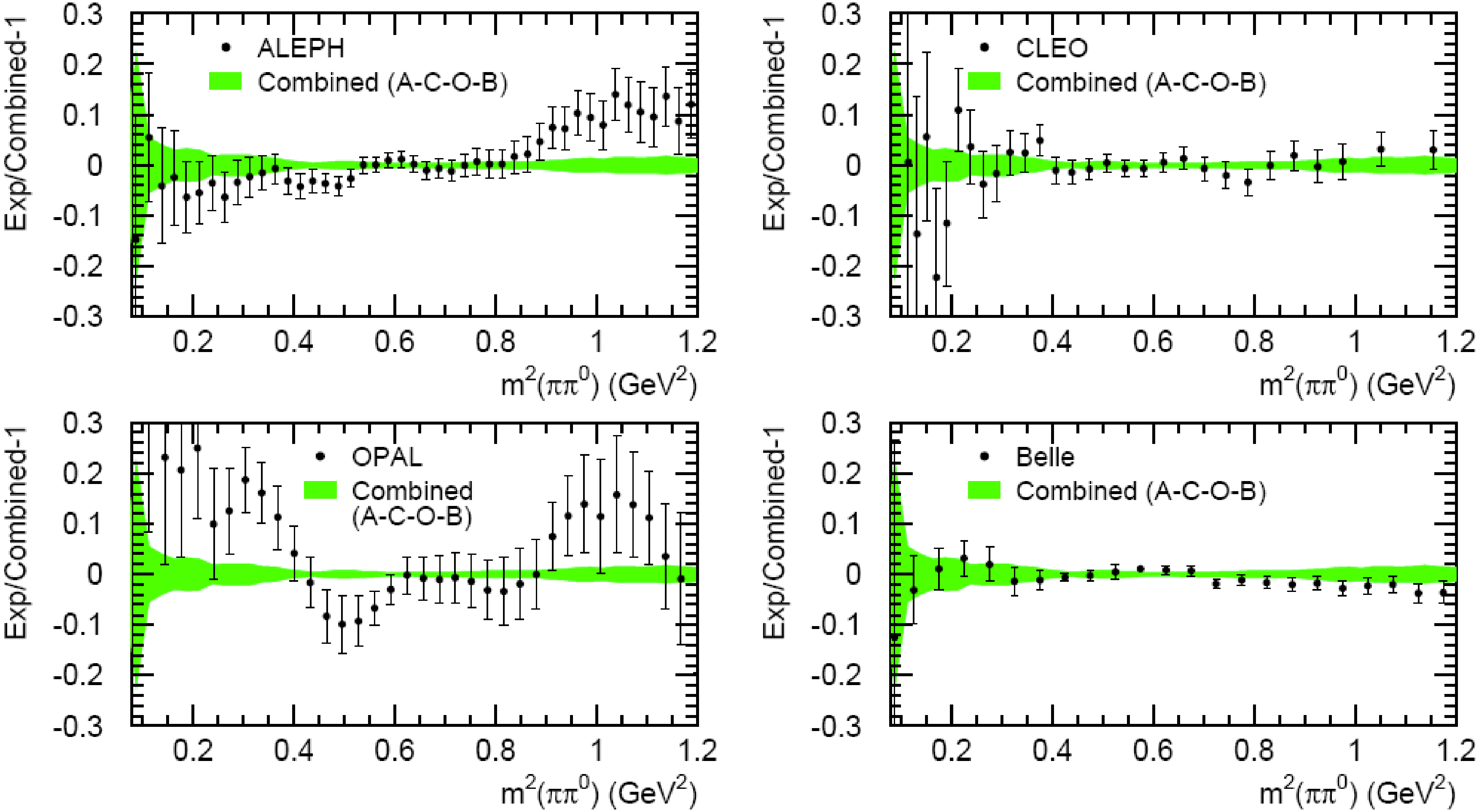
What's New?

- **New tau data from Belle** arXiv:0805.3773 [hep-ex]

  Largest sample $\tau \rightarrow h\pi^0\nu_\tau$ ($5.4\cdot10^6$ Belle $\leftarrow\rightarrow$ $81\cdot10^3$ ALEPH)

- **Isospin breaking corrections revisited**

Relative Comparison of Tau Mass Spectrum



LEP experiments at Z pole:
+ High acceptance, low non-tau background
- τ highly boosted → collimated final state

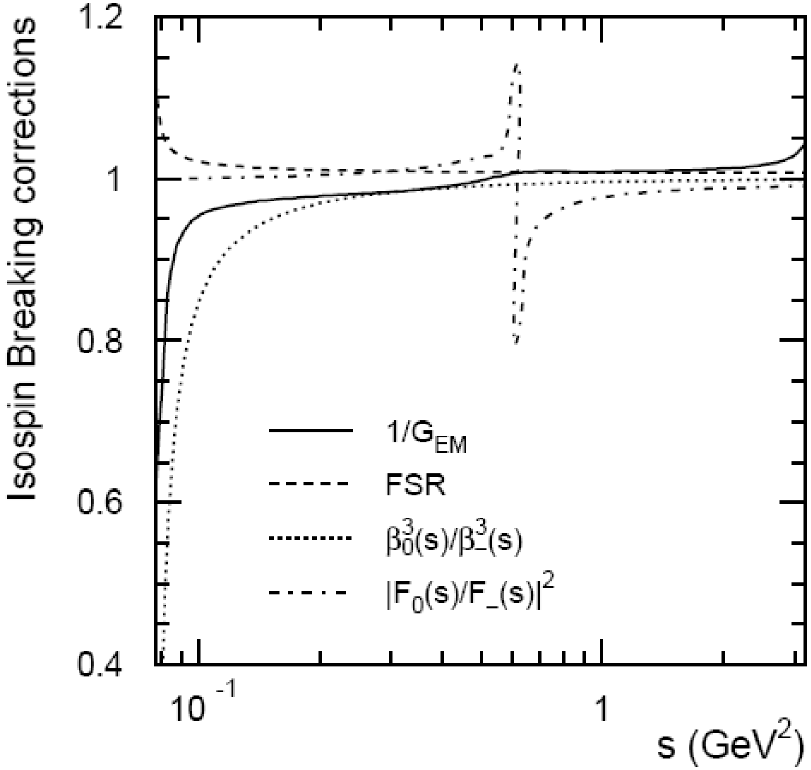
Low energy experiments at $\Upsilon(4s)$:
+ separated final state → easy π^0 recons.
- Important non-tau background

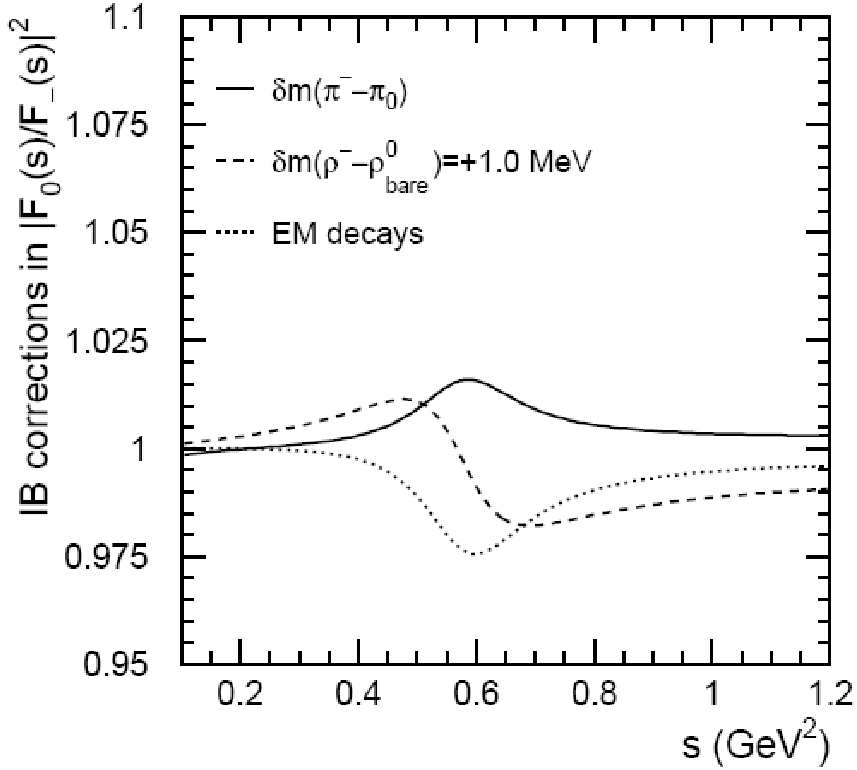
Isospin Breaking (IB) Corrections Revisited

$$\Delta^{\rm IB} a_\mu^{\rm LO,had}[\pi\pi,\tau] = \frac{\alpha^2 m_\tau^2}{6\,|V_{ud}|^2\pi^2}\,\frac{\mathcal{B}_{\pi^-\pi^0}}{\mathcal{B}_{e^-\overline{\nu}_e\nu_\tau}}\int_{4m_\pi^2}^{m_\tau^2} ds\,\frac{K(s)}{s}\,\frac{dN_X}{N_X\,ds}\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\left[\frac{R_{\rm IB}(s)}{S_{\rm EW}}-1\right]$$

$S_{EW}=1.0235\pm0.0006$

$$R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)}\,\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2$$





Isospin Breaking (IB) Corrections

Source	$\Delta a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (10^{-10}) GS model	KS model	Old (DEHZ 02)
$S_{\rm EW}$	-12.19 ± 0.15		-12.1 ± 0.3
$G_{\rm EM}$	-1.86 ± 0.88		-1.0
FSR	$+4.64 \pm 0.46$		
ρ–ω interference	$+2.40$	$+2.05$	$+3.5 \pm 0.6$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.71		-7.0
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.11$	$+3.71$	$+4.2$
$m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$	$-0.08^{+0.06}_{-0.02}$	$-0.35^{+0.31}_{-0.22}$	0 ± 2.0
$\pi\pi\gamma$, electrom. decays	-5.94 ± 0.59		-1.4 ± 1.2
Total	-16.55 ± 1.55		-13.8 ± 2.4

The old correction does not include the applied FSR correction
➔ Real change $7.4 \cdot 10^{-10}$

GS: Gounaris-Sakurai, PRL 21 (68); **KS**: Kühn-Santamaria, ZPC48 (90)

The Main Change on IB Corrections

New: $\pi\pi\gamma$, electrom. decays corrections include soft/virtual & hard photons

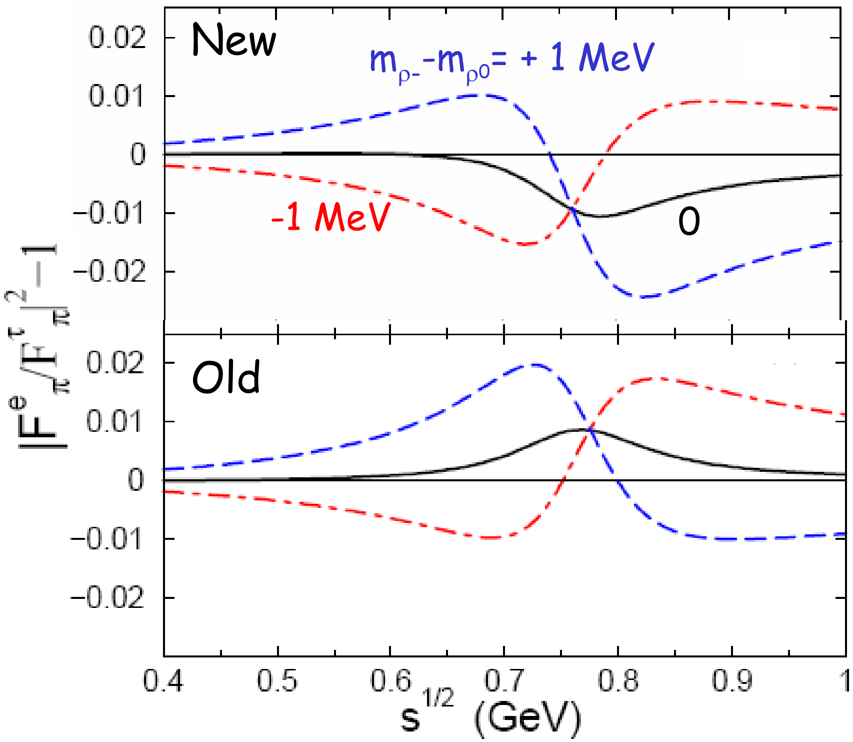
$$\Gamma_{\rho^i}(s) = \Gamma_{\rho^i\to\pi\pi(\gamma)} + \Gamma_{\rho^i\to\pi\pi\gamma}(s) = \Gamma^0_{\rho^i\to\pi\pi}(s)(1+\sigma_{\rho^i})$$

Flores-Baez, Lopez Castro, Toledo Sanchez, PRD76 (07)

with $\sigma_{\rho0}$=+8.4×10^{-3}, $\sigma_{\rho-}$=-4.0×10^{-3}

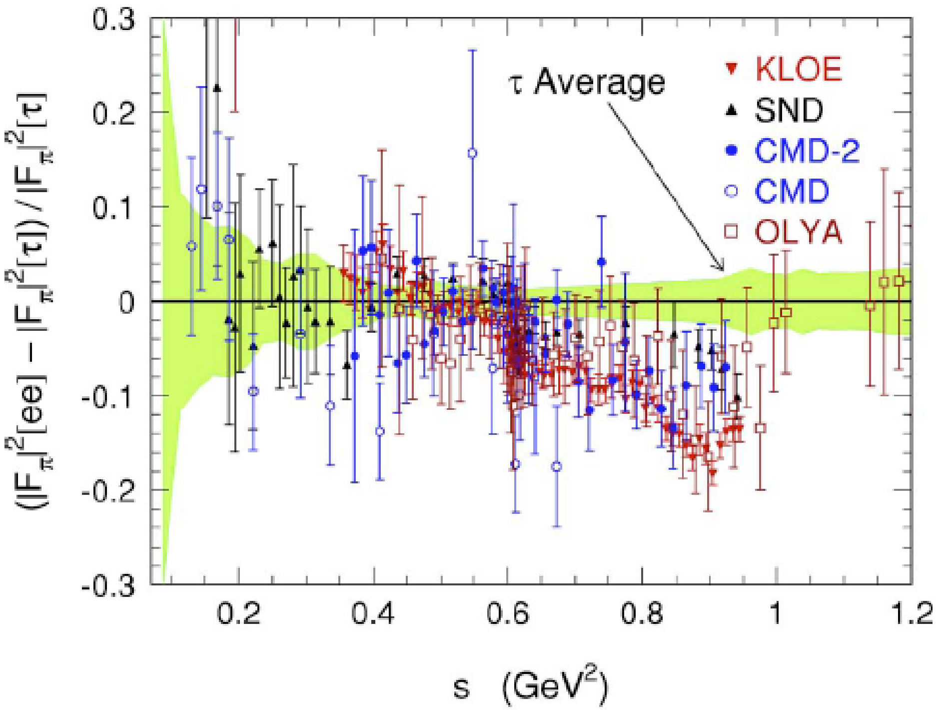
Old: only contribution with hard photons taken into account and assume:

$\Gamma_{\rho0} - \Gamma_{\rho-}$ = 0.45 ± 0.45 MeV

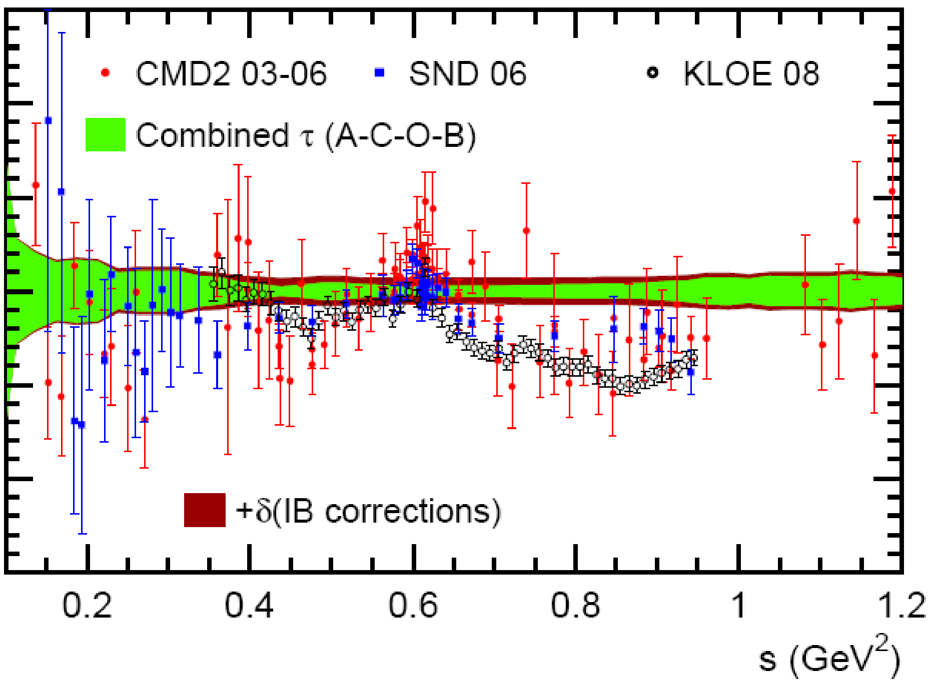


Relative Comparison of ee & IB-corrected τ Data

Old (Davier-Hoecker-Zhang 05)



New (arXiv:0906.5443)



KLOE 08 data show some discrepancy from CMD2, SND and tau

➔ Need independent ee data for cross-check and clarification (e.g. BaBar)

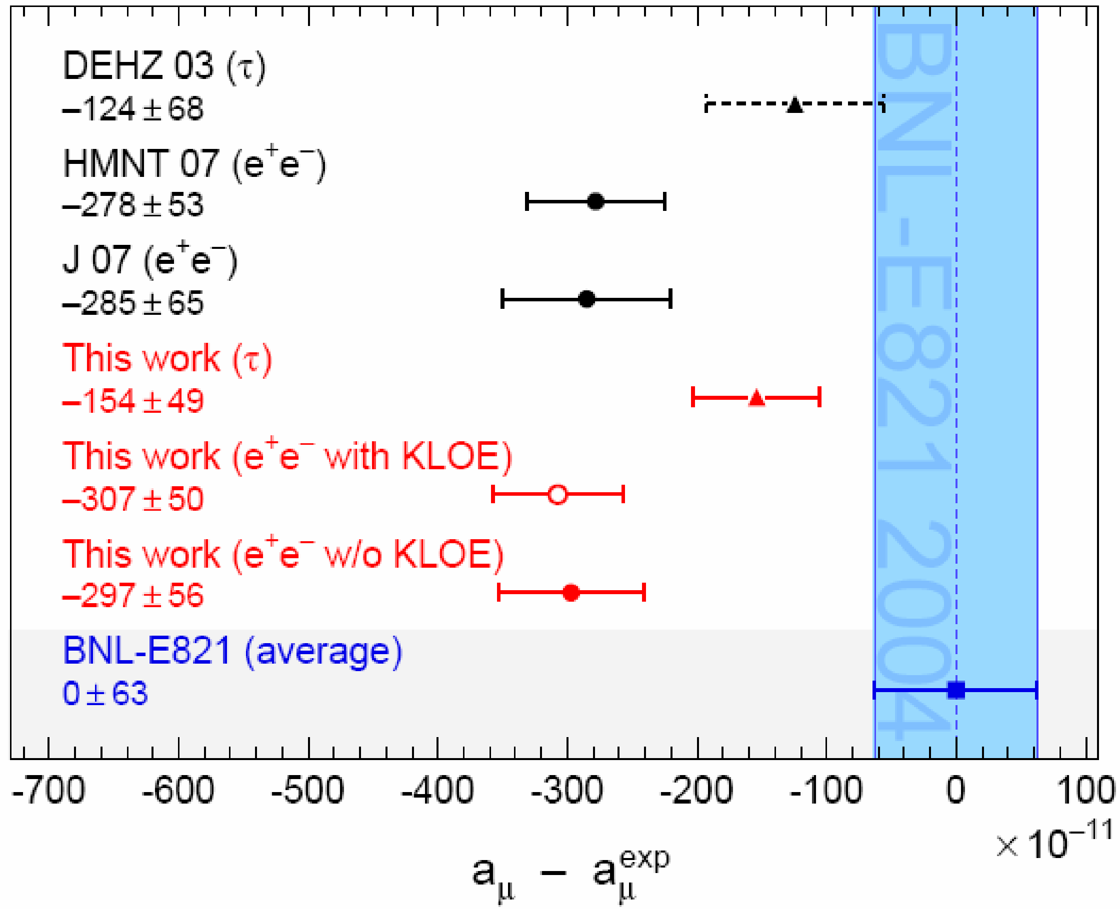
e^+e^- based $a_\mu^{had,LO}$ versus tau based $a_\mu^{had,LO}$

Modes	Energy [GeV]	e^+e^- [10^{-10}]	τ [10^{-10}]
Low s expansion	$2m_\pi$ – 0.36	$9.6 \pm 0.1 \pm 0.1_{rad}$	$9.8 \pm 0.1 \pm 0.1_{IB}$
$\pi^+\pi^-$ (+KLOE 08)	0.36 – 1.8	$494.2 \pm 3.9 \pm 0.9_{rad}$ ($493.2 \pm 3.1 \pm 0.9_{rad}$)	$504.5 \pm 2.5 \pm 1.6_{IB}$
$\pi^+\pi^-2\pi^0$	$2m_\pi$ – 1.8	$16.8 \pm 1.3 \pm 0.2_{rad}$	$21.4 \pm 1.3 \pm 0.6_{IB}$
$2\pi^+2\pi^-$ (+BaBar)	$2m_\pi$ – 1.8	$13.1 \pm 0.4 \pm 0.0_{rad}$	$12.3 \pm 1.0 \pm 0.4_{IB}$
ω (782)	0.3 – 0.81	$38.0 \pm 1.0 \pm 0.3_{rad}$	–
ϕ (1020)	1.0 – 1.055	$35.7 \pm 0.8 \pm 0.2_{rad}$	–
Other excl. (+BaBar)	$2m_\pi$ – 1.8	$24.3 \pm 1.3 \pm 0.2_{rad}$	–
J/ψ, ψ(2S)	3.08 – 3.11	$7.4 \pm 0.4 \pm 0.0_{rad}$	–
R [QCD]	1.8 – 3.7	$33.9 \pm 0.5_{theo}$	–
R [data]	3.7 – 5.0	$7.2 \pm 0.3 \pm 0.0_{rad}$	–
R [QCD]	5.0 – ∞	$9.9 \pm 0.2_{theo}$	–
Sum (+KLOE 08)	$2m_\pi$ – ∞	$690.1 \pm 4.4 \pm 1.9_{rad} \pm 0.7_{QCD}$ ($689.1 \pm 3.8 \pm 1.9_{rad} \pm 0.7_{QCD}$)	$704.4 \pm 3.5 \pm 0.7_{rad} \pm 2.7_{IB}$

a_μ Measurement versus SM Predictions

Measurement (BNL-E821) PRD73(06)072003, hep-ex/0602035	$11\,659\,208.0 \pm 5.4_{stat} \pm 3.3_{syst}$ [10^{-10}]
SM predictions:	
QED	$11\,558\,471.809 \pm 0.014_{5th\ order} \pm 0.008_{\delta\alpha}$ [10^{-10}]
HAD	
- LO	e^+e^-: $690.1 \pm 4.4 \pm 1.9_{rad} \pm 0.7_{QCD}$ [10^{-10}]
	e^+e^-(+KLOE08): $689.1 \pm 3.8 \pm 1.9_{rad} \pm 0.7_{QCD}$ [10^{-10}]
	τ: $704.4 \pm 3.5 \pm 0.7_{rad} \pm 2.7_{IB}$ [10^{-10}]
- HO	-9.8 ± 0.1 [10^{-10}]
- LBL	10.5 ± 2.6 [10^{-10}] (Prades-de Rafael-Vainshtein, 09)
Weak	15.4 ± 0.2 [10^{-10}]

a_μ Measurement versus SM Predictions



Measurement/predictions discrepancy:

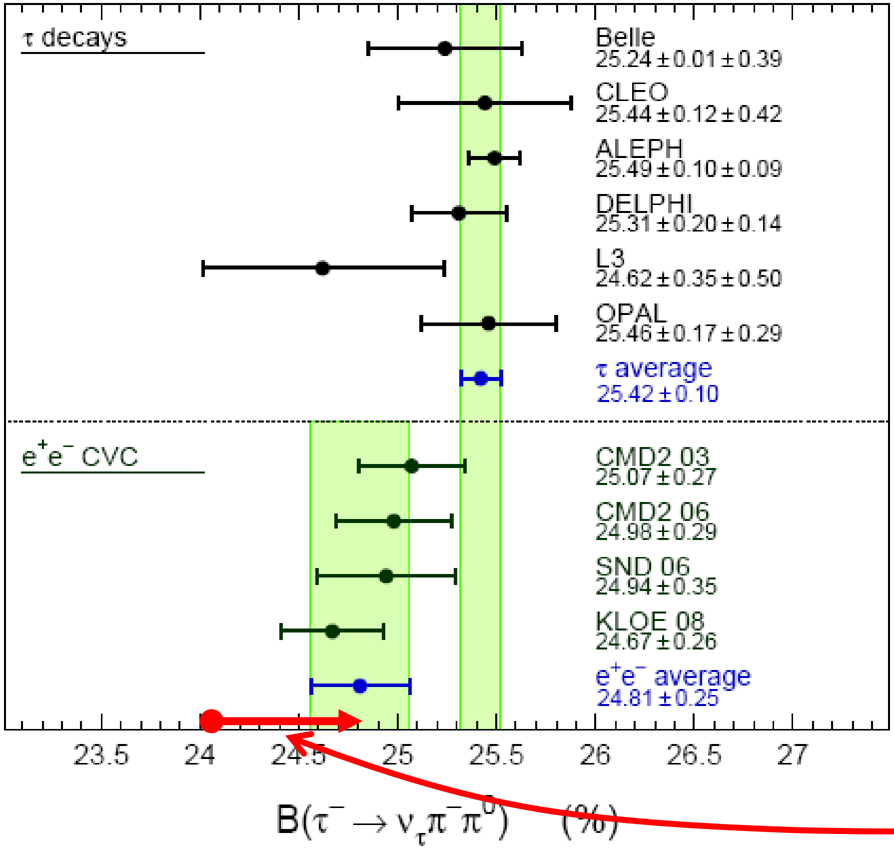
τ: 1.9σ,

e^+e^-: 3.5σ, 3.8σ (+KLOE 08)

Alternative Way of Comparing e⁺e⁻ & tau Data

tau: measured BR($\tau^- \to \pi^-\pi^0\nu_\tau$) [free from uncertainty in unfolding detector effects in SF]
e⁺e⁻: apply isospin breaking corrections and integrate data over s:

$$\mathrm{BR}_{\mathrm{CVC}}\left(\tau^- \to \pi^-\pi^0\nu_\tau\right) = \frac{6\pi\,|V_{ud}|^2\,S_{EW}}{m_\tau^2}\int_{s_{\min}}^{m_\tau^2} ds\ \mathrm{kin}(s)\cdot \upsilon^{\text{IB-corrected}}(s)$$



Source	$\Delta\mathcal{B}^{\mathrm{CVC}}_{\pi^-\pi^0}$ (10^{-2}) GS model	KS model
S_{EW}	$+0.57 \pm 0.01$	
G_{EM}	-0.07 ± 0.17	
FSR	-0.19 ± 0.02	
ρ–ω interference	0.01	0
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	$+0.19$	
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	-0.22	-0.20
$m_{\rho^\pm} - m_{\rho^0_{\mathrm{bare}}}$	$+0.10 \pm 0.09$	$+0.11 \pm 0.10$
$\pi\pi\gamma$, electrom. decays	$+0.34 \pm 0.03$	
Total	$+0.73 \pm 0.19$	

Summary and Prospects

- New IB corrections + Belle data ➔ new τ-based g-2 prediction
- Published CMD2, KLOE08 data ➔ new ee-based g-2 prediction
- The new IB corrections reduced the discrepancy
  - in the τ and ee spectral functions
  - $B_{measured}$ and B_{CVC} for 2π channel

Both predictions now deviate from the direct measurement

➔ A hint for new physics?

High precision data from BaBar (short term) and KLOE & VEPP2000 may help to resolve the remaining difference in τ and ee based predictions