Why dispersion relations help in description of pion-pion amplitudes and lead to precise determination of the $f_0(600)$ (sigma) parameters?

Robert Kamiński[a],
R. Garcia-Martin[b], J. Pelaez[b] *and*
F. Yndurain[c]

[a]Institute of Nuclear Physics Polish Academy of Sciences, Kraków, Poland
[b]Departamento de Physica Teorica II, Universidad Complutense de Madrid, Spain
[c]Departamento de Physica Teorica, Universidad Autonoma de Madrid, Spain

Outline

1. **$\pi\pi$ amplitudes from experimental data only**
   - phase shifts, inelasticities and cross sections for the $S0$ wave

2. **Dispersion relations with imposed crossing symmetry condition**
   - what do we need and what we propose?
   - historical review
   - threshold behavior of output amplitudes

3. **Example of numerical results**
   - numerical results for recent fits
   - coupling of resonances ($S0$ wave: σ, $f_0(980)$, $f_0(1400)$)

1. **Conclusions**

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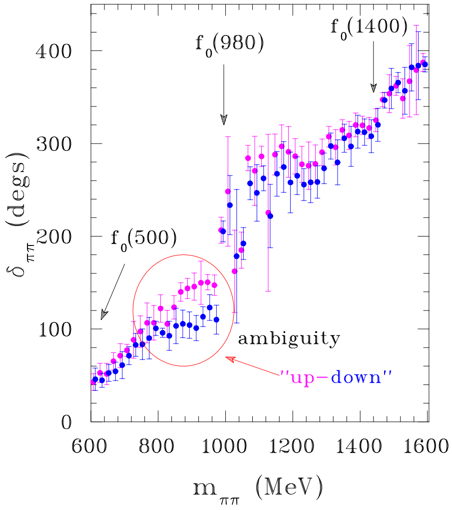
phase shifts from $\pi\pi$ thrshold to $\sim$ 1600 MeV



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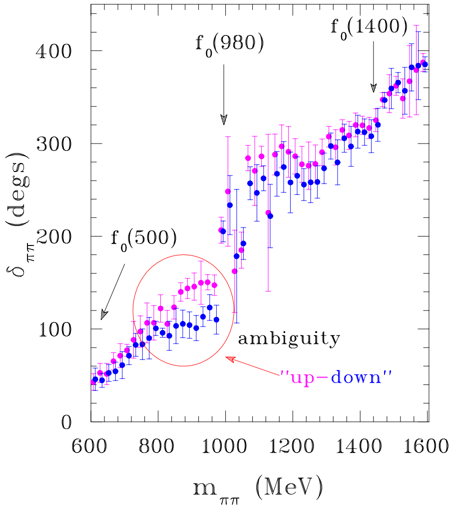

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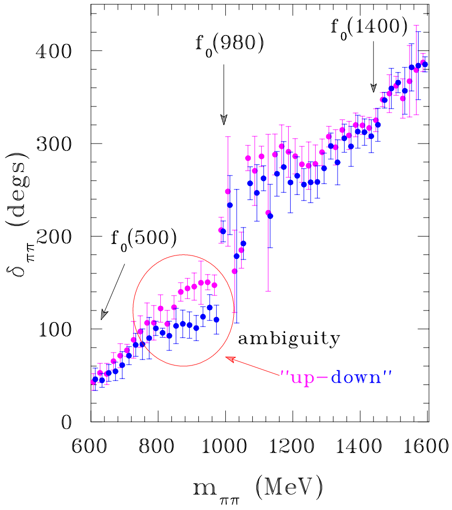

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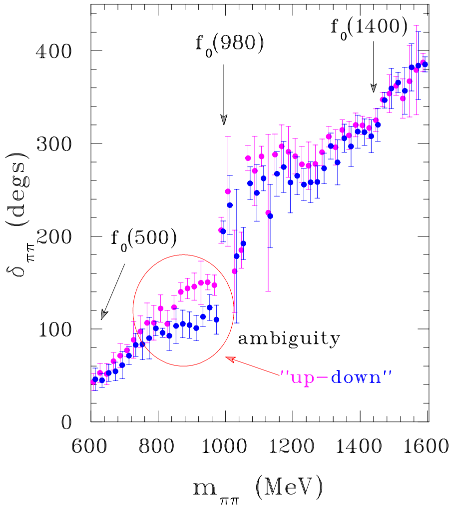

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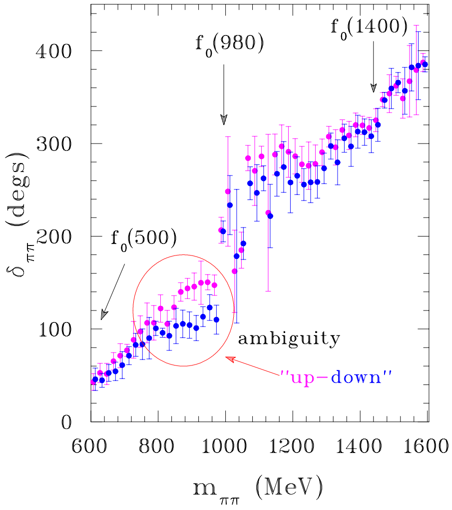

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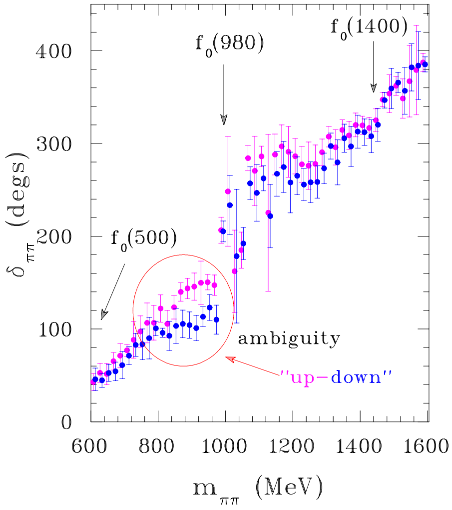

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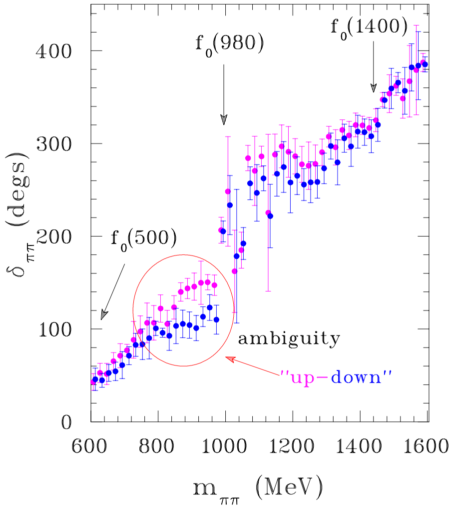

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phase shifts below 1 GeV ($S0$ wave)

The S0 wave. Different sets

he fits to different sets follow two behaviors compared with that to Kl4 data only

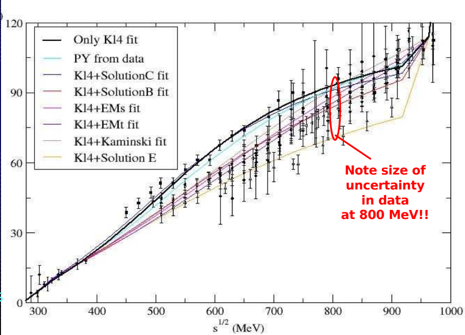
hose close to the pure Kl4 fit display a "shoulder" in the 500 to 800 MeV region

These are:
pure Kl4, SolutionC
and the global fits

Other fits do not
have the shoulder
and are separated
from pure Kl4

Kaminski et al.
lies in between
with huge errors

Solution E
deviates strongly
from the rest but has
huge error bars



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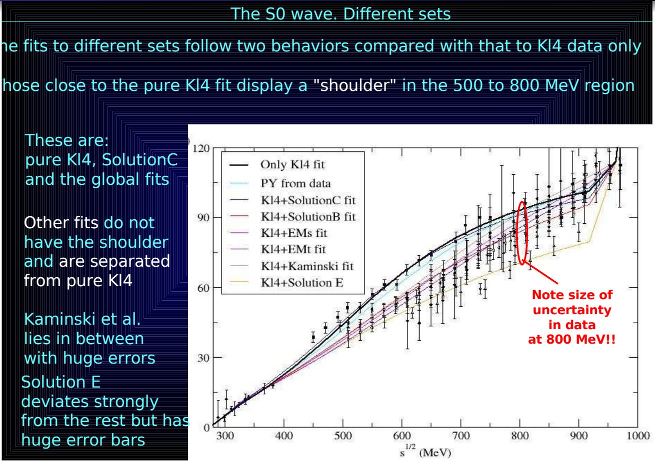

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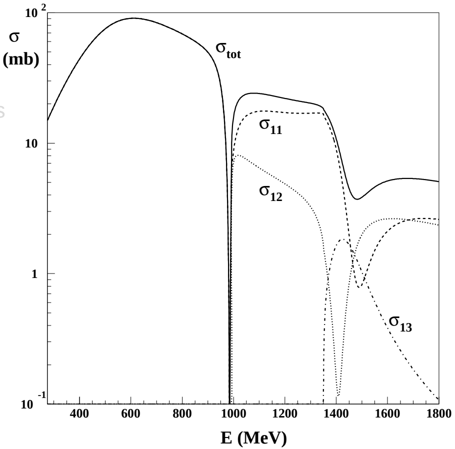
cross sections for the $S0$ wave

- σ_{11} : $\pi\pi \longrightarrow \pi\pi$
  σ_{12} : $\pi\pi \longrightarrow K\bar{K}$
  σ_{13} : $\pi\pi \longrightarrow \sigma\sigma$
- completely not intuitive behaviour of cross sections,
- Breit-Wigner approximations: Γ_σ from nonrelativistic and relativistic BW can differ by 300-400 MeV
- σ state disappeared from PDG Tables in 1976, back in 1996
- continuation of amplitudes into complex energy plane $\rightarrow$ $M = Re(s_{pole})$, $\Gamma = -2Im(s_{pole})$,



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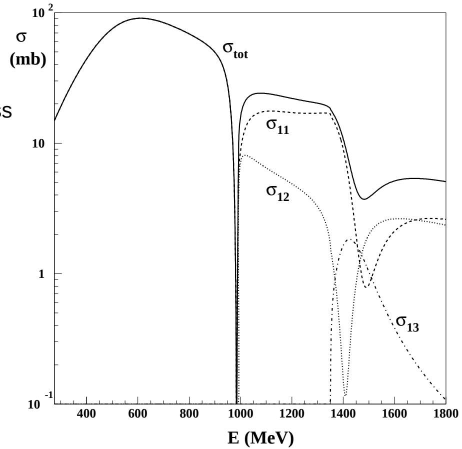

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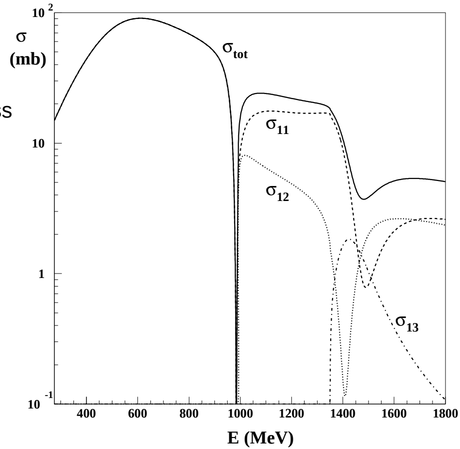

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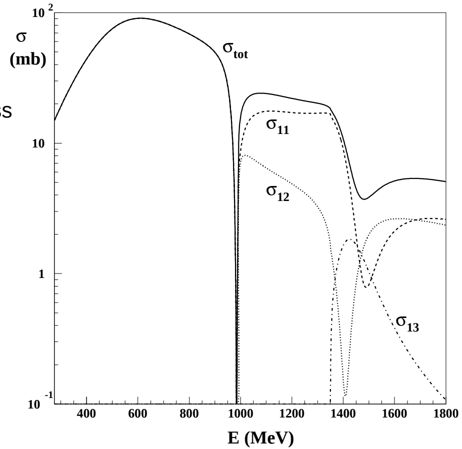

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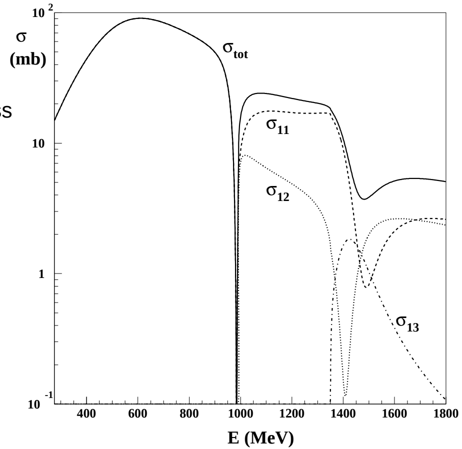

what do we need:

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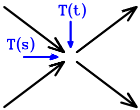

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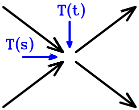

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We propose: twice subtracted dispersion relations (Roy's equations)

- $\text{Re } f_\ell^I(s) = ST(s) + KT(s) + DT(s)$ where
- "subtracting term" $ST(s) = a_0^0\delta_{I0}\delta_{\ell 0} + a_0^2\delta_{I2}\delta_{\ell 0} + \frac{s-4}{12}(2a_0^0 - 5a_0^2)(\delta_{I0}\delta_{\ell 0} + \frac{1}{6}\delta_{I1}\delta_{\ell 1} - \frac{1}{2}\delta_{I2}\delta_{\ell 0})$ with a_0^0 and a_0^2 - the $\pi\pi$ scattering lengths in the $S0$- and $S2$-wave,
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- "kernel term" $KT(s) = \sum_{I'=0}^{2} \sum_{\ell'=0}^{1} \int_4^{s_{max}} ds' K_{\ell\ell'}^{II'}(s,s') \mathrm{Im}\, f_{\ell'}^{I'}(s')$ with kernels $K_{\ell\ell'}^{II'}(s,s') \sim 1/(s-s')(s'-4)$ and
- "driving term" $DT(s) = d_\ell^I(s, s_{max}) \longrightarrow$ higher partial waves and high energy parts($s < s_{max} \approx 1.5$ GeV) of $S0$, P and $S2$ amplitudes (regge).
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threshold behavior of output amplitudes

- Threshold expansion:
  $Ref^I_\ell(s \approx 4) = (s-4)^\ell \left[a^I_\ell + b^I_\ell(s-4) + ...\right]$
- Let's compare the Roy's and GKPY equations:

Wave	Thr. exp	ST_{Roy}	$KT\&DT_{Roy}$	ST_{GKPY}	$KT\&DT_{GKPY}$
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P	0	$C_P(s-4)$	$\beta_{P1}(s-4)$	$a_0^0 - \frac{5}{2}a_0^2$	$\alpha_{P1} + \beta_{P1}(s-4)$
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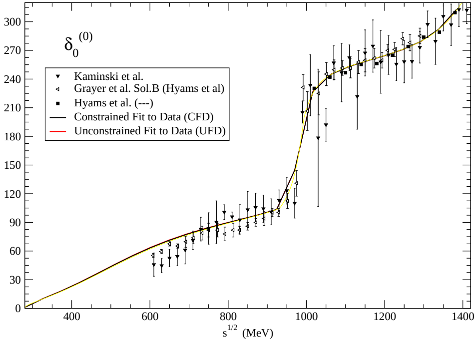

- for $m_{\pi\pi} < 932$ MeV: $cot\delta(s) = \frac{s^{1/2}}{2k} \frac{m_\pi^2}{s - \frac{1}{2}z_0^2}$ $\left[\frac{z_0^2}{m_\pi\sqrt{s}} + B_0 + B_1 w(s) + B_2 w(s)^2\right]$, $z_0 \approx m_\pi \leftarrow$ Adler zero,
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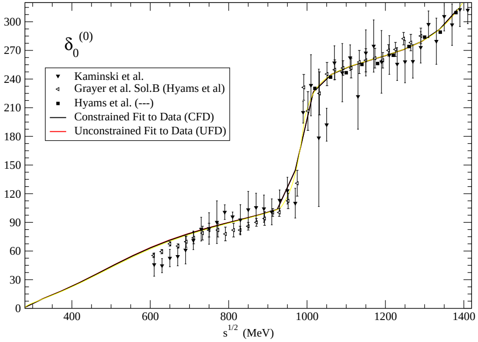

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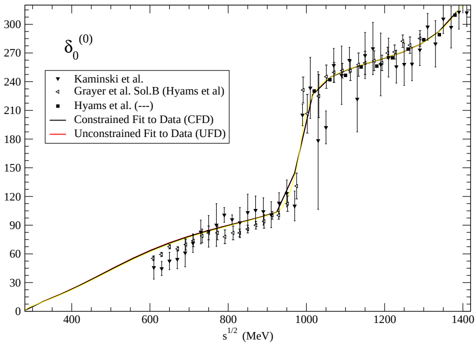

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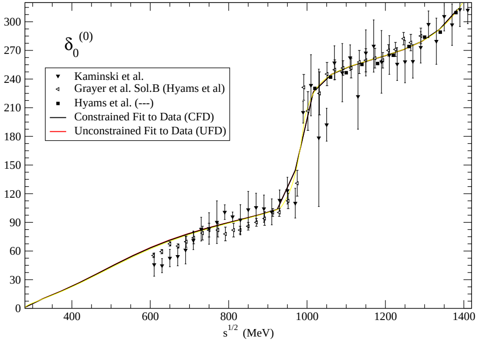

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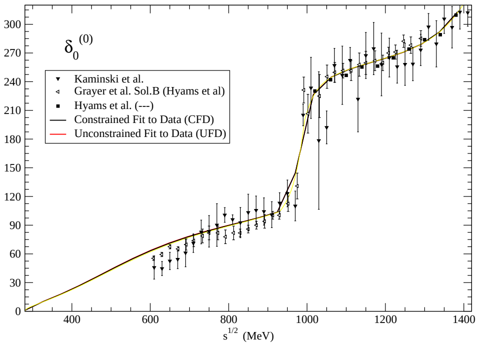

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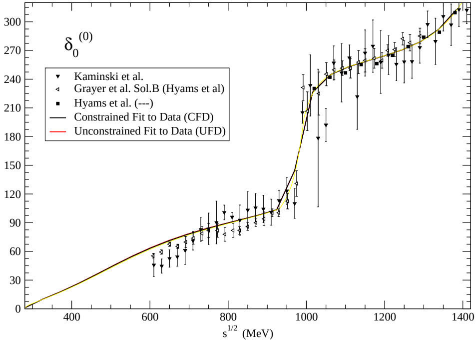

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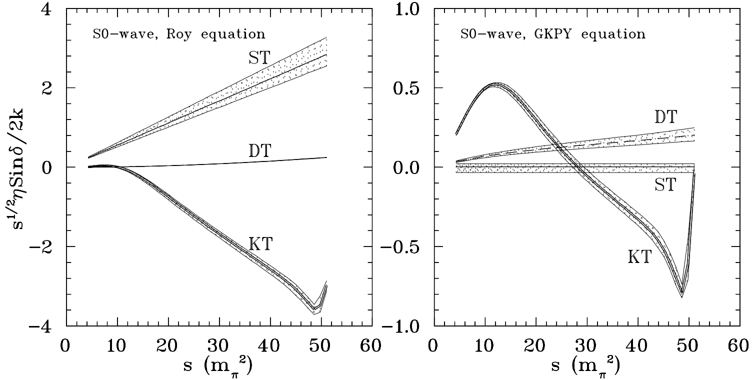
Decomposition of Roy's and GKPY eqs: $S0$-wave



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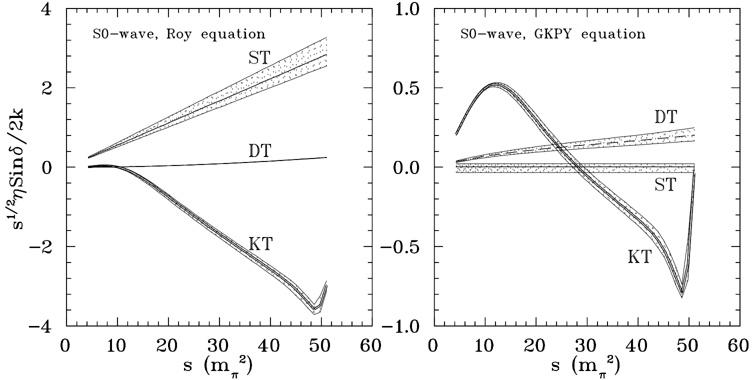

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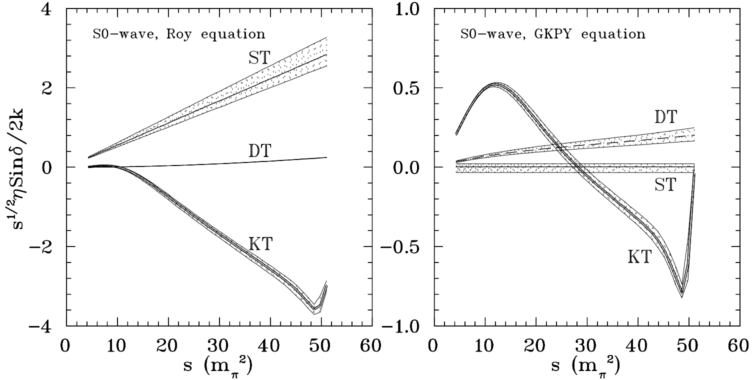

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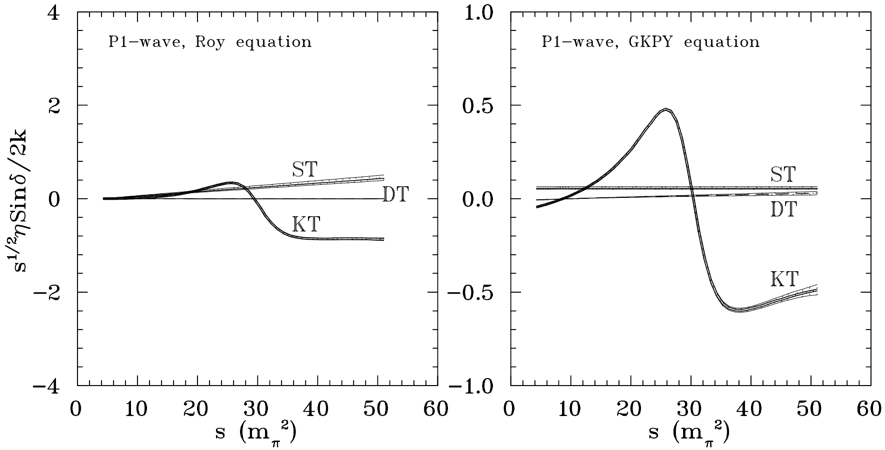

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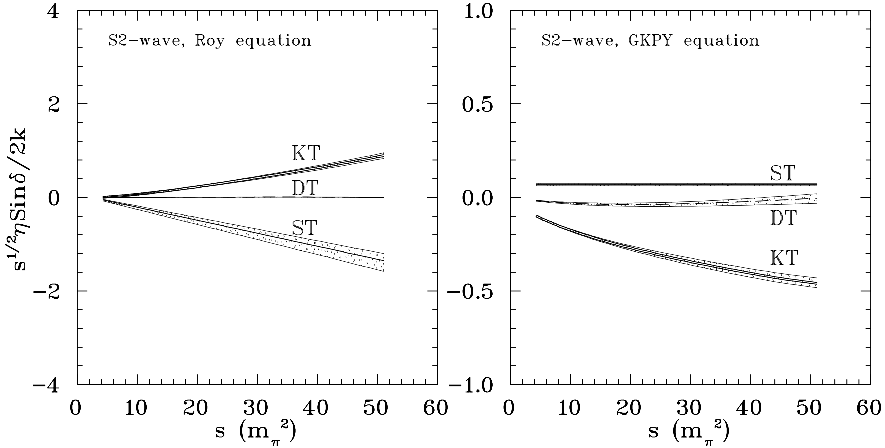
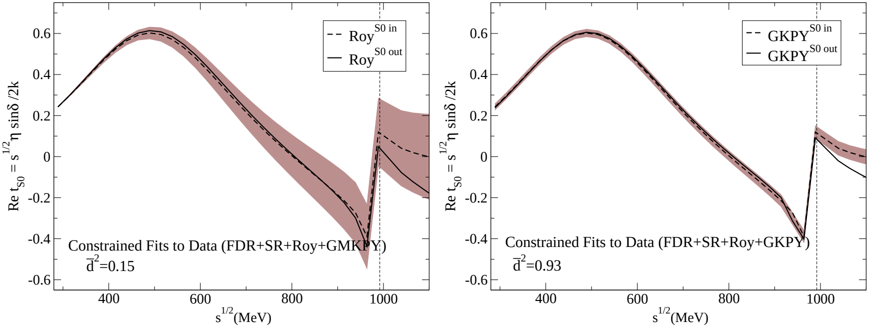


output from Roy and GKPY equations, $S0$-wave



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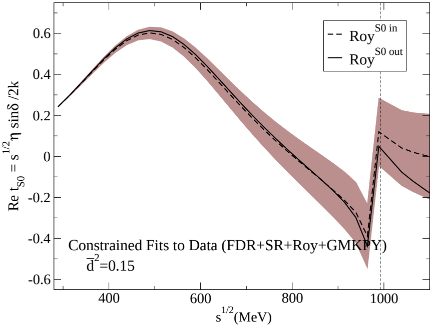

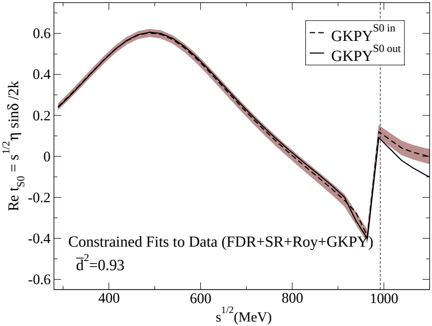

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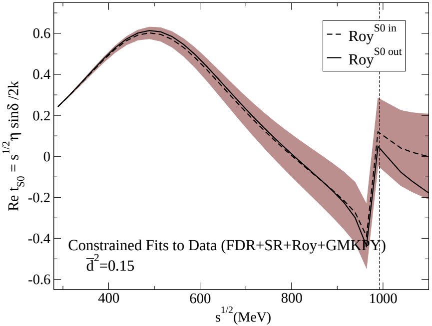

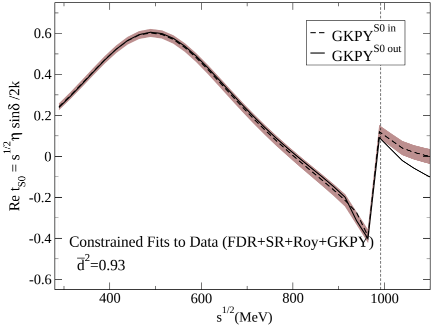

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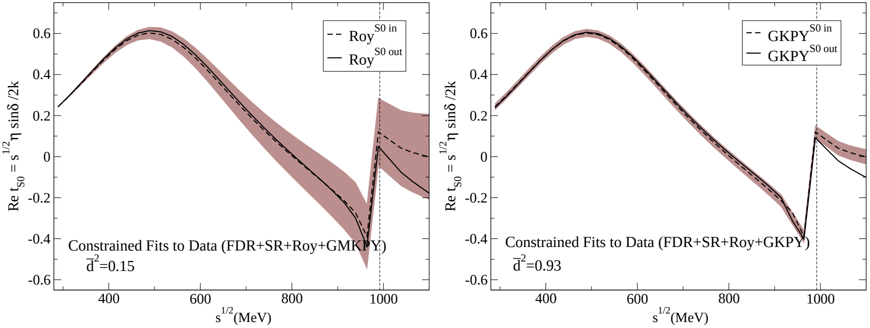

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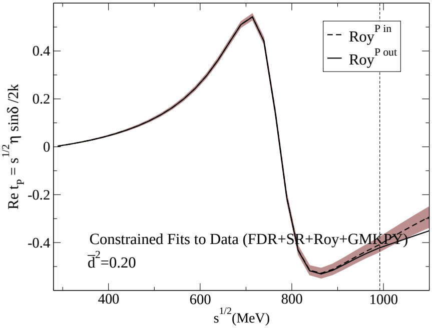

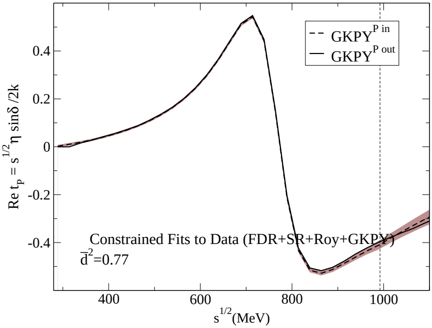

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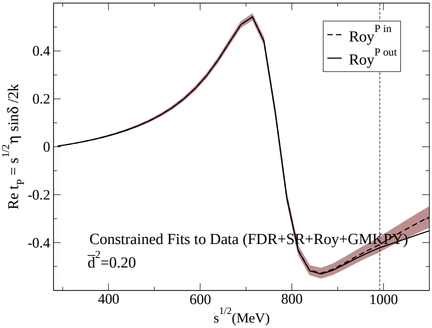
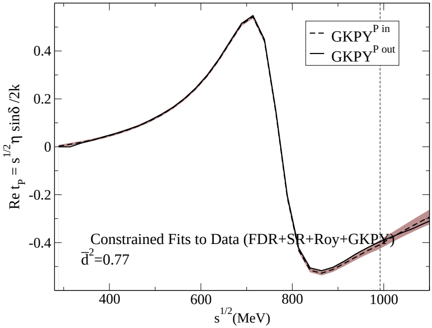
output from Roy and GKPY equations, P-wave





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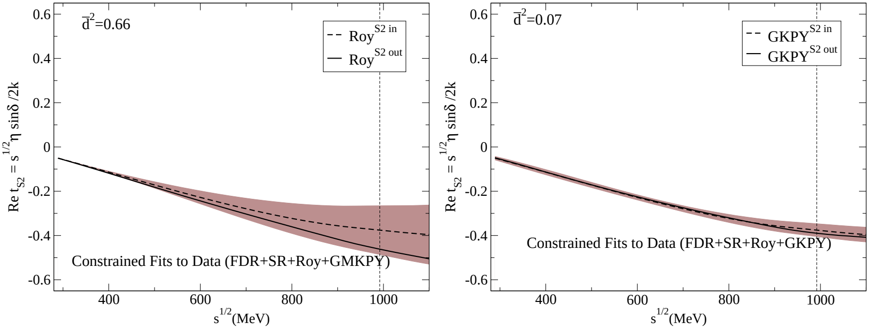
output from Roy and GKPY equations, $S2$-wave



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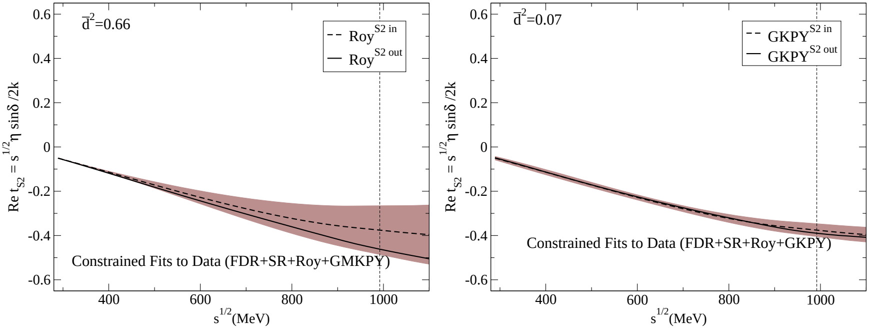

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Continuation to the complex s plane:

$Im(s_{pole})$:

- ROY: -255 ± 14 MeV
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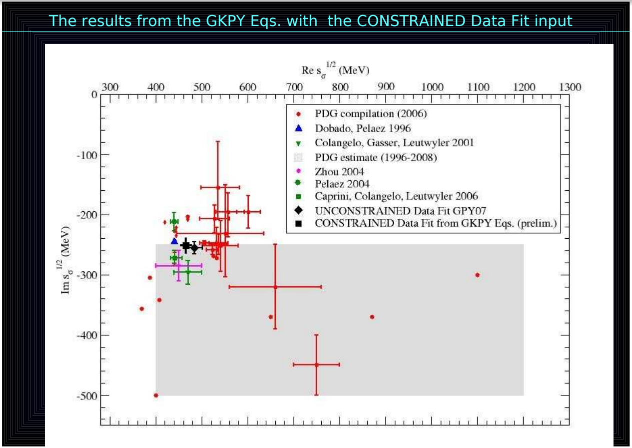

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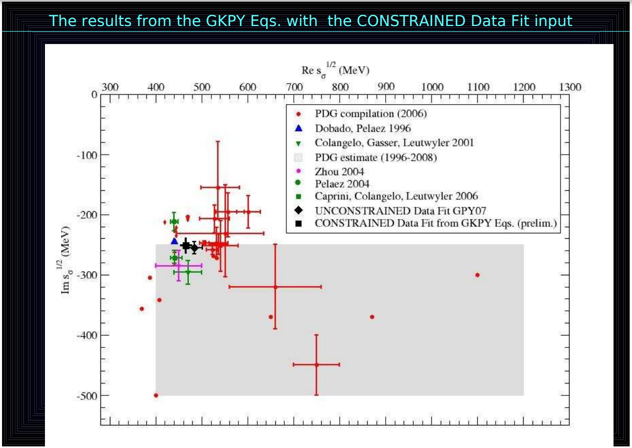

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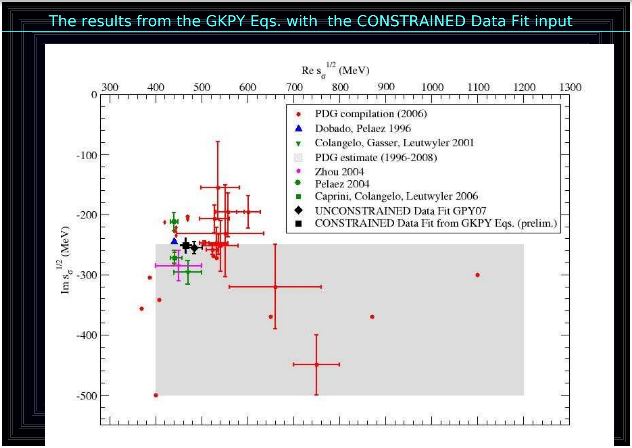

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- dispersion relation offer strong constraints for amplitudes $\rightarrow$ small errors of σ and of $a_0^0 = 0.222 \pm 0.009$, $a_0^2 = -0.045 \pm 0.008$,
- one can use them even where is no data,
- we do not use any ChPT predictions,
- only analyticity! crossing symmetry is for free,
- one can combine data from complete set of partial waves $(S - G)$,
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