$g_{B^*B\pi}$ coupling in the static heavy quark limit

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- Effective description of heavy-light systems
- Extraction of $\hat{g}$ by numerical simulations
- Outlook

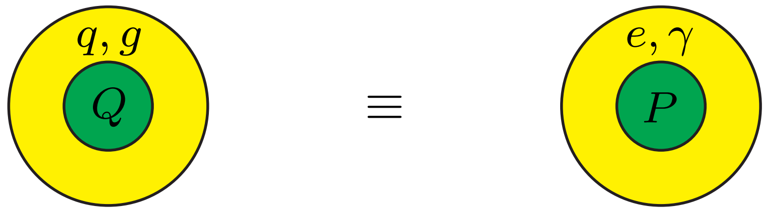
[D. Bećirević, B. B., E. Chang, B. Haas, arXiv:0905.3355]

Effective description of heavy-light systems

HQET is an effective theory whose the hard cut-off is m_b.

$$\mathcal{L}_{\rm HQET} = \bar{h}_v(iv\cdot D)h_v + \mathcal{O}(1/m_Q) \equiv \mathcal{L}^{\rm stat}_{\rm HQET} + \mathcal{O}(\Lambda_{\rm QCD}/m_Q) \quad p_Q = m_Q\, v + k$$

Heavy Quark Symmetry SU(2$\mathrm{N_h}$) for $\mathcal{L}^{\rm stat}_{\rm HQET}$: flavour $\times$ spin



Heavy-light meson   Hydrogen atom

Angular momentum: $J = \frac{1}{2} \oplus j_l$.

Spectroscopy: heavy-light mesons are put together in doublets.

$H = B, D$:

j_l^P	J^P	orbital excitation
$\frac{1}{2}^-$	0^-	H
	1^-	H^*
$\frac{1}{2}^+$	0^+	H_0^*
	1^+	H_1^*
$\frac{3}{2}^+$	1^+	H_1
	2^+	H_2^*

Computing hadronic quantities from lattice simulations at the physical point is out of reach.

One needs theoretical inputs to guide extrapolations of quantities like f_B, $F_+^{B\to\pi}$, B_B down to the chiral point because $m_q \sim 5 - 10 m_{u/d}$.

Formulae derived from HMχPT: combination of HQS and spontaneous chiral symmetry breaking.

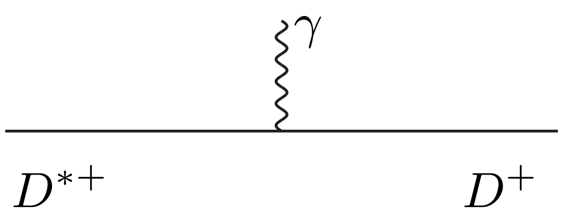
$$\mathcal{L}_{\rm heavy} = -{\rm tr}_a \mathcal{T}_r [\overline{H}_a iv \cdot D_{ba} H_b] + \hat{g}\ {\rm tr}_a \mathcal{T}_r [\overline{H}_a H_b \gamma_\mu \mathbf{A}^\mu_{ba} \gamma_5] + \cdots$$

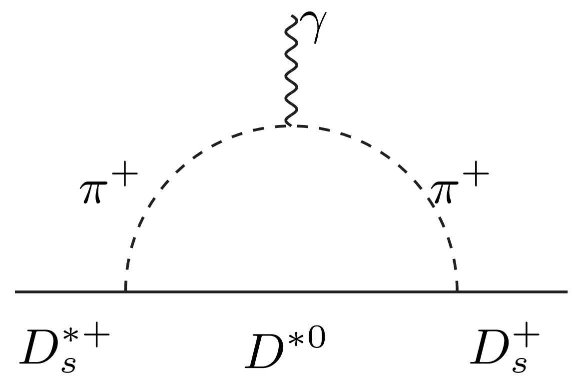
$$D^\mu_{ba} H_b = \partial^\mu H_a - H_b \frac{1}{2}[\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger]_{ba}$$

$$\mathbf{A}^{ab}_\mu = \frac{i}{2}[\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger]_{ab} \quad \xi = \exp(i\Phi/f) \quad H_a(v) = \frac{1+\not{v}}{2}\left[P^{*\ a}_\mu(v)\gamma_\mu - P^a(v)\gamma_5\right]$$

The coupling $\hat{g}$ enters in every expressions of processes involving heavy-light mesons which take into account chiral loop corrections.

Example of the D^* radiative decay with a soft photon:





$$F_u^{D^* \to D}(q^2 = 0, m_\pi^2) = \sqrt{m_D m_{D^*}} \left[\frac{2}{3}\beta \left(1 - \frac{2}{3}\frac{3+\hat{g}_c^2}{(4\pi f)^2} m_\pi^2 \log(m_\pi^2/\mu^2\right) - \frac{\hat{g}_c^2 + d_u(\mu)}{4\pi f^2} m_\pi\right]$$

It seems from an exploratory study that the mass dependence of F_u is smoother, unless assuming a huge counter-term $d_u(\mu)$ [D. Bećirević and B. Haas, '09]

Extraction of $\hat{g}$ from numerical simulations

Identifying $\hat{g}_b$ with $\hat{g}_c = \frac{g_{D^* D\pi}}{2\sqrt{m_D m_{D^*}}} f_\pi$ is dangerous in B physics phenomenology because of potentially large $\mathcal{O}(1/m_c^n)$ corrections.

$B^* \to B\pi$ is kinematically forbidden. $g_{B^* B\pi}$ delicate to estimate *via* QCD sum rules because of the use of double dispersion relations.

Our strategy: estimate of $\hat{g}$ from lattice simulations with $N_f = 2$ flavours of dynamical quarks and a static heavy quark.

$$\langle H(p)\pi(q)|H^*(p', \epsilon_\lambda)\rangle = g_{H^* H\pi}\, q\cdot\epsilon_\lambda \quad q = p' - p \quad A_\mu(x) = \bar{q}(x)\gamma_\mu\gamma^5 q(x)$$

$$\begin{aligned}\langle H(p)|A^\mu|H^*(p', \epsilon_\lambda)\rangle &= 2m_V A_0(q^2)\frac{q\cdot\epsilon_\lambda}{q^2}q^\mu + (m_H + m_H^*)A_1(q^2)\left(\epsilon_\lambda^\mu - \frac{q\cdot\epsilon_\lambda}{q^2}q^\mu\right) \\ &+ A_2(q^2)\frac{q\cdot\epsilon_\lambda}{m_H + m_H^*}\left(p^\mu + p'^\mu - \frac{m_{H^*}^2 - M_H^2}{q^2}q^\mu\right)\end{aligned}$$

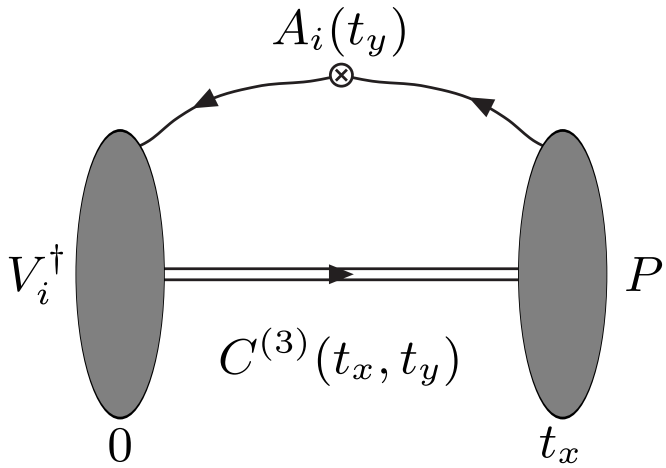
$$\langle H(p)|q_\mu A^\mu|H^*(p', \epsilon_\lambda)\rangle = g_{H^* H\pi}\frac{q\cdot\epsilon^\lambda}{m_\pi^2 - q^2} f_\pi m_\pi^2 + \cdots \quad \text{(soft pion and reduction formula)}$$

$$g_{H^* H\pi} \equiv \frac{2\sqrt{m_{H^*} m_H}}{f_\pi}\hat{g}_Q, \quad \hat{g}_Q = \hat{g} + \mathcal{O}(1/m_Q^n).$$

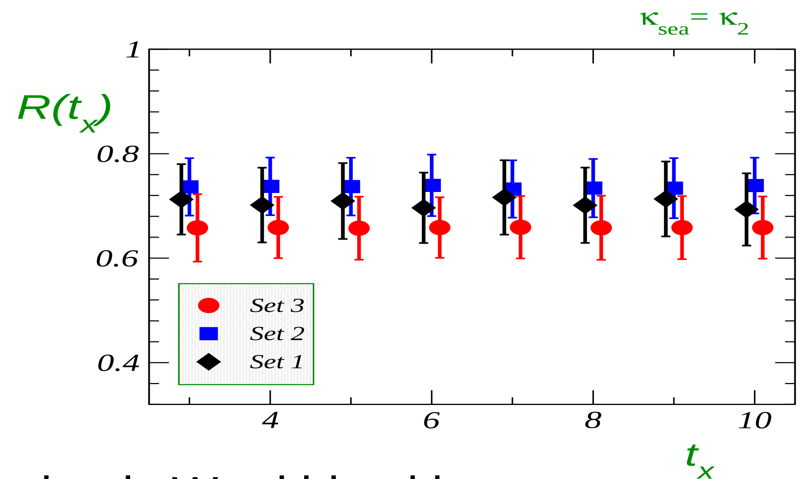
$\vec{q} = \vec{p} = \vec{p}' = 0$: $\langle H|A^i|H^*(\epsilon_\lambda)\rangle = (m_H^* + m_H)A_1(0)\epsilon_\lambda^i$

Static limit of HQET: $\hat{g} \propto \langle H|A_i|H^*\rangle$.

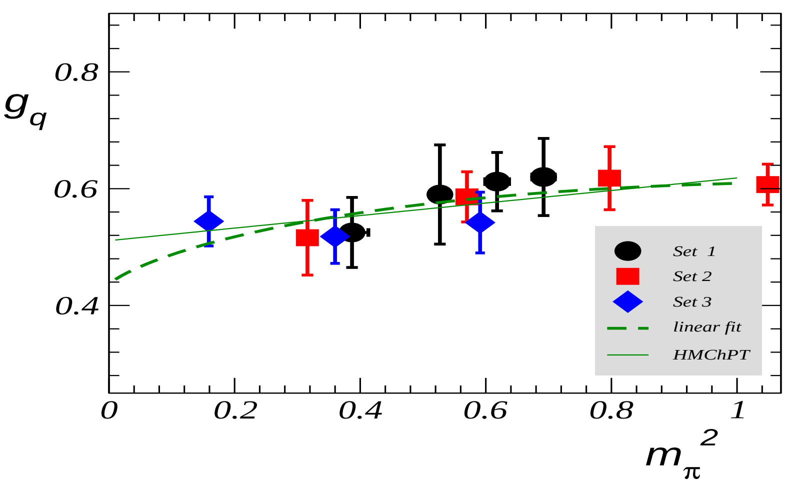
3 lattice spacings have been considered ($a \sim 0.06, 0.075, 0.1$ fm) with several sea quark masses $m_q \in [m_s/4, 1.5 m_s]$ to make chiral extrapolations. Computation performed from publically available ensembles (ILDG).



$$R(t_x) = \frac{1}{3}\frac{C^{(3)}(t_x,t_y)}{\mathcal{Z}^2 e^{-\mathcal{E}^q t_y}} \longrightarrow \hat{g}_q$$



Non perturbative renormalisation of $\vec{A}$ *via* hadronic Ward identities.



$$\hat{g}_q = \hat{g}_{\text{lin}}(1 + c_{\text{lin}} m_\pi^2)$$

$$\hat{g}_q = \hat{g}_0 \left[1 - \frac{4\hat{g}_0^2}{(4\pi f)^2} m_\pi^2 \log(m_\pi^2) + c_0 m_\pi^2\right]$$

$$\hat{g}_{\text{lin}} = 0.51 \pm 0.04 \qquad c_{\text{lin}} = (0.21 \pm 0.12)\ \text{GeV}^{-1}$$

$$\hat{g}_0 = 0.46 \pm 0.04 \qquad c_0 = (0.40 + \pm 0.12)\ \text{GeV}^{-1}$$

Discretisation effects not explicitely added in the combined fit because they are not likely to enter in competition with statistical error $\sim$ 10%.

Our result is in good agreement with a first unquenched calculation performed on coarser lattices ($a > 0.15$ fm) [H. Ohki et al, '08].

Outlook

- It is crucial to reduce theoretical uncertainties on hadronic quantities in order to fruitfully exploit coming experimental data in the flavour sector for precision tests of the Standard Model; it is particularly the case in B physics.

- Their light quark mass dependence is often described by HMχPT whose the couplings must be determined from first principle computations.

- We have extracted one of those couplings, $\hat{g}$, in the static heavy quark limit from $N_f = 2$ lattice simulations: it reveals to be quite smaller than the coupling $\hat{g}_c$ known from the experimentally measured decay rate $D^* \to D\pi$.

- There is still room for some improvement, in particular from the statistical point of view. The contribution of excited states to 3pts Green functions from which $\hat{g}$ is estimated might also be taken into account by solving a Generalised Eigenvalue Problem, as recently discussed for f_B [B. B. et al, '09].

- Some HMχPT pionic couplings have also been measured from axial charge distribution of a B meson [D. Bećirević et al, '09]: it appears that $\hat{g}$ obtained from this second approach is in very good agreement with the one presented here.