$g_{B^* B\pi}$ coupling in the static heavy quark limit

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Effective description of heavy-light systems

HQET is an effective theory whose the hard cut-off is $m_b$.

\[ \mathcal{L}_{\text{HQET}} = \bar{h}_v (i v \cdot D) h_v + \mathcal{O}(1/m_Q) \equiv \mathcal{L}_{\text{HQET}}^{\text{stat}} + \mathcal{O} (\Lambda_{\text{QCD}} / m_Q) \]

\[ p_Q = m_Q v + k \]

Heavy Quark Symmetry $\text{SU}(2N_h)$ for $\mathcal{L}_{\text{HQET}}^{\text{stat}}$: flavour × spin

Angular momentum: $J = \frac{1}{2} \oplus j_l$.

Spectroscopy: heavy-light mesons are put together in doublets.

\[
\begin{array}{|c|c|c|}
\hline
j_l^P & J^P & \text{orbital excitation} \\
\hline
\frac{1}{2}^- & 0^- & H \\
 & 1^- & H^* \\
\hline
\frac{1}{2}^+ & 0^+ & H_0^* \\
 & 1^+ & H_1^* \\
\hline
\frac{3}{2}^+ & 1^+ & H_1 \\
 & 2^+ & H_2^* \\
\hline
\end{array}
\]

Computing hadronic quantities from lattice simulations at the physical point is out of reach.

One needs theoretical inputs to guide extrapolations of quantities like $f_B$, $F_{B^+ \rightarrow \pi}$, $B_B$ down to the chiral point because $m_q \sim 5 - 10m_{u/d}$.

Formulae derived from HM$\chi$PT: combination of HQS and spontaneous chiral symmetry breaking.
\[
\mathcal{L}_{\text{heavy}} = -\text{tr}_a \text{Tr}[\overline{H}_a i \nu \cdot D_{ba} H_b] + \hat{g} \text{tr}_a \text{Tr}[\overline{H}_a H_b \gamma_\mu A_{ba}^{\mu} \gamma_5] + \cdots
\]
\[
D_{ba}^\mu H_b = \partial^\mu H_a - H_b \frac{1}{2} [\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger]_{ba}
\]
\[
A_{\mu}^{ab} = \frac{i}{2} [\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger]_{ab} \quad \xi = \exp \left( i \Phi / f \right) \quad H_a (v) = \frac{1 + \phi}{2} \left[ P_\mu^a (v) \gamma_\mu - P^a (v) \gamma_5 \right]
\]

The coupling \( \hat{g} \) enters in every expressions of processes involving heavy-light mesons which take into account chiral loop corrections.

Example of the \( D^\ast \) radiative decay with a soft photon:

\[
F_{u}^{D^\ast \rightarrow D} (q^2 = 0, m_\pi^2) = \sqrt{m_D m_{D^*}} \left[ \frac{2}{3} \beta \left( 1 - \frac{2}{3} \frac{\hat{g}_c^2}{(4\pi f)^2} m_\pi^2 \log(m_\pi^2 / \mu^2) \right) - \frac{\hat{g}_c^2 + d_u (\mu)}{4\pi f^2} m_\pi \right]
\]

It seems from an exploratory study that the mass dependence of \( F_u \) is smoother, unless assuming a huge counter-term \( d_u (\mu) \) [D. Bećirević and B. Haas, '09]
Extraction of $\hat{g}$ from numerical simulations

Identifying $\hat{g}_b$ with $\hat{g}_c = \frac{g_{D^*D\pi}}{2\sqrt{m_D m_{D^*}}} f_\pi$ is dangerous in $B$ physics phenomenology because of potentially large $O(1/m^n)$ corrections.

$B^* \rightarrow B\pi$ is kinematically forbidden. $g_{B^*B\pi}$ delicate to estimate via QCD sum rules because of the use of double dispersion relations.

Our strategy: estimate of $\hat{g}$ from lattice simulations with $N_f = 2$ flavours of dynamical quarks and a static heavy quark.

\[ \langle H(p)\pi(q)|H^*(p',\epsilon_\lambda) \rangle = g_{H^*H\pi} \ q \cdot \epsilon_\lambda \ q = p' - p \ A_\mu(x) = \bar{q}(x)\gamma_\mu\gamma^5 q(x) \]

\[
\langle H(p)|A^\mu|H^*(p',\epsilon_\lambda) \rangle = 2m_V A_0(q^2) \frac{q \cdot \epsilon_\lambda}{q^2} q^\mu + (m_H + m_H^*) A_1(q^2) \left( \epsilon_\lambda^\mu - \frac{q \cdot \epsilon_\lambda}{q^2} q^\mu \right) 
\]

\[
+ A_2(q^2) \frac{q \cdot \epsilon_\lambda}{m_H + m_H^*} \left( p^\mu + p'^\mu - \frac{m_H^2 - M_H^2}{q^2} q^\mu \right)
\]

\[
\langle H(p)|q_\mu A^\mu|H^*(p',\epsilon_\lambda) \rangle = g_{H^*H\pi} \frac{q \cdot \epsilon_\lambda}{m_H^2 - q^2} f_\pi m_\pi^2 + \cdots \quad \text{(soft pion and reduction formula)}
\]

$g_{H^*H\pi} \equiv \frac{2\sqrt{m_H^* m_H}}{f_\pi} \hat{g}_Q, \quad \hat{g}_Q = \hat{g} + O(1/m_Q^n)$.

$\vec{q} = \vec{p} = \vec{p}' = 0: \langle H|A^i|H^*(\epsilon_\lambda) \rangle = (m_H^* + m_H) A_1(0) \epsilon_\lambda^i$

Static limit of HQET: $\hat{g} \propto \langle H|A_i|H^* \rangle$. 

3 lattice spacings have been considered \( (a \sim 0.06, 0.075, 0.1 \text{ fm}) \) with several sea quark masses \( m_q \in [m_s/4, 1.5m_s] \) to make chiral extrapolations. Computation performed from publically available ensembles (ILDG).

\[
R(t_x) = \frac{1}{3} \frac{C^{(3)}(t_x, t_y)}{Z^2 e^{-E_{t_y}}} \rightarrow \hat{g}_q
\]

Non perturbative renormalisation of \( \vec{A} \) via hadronic Ward identities.

\[
\hat{g}_q = \hat{g}_{\text{lin}} (1 + c_{\text{lin}} m_{\pi}^2)
\]

\[
\hat{g}_q = \hat{g}_0 \left[ 1 - \frac{4\hat{g}_0^2}{(4\pi f)^2} m_{\pi}^2 \log(m_{\pi}^2) + c_0 m_{\pi}^2 \right]
\]

\( \hat{g}_{\text{lin}} = 0.51 \pm 0.04 \)

\( c_{\text{lin}} = (0.21 \pm 0.12) \text{ GeV}^{-1} \)

\( \hat{g}_0 = 0.46 \pm 0.04 \)

\( c_0 = (0.40 \pm 0.12) \text{ GeV}^{-1} \)

Discretisation effects not explicitly added in the combined fit because they are not likely to enter in competition with statistical error \( \sim 10\% \).

Our result is in good agreement with a first unquenched calculation performed on coarser lattices \( (a > 0.15 \text{ fm}) \) [H. Ohki et al, '08].
Outlook

• It is crucial to reduce theoretical uncertainties on hadronic quantities in order to fruitfully exploit coming experimental data in the flavour sector for precision tests of the Standard Model; it is particularly the case in $B$ physics.

• Their light quark mass dependence is often described by HM$\chi$PT whose the couplings must be determined from first principle computations.

• We have extracted one of those couplings, $\hat{g}$, in the static heavy quark limit from $N_f = 2$ lattice simulations: it reveals to be quite smaller than the coupling $\hat{g}_c$ known from the experimentally measured decay rate $D^* \rightarrow D\pi$.

• There is still room for some improvement, in particular from the statistical point of view. The contribution of excited states to 3pts Green functions from which $\hat{g}$ is estimated might also be taken into account by solving a Generalised Eigenvalue Problem, as recently discussed for $f_B$ [B. B. et al, '09].

• Some HM$\chi$PT pionic couplings have also been measured from axial charge distribution of a $B$ meson [D. Bećirević et al, '09]: it appears that $\hat{g}$ obtained from this second approach is in very good agreement with the one presented here.